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The Stockholm "Förmedlings" Bureau.

The Telephone Commissions Office.

By *A. Lignell.*

As the use of the telephone has gradually extended from large houses and offices to the very smallest and is now found in almost every residence, efforts have been made to enlarge its scope, to make it a servant of the public not only for direct intercommunication but also for enabling the subscriber to benefit from his subscription in other ways also, e. g. during temporary absence.

For this purpose the Telephone Commissions Office, which for a small fee will accept various commissions from the subscribers, have been established in the larger towns of Sweden for many years.

Their usefulness to clients is best illustrated by a few examples.

A doctor is going away for a month's holiday. He informs the Commissions Office of the name, address, consulting hours, and telephone number of his locum tenens, his own postal address and perhaps telephone number, and the date of his return.

When the doctor's number is called, the Office will reply, and give the necessary information to the caller.

A midwife has her telephone connected to the Office, to which she gives full details of where she can be found at various times of the day. Should she go to a cinema, for instance, she would inform the Office of its name and the row and number of her seat.

When a business man with no office staff goes out for lunch, the Bureau can say when he expects to return and, if desired, inform him of the name and telephone number of any person ringing him up in his absence.

A subscriber leaves a message for a certain person, who will inquire for it by telephone. To avoid any unauthorized person receiving the message, a code word may be agreed on between the person who leaves the message and the recipient, without which it will not be given.

Certain local or trunk calls may be re-directed for any desired period to another number, and when his office is closed, for instance, a subscriber can have incoming calls distributed to various residential numbers.

Waking by telephone is also extensively used, both as a standing order and on odd occasions.

Some people get woken up every weekday—perhaps at different times each day according to their own time-table. Even a person who wishes to have an occasional nap can let the Office see that he does not oversleep.

If your watch has stopped or you wish to put it right, the Office will furnish the correct time.

Information is also supplied regarding what doctors are on duty at night or on Sundays and holidays in the various districts of the town.

As we see, the Commissions Office can be exceedingly useful to subscribers, and the most varied professions are increasingly availing themselves of its services.

Commissions.

The Office supervisor accepts orders for and cancellation of commissions. When ringing, the subscriber asks for "Förmedlingsbyrån" by name, and can then give his instructions by telephone. If a commission is for more than 7 days at a stretch, a written confirmation is required, experience having proved this necessary.

When receiving a commission, the supervisor notes the necessary information on an order card, which is put in a pocket in the switchboard, immediately above the subscriber's connecting jacks, easily accessible to the operator. The card is illustrated in fig. 1; on the back, all executed commissions are recorded.

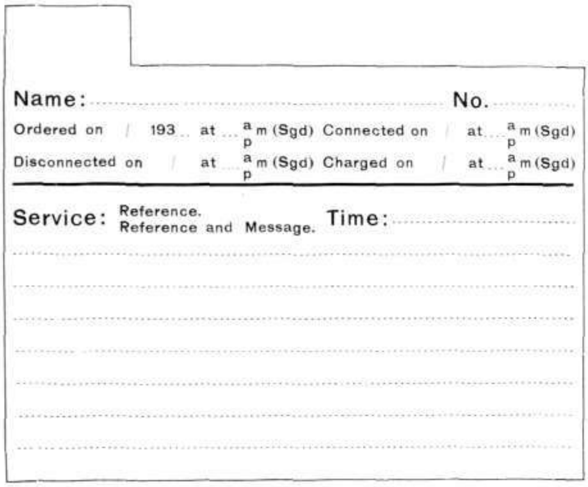

Name: .... No. ....
Ordered on / 193.. at ... a/p m (Sgd) Connected on / at ... a/p m (Sgd)
Disconnected on / at ... a/p m (Sgd) Charged on / at ... a/p m (Sgd)
Service: Reference. Reference and Message. Time: ....

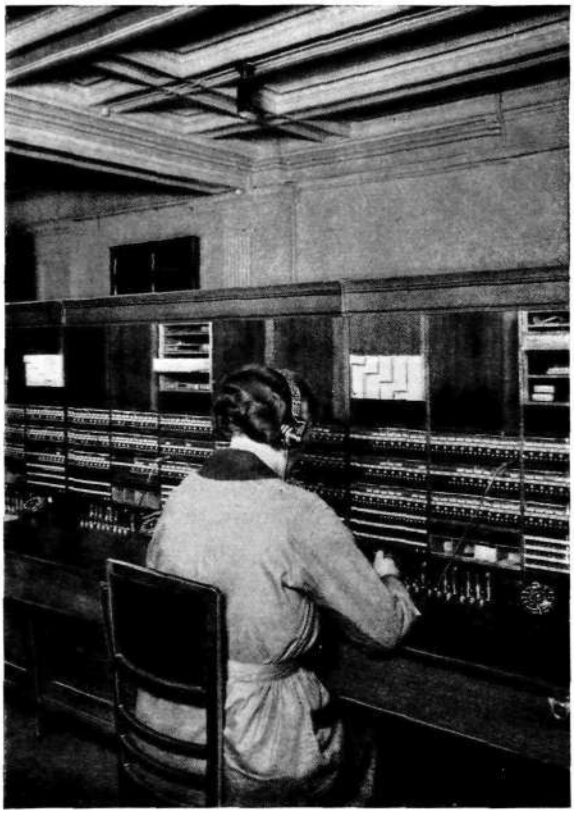
Fig. 1.

Connexion of a Subscriber to the Bureau.

Having noted the subscriber's number and order on an order card, the supervisor instructs the exchange concerned to put his line through to the Bureau. Normally, this line is connected direct to the internal wire through the cutting-in switch, but in this case that connexion will be broken at the cutting-in switch, and each section of the subscriber's line will be connected to an extra calling-device in the "Commissions Office".

Operator's Positions. (See figs. 2 and 3.)

Each subscriber's line has three jacks in the Office multiple field: one for the exchange line, one for the subscriber's line and one for connecting these two lines by a plug when required.


R 4048 Fig. 3.

The first two jacks have each a call lamp and the last one a supervisory lamp, which can be connected or disconnected as desired.

The multiple field is also provided with trunk-line jacks and order wire speaking-keys for the various exchanges, as well as jacks and call lamps for record operator's circuits.

The boards have 3 cord-groups per operator, each consisting of 3 cords, one for replying to calls from the exchange, one for replying to calls from the subscriber, and one for


R 4077 Fig. 2.

The Stockholm Telephone Commissions Office.

connexions to the various exchanges. For each cord there is a combined listening and speaking key, and each operator's position has also 3 ringing keys, one for each kind of cord.

The Office, open day and night, will undertake the following services.

Reference Service.

The subscriber's line is connected to the Bureau, which will request calling parties to ring another number, or inform them of the subscriber's absence, his address while away, time of return, etc. all without making any note of the calling party's number or business, for a fee of,

per day or part of a day .... kr. 1: —
for 10 *consecutive* days or more,
up to a month .... ,, 10: —

Reference and Message Service.

The subscriber's line is connected to the Bureau, and short messages are given or received on his behalf, for a fee of,

per day or part of a day .... kr. 1: 50
for 15 *consecutive* days or more,
up to a month .... ,, 20: —

The Office keeps any notes intended for the subscriber *until asked for,* or forwards them by post at his expense, but does *not undertake to ring him up specially to give him the messages.*

Message Service.

Short messages (not exceeding 20 words in length) are communicated by telephone from a subscriber to one or more stated subscribers.

The Office will either keep the message until it is asked for, or ring a specified number to deliver the message, either immediately or after a stated time. If there is no reply from the addressee number when first called, the Bureau will ring up again at 30-minute intervals for two hours. After that time no further call is made.

For a message kept until called for, a fee is charged irrespective of the number of inquiries, of .... kr. 0: 50
and for messages to be telephoned, a fee for each addressee of .... kr. 0: 50

Calling Service.

The Office can be made use of to connect the subscriber's instrument with another specified number (a person cannot be inquired for by name) either immediately or at a certain definite time, for a fee each time of .... kr. 0: 10

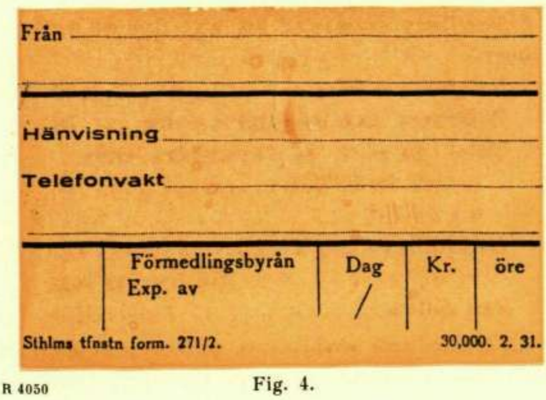
If there is no reply from the number called, the Office will on special request undertake to repeat the call, though not more than four times, either at certain specified times or at 30-minute intervals for the succeeding two hours. An extra fee is charged for this of .... kr. 0: 20

Waking.

The Bureau will wake a subscriber by calling his number at a certain time, for a fee each time of .... kr. 0: 20
(That the number given is correct is checked on receipt of the order.)

Time-giving.

The official time will be given, for a fee each time of .... kr. 0: 10
(That the number given is correct is checked on receipt of the order.)

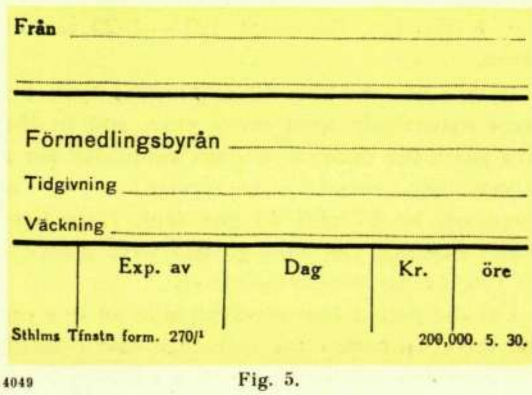

Från

Hänvisning
Telefonvakt

	Förmedlingsbyrån Exp. av	Dag	Kr.	öre
		/		

Sthlms tfnstn form. 271/2. 30,000. 2. 31.

R 4050 Fig. 4.


Från

Förmedlingsbyrån
Tidgivning
Väckning

	Exp. av	Dag	Kr.	öre

Sthlms Tfnstn form. 270/1 200,000. 5. 30.

R 4049 Fig. 5.

The Stockholm Telephone Commissions Office.

The Telephone Commissions Bureau 1930.

Month	Reference Service			Reference and Message service			Message Service		Waking		Time giving		Total		
	days con-nected	debit cards	fees paid	days con-nected	debit cards	fees paid	debit cards	fees paid	debit cards	fees paid	debit cards	fees paid	days connec-ted	debit cards	fees paid
Jan.	5 781	1 060	2 787: —	5 755	825	4 417: 90	8	3: 60	7 198	1 439: 60	7 369	736: 90	11 536	16 460	9 385: —
Feb.	4 444	897	2 074: 90	5 114	814	5 170: 10	8	3: 70	5 675	1 135: —	7 095	709: 50	9 588	14 489	9 093: 20
March ..	4 610	938	2 113: 10	5 784	918	4 513: 50	19	8: —	5 859	1 171: 80	7 451	745: 10	10 394	15 245	8 551: 50
April ..	5 435	1 220	2 674: —	6 150	1 051	4 953: 60	23	9: 60	5 159	1 031: 80	6 632	663: 20	11 585	14 085	9.332: 20
May	5 832	1 241	2 622: 20	6 505	1 105	5 050: 20	17	7: 90	6 459	1 291: 80	6 941	694: 10	12 341	15 763	9 666: 20
June	9 282	1 667	4 224: 70	9 746	1 417	7 387: —	13	4: 30	9 768	1 953: 60	6 504	650: 40	19 048	19 369	14 220: —
July	15 082	1 600	6 047: 30	13 606	1 783	10 218: —	10	4: 20	12 061	2 412: 20	6 226	622: 60	28 688	21 680	19 304: 30
Aug.	11 985	1 531	5 054: 50	12 434	1 572	9 552: 30	6	3: —	10 512	2 102: 40	6 704	670: 40	24 419	20 325	17 382: 60
Sept.	6 285	1 177	2 842: 30	7 568	1 110	5 869: —	14	5: 50	7 073	1 414: 60	6 948	694: 80	13 853	16 322	10 853: 20
Oct.	7 203	1 159	3 414: 70	6 917	1 007	5 288: 90	12	4: 40	6 120	1 224: —	7 103	710: 30	14 120	15 401	10 642: 30
Nov.	5 489	986	2 391: 90	6 075	800	4 570: 90	15	6: 70	6 709	1 341: 80	7 600	760: —	11 564	16 110	9 071: 30
Dec.	6 379	1 234	2 811: 50	6 269	908	4 668: 10	15	5: 90	9 410	1 882: —	8 392	839: 20	12 648	19 959	10 206: 70
Total	87 807	14 770	39 058: 10	91 957	13 310	71 686: 50	160	66: 80	92 008	18 400: 60	84 965	8 496: 60	179 764	205 208	137 708: 50
1929	82 458	12 981	38 933: 20	83 533	11 343	60 633: 10	86	43: —	79 035	15 767: 40	77 163	7 716: 30	165 991	180 608	123 093: —
1928	75 668	12 173	35 297: 60	78 295	10 709	59 066: 10	76	38: —	72 682	14 536: 40	73 723	7 372: 30	153 963	169 363	116 310: 40
1927	72 132	11 548	31 745: 55	77 935	10 252	50 220: 40	49	24: 50	66 417	13 283: 40	69 142	6 914: 20	150 067	157 408	111 188: 05
1926	66 762	10 754	29 520: 50	73 307	10 168	55 874: 20	—	—	62 157	12 431: 40	62 326	6 232: 60	140 060	145 395	104 058: 70
1925	69 209	10 329	29 344: 90	65 303	9 205	50 204: 25	—	—	54 714	10 967: 20	54 728	5 472: 90	134 599	129 026	95 989: 25

Fig. 6.

Debiting.

The supervisor debits the charge for the service when this is performed or, for a standing order, at the end of every month. Specification of the amount (debit card for reference, reference and message, and message services, and debit card for time-giving and waking, are illustrated in figs. 4 and 5) is sent to the subscriber with his ordinary telephone bill.

Extent of the Work.

Fig. 6 is a table giving the numbers of days connected, debit cards, wakings, time-givings and messages, and the fees debited for these during each month of the year 1930. In each column the totals for the years 1925—1929 are also given.

It will be seen that these services have been more extensively used every year, and in these five years the number of days connected for reference and reference and message services has increased by 27 and 40 per cent. respectively, while wakings and time-givings have grown by 68 and 44 per cent. respectively.

For the period January-September of this year, the days connected for reference and reference and message services have, in comparison with the same period of 1930, increased by 10 and 5 per cent. respectively, wakings by 6 per cent., and time-givings by 4 per cent.

Messages not combined with reference and messages are, on the other hand, but slightly used.

It may be of interest to see how the various commissions accepted by the Information Bureau are distributed on one day, and an investigation undertaken to ascertain this shows that on July 20th of this year these were:

Subscribers connected for extended periods of time:

Reference service 559
Reference and message service 450

Subscribers connected for one day or part of a day:

Reference service 23
Reference and message service .. 34
Total number of subscribers connected on 20/7/31 .... 1 066
on 20/7/31 .... 1 066
In addition there were .... 449 wakings
216 time-givings
and delivered .... 3 messages

All trades and professions were represented in the Office clientele. Some of the professions represented most numerously on that date are enumerated below:

Doctors	131
Business men	105
Dentists	94
Limited Companies	81
Engineers	60
Painters and Decorators	49
Directors and Managers	43
Wholesale Merchants	40
Architects	22
Lawyers	16
Professors	11
etc.	

The cost of the staff amounted to about 42 per cent. of the gross receipts.

R 1062 *Home of the Royal Swedish Telegraph Administration, Brunkebergstorg, Stockholm.*

The Stockholm Telephone Commissions Office.

Simplified methods of designing electric wave filters and a contribution to the theory of matching filter quadripoles.

Communication from the Research and Development Department.[1]

By *H. Sterky.*

Contents.

[1] Ms. received by the Editor Aug. 7th 1931.

Definitions.

f = arbitrary frequency.

f_1 = upper cut-off frequency, f_2 = lower cut-off frequency; $f_1 f_2 = f_{00}^2$.

$f_{00} = \sqrt{f_1 f_2}$ = geometric mean frequency.

f' = arbitrary frequency $> f_{00}$, f'' = „ „ $< f_{00}$; $f' f'' = f_{00}^2$

$\triangle = f_1 - f_2$ = absolute band width.

$x = \frac{f_1}{f_2}$ = relative band width.

$p = \frac{\triangle}{f_{00}}$ = percentual band width.

$\triangle' = f_1 - f_2$.

$y = \frac{f'}{f''}$.

$p' = \frac{\triangle'}{f_{00}}$.

Θ = quadripole propagation constant.

$\gamma = \beta + j\alpha$ = filter „ „ for one section.

β = attenuation constant for one section.

α = phase constant for one section.

b_D = effective attenuation.

a_1, a_2, b and c = quadripole quantities.

I_1 = open-circuit impedance at ′ end of quadripole.

I_2 = „ „ „ ″ end „ „

R_1 = short-circuit „ „ ′ end „ „

R_2 = „ „ „ ″ end „ „

A = quadripole impedance.

$\sigma, \sigma' \sigma''$ = position angles.

$\mathfrak{Z}'$ = image impedance at ′ end of quadripole.

$\mathfrak{Z}''$ = „ „ at ″ end „ „

R' = terminal load impedance at ′ end of quadripole.

R'' = „ „ „ at ″ end „ „

$\mathfrak{Z}'_B$ = impedance of loaded quadripole at ′ end.

$\mathfrak{Z}''_B$ = „ „ „ „ at ″ end.

ε = reflection coefficient.

z_1 = impedance of series arm.
z_2 = „ „ shunt arm.
$k = \sqrt{z_1 z_2}$ for "constant k" filter.
$U + jV = \frac{z_1}{4 z_2}$.
$U_k = \frac{z_1}{4 z_2}$ for "constant k" filter without losses.
$\mathfrak{Z}_T$ = midseries image impedance.
$\mathfrak{Z}_\pi$ = midshunt „ „
$\mathfrak{Z}_{T00}$ = value of $\mathfrak{Z}_T$ at f_{00}.
$\mathfrak{Z}_{\pi 00}$ = „ „ $\mathfrak{Z}_\pi$ „ „
R_T = load impedance at midseries termination.
R_π = „ „ „ midshunt „
$\mathfrak{Z}_{BT}$ = midseries impedance of loaded filter.
$\mathfrak{Z}_{B\pi}$ = midshunt impedance of loaded filter.
$a = \frac{R_\pi}{\mathfrak{Z}_{\pi 00}} = \frac{\mathfrak{Z}_{T00}}{R_T}$ = matching coefficient ($a > 1$).
m = m-derivation coefficient according to Zobel ($m < 1$).

1. Introductory Resumé.

The general equations determining the properties of a quadripole* have long been known from the theoretical works of among others Breisig, Campbell, Pleijel and Wagner. A special class of quadripoles, so called wave filters, are increasingly employed in telephony and wireless. A reference list of literature on the subject is given at the end of this article, for the benefit of readers who wish to study more closely the historical development and mathematical theories of filters. Wagner, Campbell and Zobel are the chief contributors to the practical computation methods for this kind of quadripoles, and have thus made a wider use of filters in electrotechnics possible.

The object of the present paper is to submit the results of a special investigation on the matching of filter image impedances, made by the author in designing apparatus for carrier current telephony and telegraphy. Zobel has published a method for matching filter image impedance and terminal load resistances, the method of "m-derivation", in which a number of elements—e. g. inductances or capacities or a combination of both—beyond the original number of the prototype filter must be introduced.

* Or four terminal network (U. S. A.).

According to the method described here, which has been designated a-matching, the filter image impedances and the terminal load resistances are more or less well matched without any increase of the number of elements in the filter prototype.

The importance of the results attained will hardly be made sufficiently evident by a mere account of the investigation. The first two chapters of this article will therefore be devoted to a resumé of the derivation of the fundamental equations on which all filter computations are based, although this will naturally involve a repetition of much that has been previously published.

In a following chapter, two fundamental filter conceptions, viz. load impedance and effective attenuation, are derived with the assistance of Kennelly's definitions of position angles; new formulæ are deduced for the computation of the effective attenuation of two different kinds of filters, which formulæ are characterized by the same simplicity and lucidity as those given by Kennelly for the relation of voltages and currents in recurrent networks. The new formulæ for effective attenuation are well adapted for practical use.

In a later chapter, the author has had an opportunity of deducing a simple formula for the computation of an auxiliary quantity in a certain type of band pass filter, the so-called "constant k" type, which is largely used by American, German and Swedish telephone concerns. This auxiliary quantity is of fundamental importance in the computation of attenuations, image impedances and load impedances in all-filters of this class. A mathematically simple and general method has thus been substituted for a previously tedious and troublesome calculation.

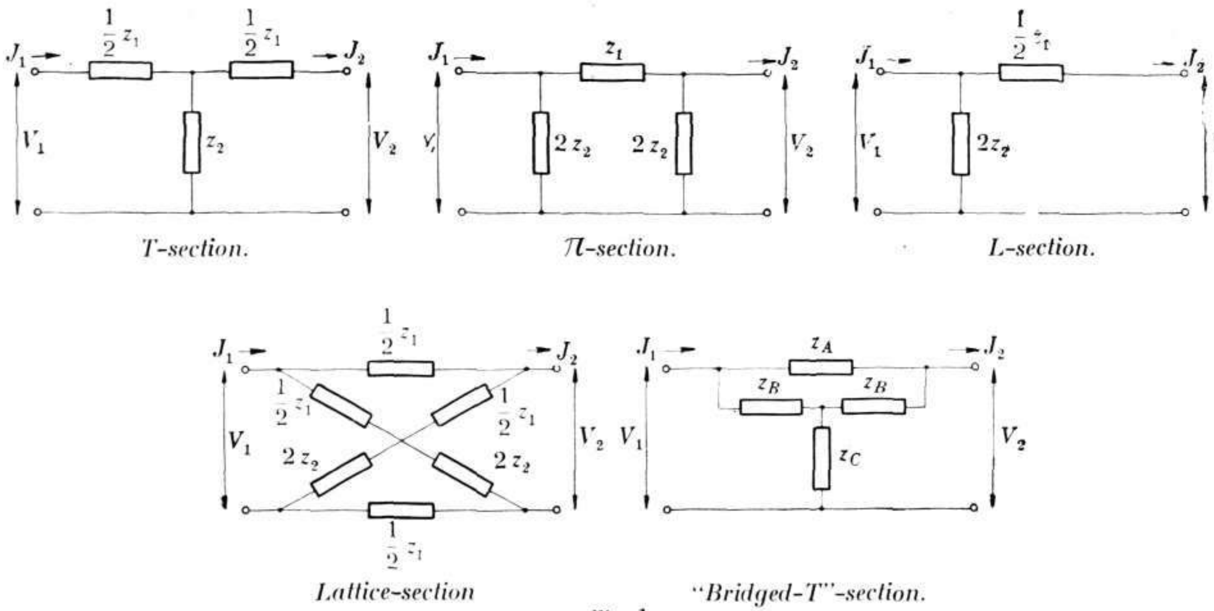
The later, and main part of the paper deals with a-matching, and formulæ are deduced for effective attenuation, load impedance and reflection coefficient in a-matched "constant k" filters. Finally, the advantages and drawbacks of a-matching are discussed, and suggestions made for further work on this subject.

The grateful thanks of the author are due to Messrs. S. Herlitz and S. Rodhe, for their kind assistance in making certain numerical calculations.

2. Different forms of general equations for quadripoles.

All calculations regarding filters are based on the general quadripole equations. By a quadripole we mean an electrical network consisting of general impedances with or without E. M. F:s, and joined together at a number of points, of which four are accessible for measurements. The characteristic properties of a quadripole for two pairs of these points can be measured and computed if certain constants are known; these we will now determine.

From the beginning we will confine ourselves to a consideration of quadripoles of which no E. M. F:s or unidirectional impedances (thermionic valves) form part, but only ordinary impedances connected so as to form what is called *T*-, *Π*- or *L*- sections (see fig. 1), in which form electric wave-filters generally occur. This does ,not however, diminish the general validity of the deduction for other types of filter, for it is possible to show that other forms also, e. g. Lattice- and "Bridged-*T*"-sections, are equivalent to the *T*- and *Π*-sections[1] mentioned above. The *L*-section is to be regarded as half a *T*-or *Π*-section, and a knowledge of its characteristic properties will be useful for calculating those of an odd number of half *T*-or *Π*-sections.



T-section. *Π-section.* *L-section.*

Lattice-section *"Bridged-T"-section.*

Fig. 1.

[1] See Bibliography, 4.

A rectangle according to figure 2 is used as a general symbol for a filter quadripole. We

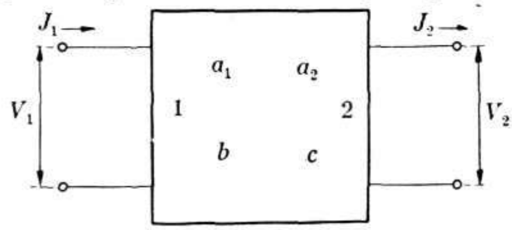


Fig. 2.

will now give a short summary of the German, Swedish, and Anglo-Saxon methods of designating the characteristic quantities of the quadripole and deducing its properties.

2 A. According to German praxis—Breisig and others.

Fig. 2 shows the quadripole with the four characteristic coefficients introduced by Breisig, a_1, a_2, b and c. If this quadripole is passive, i. e. contains no E. M. F:s or unidirectional impedances, the following three equations will give the relation between voltage and current in each of the two pairs of terminals 1 and 2.

$$\left.\begin{aligned} V_1 &= a_1\, V_2 + b\, J_2 \\ J_1 &= a_2\, J_2 + c\, V_2 \\ 1 &= a_1\, a_2 - bc \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Knowing the values of a_1, a_2 b, and c, and how the quadripole is connected, e. g. between an E. M. F. E with an internal impedance R' and a terminal load of impedance R'', we can obtain from (1) the voltages and currents V_1 and J_1 and V_2 and J_2 respectively. a_1, a_2, b, and c are calculated by Kirchoff's laws from the impedances forming the quadripole. As an example we will compute the impedance $\mathfrak{Z}_B'$ of the loaded quadripole, or the ratio of V_1 to J_1, when the quadripole is connected according to fig. 3. $\mathfrak{Z}_B'$ will in future be called the load impedance for short. We then have

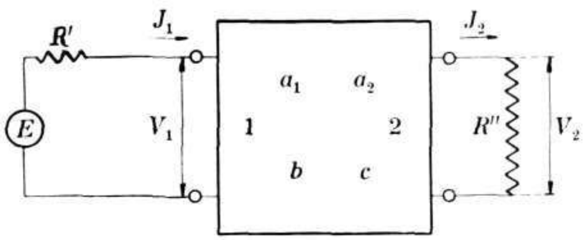


Fig. 3.

$$\left.\begin{aligned} V_1 &= a_1 V_2 + b J_2 \\ J_1 &= a_2 J_2 + c V_2 \\ E &= V_1 + R' J_1 \\ V_2 &= R'' J_2 \\ 1 &= a_1 a_2 - bc \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From this we get

$$\mathfrak{Z}_B' = \frac{V_1}{J_1} = \frac{a_1 V_2 + b J_2}{a_2 J_2 + c V_2} = \frac{a_1 R'' + b}{a_2 + c R''} \quad \ldots\ldots\ldots (3)$$

The ratio of E to V_2—which is of great importance for calculating the so-called effective attenuation, of which more below—is also easily obtained from (2)

$$\frac{E}{V_2} = \frac{V_1 + R' J_1}{V_2} = \frac{a_1 V_2 + b J_2 + R'(a_2 J_2 + c V_2)}{V_2}$$

whence

$$\frac{E}{V_2} = a_1 + a_2 \frac{R'}{R''} + \frac{b}{R''} + c R_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

2 B. According to Swedish praxis—Pleijel.

According to (82) in Pleijel's "Telefonledningars elektriska egenskaper",[1] and with symbols as in fig. 4, the general equations for a passive asymmetrical quadripole will be as follows:

$$\left.\begin{aligned} V_1 &= I_1 J_1 - A J_2 \\ V_2 &= A J_1 - I_2 J_2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

[1] See Bibliography, 2.

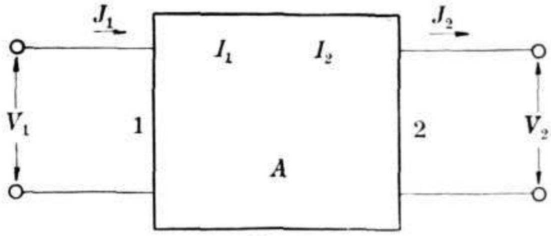


Fig. 4.

or, if V_1 and J_1 are solved from (5)

$$\left.\begin{aligned} V_1 &= \frac{I_1}{A} V_2 + \left(\frac{I_1 I_2}{A} - A\right) J_2 \\ J_1 &= \frac{I_2}{A} J_2 + \frac{1}{A} V_2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

In these equations I_1 and I_2 are the open circuit impedances of the quadripole measured from the input side 1 or output side 2. This can easily be verified by making $J_1 = 0$ or $J_2 = 0$, when equation 5 gives us

$$\left.\begin{aligned} \left(\frac{V_2}{-J_2}\right)_{J_1=0} &= I_2 \\ \left(\frac{V_1}{J_1}\right)_{J_2=0} &= I_1 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

(5) and (6) are further simplified by introducing the short circuit impedances R_1 and R_2. These are obtained from (6) by making $V_1 = 0$ and $V_2 = 0$.

$$\left.\begin{aligned} \left(\frac{V_2}{-J_2}\right)_{V_1=0} &= R_2 = I_2 - \frac{A^2}{I_1} \\ \left(\frac{V_1}{J_1}\right)_{V_1=0} &= R_1 = I_1 - \frac{A^2}{I_2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

whence also the formula, important for asymmetrical quadripoles

$$\frac{I_1}{I_2} = \frac{R_1}{R_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Thus instead of (6) we get the following simplified pair of equations:

$$\left.\begin{aligned} V_1 &= \frac{I_1}{A} V_2 + \frac{I_1 R_2}{A} J_2 \\ J_1 &= \frac{I_2}{A} J_2 + \frac{1}{A} V_2 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

and are able to find the relation between a_1, a_2, b, and c in (1) and I_1, I_2, R_1, R_2 and A in (10).

We further introduce the propagation constant Θ of the quadripole and the two impedances $\mathfrak{Z}'$ and $\mathfrak{Z}''$ which we define as the image impedances of the quadripole at the input (′) and output (″) ends. According to p. 118 et seq. in "Telefonledningars elektriska egenskaper" we have:

$$\left.\begin{aligned} \mathfrak{Z}' &= \sqrt{R_1 I_1} \\ \mathfrak{Z}'' &= \sqrt{R_2 I_2} \\ \operatorname{tgh} \Theta &= \sqrt{\frac{R_1}{I_1}} = \sqrt{\frac{R_2}{I_2}} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (11)$$

From (8), (9), and (11) we get:

$$A_2 = I_1 (I_2 - R_2) = \frac{\sqrt{R_2 I_2}\,\sqrt{R_1 I_1}}{\sqrt{R_2 I_2 R_1}} \sqrt{I_1}\,(I_2 - R_2)$$

$$= \frac{\mathfrak{Z}'\,\mathfrak{Z}''}{\sqrt{R_1 R_2}} \sqrt{\frac{R_1}{R_2}}\,(I_2 - R_2) = \mathfrak{Z}'\,\mathfrak{Z}''\,\frac{I_2}{R_2}\left(1 - \frac{R_2}{I_2}\right)$$

$$= \mathfrak{Z}'\,\mathfrak{Z}''\,\frac{1 - \operatorname{tgh}^2 \Theta}{\operatorname{tgh}^2 \Theta}$$

or

$$A = \frac{\sqrt{\mathfrak{Z}'\,\mathfrak{Z}''}}{\sinh \Theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

and

$$\left.\begin{aligned} \cosh \Theta &= \frac{\sqrt{I_1 I_2}}{A} \\ \sinh \Theta &= \frac{\sqrt{I_1 R_2}}{A} = \frac{\sqrt{I_2 R_1}}{A} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (13)$$

2 C. According to Anglo-Saxon praxis—Kennelly and others.

Using hyperbolic functions, the general equations for an asymmetrical passive quadripole can, according to Kennelly, be written as follows:

$$\left.\begin{aligned} V_1 &= \sqrt{\frac{\mathfrak{Z}'}{\mathfrak{Z}''}} \cosh \Theta\, V_2 + \sqrt{\mathfrak{Z}'\,\mathfrak{Z}''} \sinh \Theta\, J_2 \\ J_1 &= \sqrt{\frac{\mathfrak{Z}''}{\mathfrak{Z}'}} \cosh \Theta\, J_2 + \frac{1}{\sqrt{\mathfrak{Z}'\,\mathfrak{Z}''}} \sinh \Theta\, V_2 \end{aligned}\right\} \ \ldots (14)$$

When a quadripole is connected between two impedances R' and R'' as fig. 5, the calculations are in certain cases simplified by introducing what are called position angles. The position angle

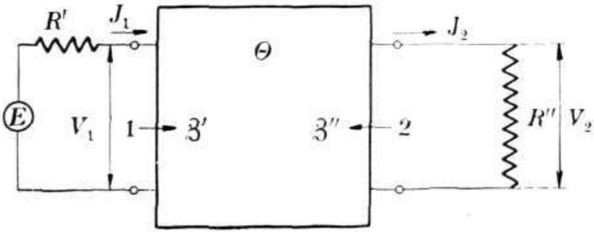


Fig. 5.

σ for the terminal load impedance R'', for instance, is defined by the equation.

$$\operatorname{tgh} \sigma = \frac{R''}{\mathfrak{Z}''} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

where $\mathfrak{Z}''$ is the image impedance of the quadripole at the ″-end.

If σ is introduced in (14), the following simple expression is obtained for voltages and currents:

$$\left.\begin{aligned} \frac{V_1}{V_2} &= \sqrt{\frac{\mathfrak{Z}'}{\mathfrak{Z}''}}\,\frac{\sinh(\Theta + \sigma)}{\sinh \sigma} \\ \frac{J_1}{J_1} &= \sqrt{\frac{\mathfrak{Z}''}{\mathfrak{Z}'}}\,\frac{\cosh(\Theta + \sigma)}{\cosh \sigma} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

2 D. Relation between quadripole quantities according to different methods.

The following table I has been drawn up to help in the study of the available literature on filters from different countries. In this the various quantities characterizing the quadripole according to German, Swedish, and Anglo-Saxon methods described above are given, with the relations existing between them.

3. Deduction of filter equations.

Fig. 6 represents a *T*-section and a *Π*-section, and the filter chains of infinite length that can be formed from them.

When the open circuit and short circuit impedances of the *T*- and *Π*-sections respectively are computed, in terms of the impedances z_1 of the series arms and z_2 of the shunt arms, it is also possible to obtain the value of the quantity A from (8). All the quantities in (10) are then known and can be put in. If these calculations are made, the general equations for the *T*- and *Π*-sections are obtained.

T-section.

$$\left.\begin{aligned} I &= I_1 = I_2 = \frac{z_1}{2} + z_2 \\ R &= R_1 = R_2 = \frac{z_1\, z_2 \left(1 + \dfrac{z_1}{4 z_2}\right)}{\dfrac{z_1}{2} + z_2} \\ A &= z_2 \end{aligned}\right\} \quad \ldots\ldots\ldots (17)$$

TABLE I.

The relation between various quadripole quantities.

	German method	Swedish method	Anglo-Saxon method
German method	a_1	$\frac{I_1}{A}$	$\sqrt{\frac{\mathfrak{Z}'}{\mathfrak{Z}''}}\cosh\Theta$
	a_2	$\frac{I_2}{A}$	$\sqrt{\frac{\mathfrak{Z}''}{\mathfrak{Z}'}}\cosh\Theta$
	b	$\frac{I_1 R_2}{A}=\frac{I_2 R_1}{A}$	$\sqrt{\mathfrak{Z}'\mathfrak{Z}''}\sinh\Theta$
	c	$\frac{1}{A}$	$\frac{1}{\sqrt{\mathfrak{Z}'\mathfrak{Z}''}}\sinh\Theta$
	$a_1 a_2 - bc = 1$	$\begin{cases} I_1(I_2 - R_2) = A^2 \\ I_2(I_1 - R_1) = A^2 \end{cases}$	$\cosh^2\Theta - \sinh^2\Theta = 1$
Swedish method	$\frac{a_1}{c}$	I_1	$\frac{\mathfrak{Z}'}{\mathrm{tgh}\,\Theta}$
	$\frac{a_2}{c}$	I_2	$\frac{\mathfrak{Z}''}{\mathrm{tgh}\,\Theta}$
	$\frac{b}{a_2}$	R_1	$\mathfrak{Z}'\,\mathrm{tgh}\,\Theta$
	$\frac{b}{a_1}$	R_2	$\mathfrak{Z}''\,\mathrm{tgh}\,\Theta$
	$\frac{1}{c}$	A	$\frac{\sqrt{\mathfrak{Z}'\mathfrak{Z}''}}{\sinh\Theta}$
Anglo-Saxon method	$\sqrt{\frac{a_1}{a_2}}\sqrt{\frac{b}{c}}$	$\sqrt{R_1 I_1}$	$\mathfrak{Z}'$
	$\sqrt{\frac{a_2}{a_1}}\sqrt{\frac{b}{c}}$	$\sqrt{R_2 I_2}$	$\mathfrak{Z}''$
	$\sqrt{bc}$	$\frac{\sqrt{I_1 R_2}}{A}=\frac{\sqrt{I_2 R_1}}{A}$	$\sinh\Theta$
	$\sqrt{a_1 a_2}$	$\frac{\sqrt{I_1 I_2}}{A}$	$\cosh\Theta$
	$\sqrt{\frac{bc}{a_1 a_2}}=\sqrt{1-\frac{1}{a_1 a_2}}$	$\sqrt{\frac{R_1}{I_1}}=\sqrt{\frac{R_2}{I_2}}$	$\mathrm{tgh}\,\Theta$

$$\left.\begin{aligned} V_1 &= \left(1 + \frac{z_1}{2 z_2}\right) V_2 + z_1 \left(1 + \frac{z_1}{4 z_2}\right) J_2 \\ J_1 &= \left(1 + \frac{z_1}{2 z_2}\right) J_2 + \frac{1}{z_2} V_2 \end{aligned}\right\} \quad \ldots (18)$$

chain of similar T-sections, as in fig. 6. The load on such a link is $\mathfrak{Z}_T$ the mid-series image impedance of the infinite chain. The insertion of the filter link under consideration will not change

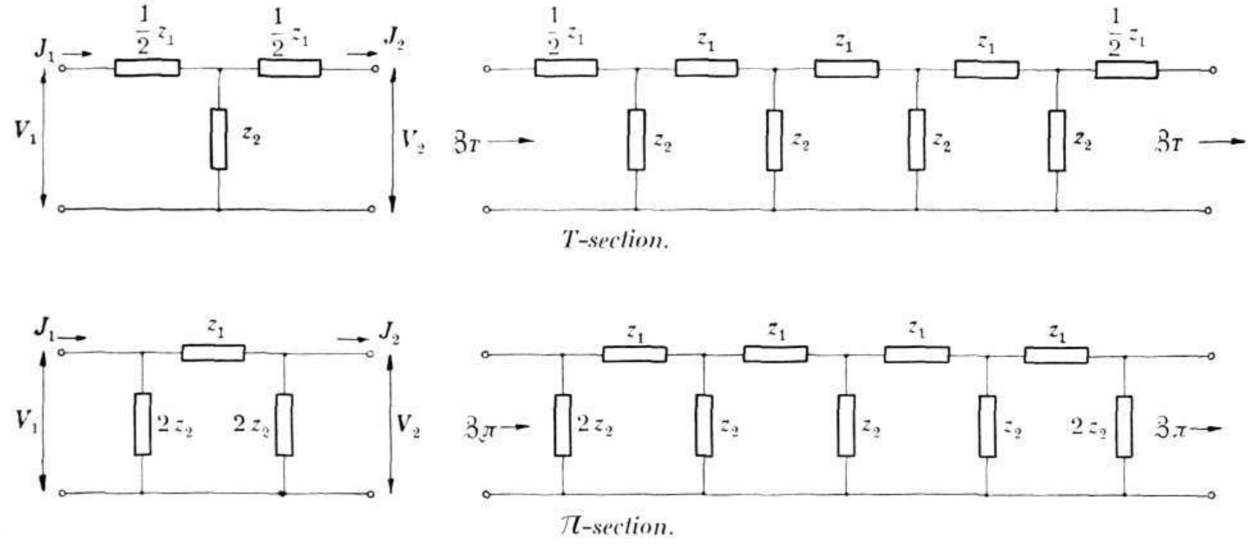


Fig. 6.

the mid-series image impedance of the chain, and hence we get

$$\mathfrak{Z}_T = \frac{z_1}{2} + \frac{\left(\frac{z_1}{2} + \mathfrak{Z}_T\right) z_2}{\frac{z_1}{2} + \mathfrak{Z}_T + z_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

from which we solve

$$\mathfrak{Z}_T = \sqrt{z_1 z_2} \sqrt{1 + \frac{z_1}{4 z_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

For the π-section we get in the same way

$$\mathfrak{Z}_\pi = \frac{\sqrt{z_1 z_2}}{\sqrt{1 + \frac{z_1}{4 z_2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Π-section.

$$\left.\begin{aligned} I &= I_1 = I_2 = \frac{\frac{z_1}{2} + z_2}{1 + \frac{z_1}{4 z_2}} \\ R &= R_1 = R_2 = \frac{z_1 z_2}{\frac{z_1}{2} + z_2} \\ A &= \frac{z_2}{1 + \frac{z_1}{4 z_2}} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (19)$$

$$\left.\begin{aligned} V_1 &= \left(1 + \frac{z_1}{2 z_2}\right) V_2 + z_1 J_2 \\ J_1 &= \left(1 + \frac{z_1}{2 z_2}\right) J_2 + \frac{1 + \frac{z_1}{4 z_2}}{z_2} V_2 \end{aligned}\right\} \quad \ldots\ldots\ldots (20)$$

With the help of the transformation formulæ in Table I, it is possible also to calculate, from (17) and (19), the mid-series and mid-shunt image impedances $\mathfrak{Z}_T$ and $\mathfrak{Z}_\pi$ respectively, of the T- and Π-sections, and the filter propagation constant γ. There is however, a simpler and more direct way, which is given in detail by Zobel and others. In making this calculation for, say, the T-section, we assume this to form one link in an infinite chain of similar T-sections, as in fig. 6.

The filter propagation constant γ for one section is defined as the natural logarithm of the square root of the ratio $\frac{P_1}{P_2}$, where P_1 is the power fed into, and P_2 the power taken out of, the filter section.

$$\therefore \gamma = \log_e \sqrt{\frac{P_1}{P_2}} \quad \ldots\ldots\ldots\ldots\ldots (24)$$

As $P_1 = \frac{V_1^2}{\mathfrak{Z}_T}$ and $P_2 = \frac{V_2^2}{\mathfrak{Z}_T}$ we get $e^\gamma = \frac{V_1}{V_2}$

The ratio $\frac{V_1}{V_2}$ is obtained directly by applying Kirchhoff's laws to the T-section in fig. 6.

$$V_1 = V_2 + \frac{z_1}{2}\frac{V_2}{\mathfrak{Z}_T} + \frac{z_1}{2}\left[\frac{V_2}{\mathfrak{Z}_T} + \frac{V_2 + \frac{z_1}{2}\frac{V_2}{\mathfrak{Z}_T}}{z_2}\right]$$

and hence, using also (22)

$$e^{\gamma} = \frac{V_1}{V_2} = 1 + \frac{z_1}{2z_2} + \sqrt{\frac{z_1}{z_2}}\sqrt{1 + \frac{z_1}{4z_2}} \quad \ldots \; (25)$$

A simpler expression for γ is obtained with hyperbolic functions. We have

$$\cosh \gamma = \frac{e^{\gamma} + e^{-\gamma}}{2} \quad \ldots\ldots \; (26)$$

By comparing equations (25) and (26) we get, after some intermediate calculations,

$$\cosh \gamma = 1 + \frac{z_1}{2z_2} \quad \ldots\ldots \; (27)$$

The same expression is obtained in an analogous way for a Π-section.

We will now introduce some expressions which in certain cases may considerably simplify the calculations for filters; for this purpose we make

$$\frac{z_1}{4z_2} = U + jV. \quad \ldots\ldots \; (28)$$

for in the general case the relation $\frac{z_1}{z_2}$ is complex, and we then obtain

$$\left.\begin{aligned} \mathfrak{Z}_T &= \sqrt{z_1 z_2}\sqrt{1 + U + jV} \\ \mathfrak{Z}_\pi &= \frac{\sqrt{z_1 z_2}}{\sqrt{1 + U + jV}} \\ \cosh \gamma &= 1 + 2(U + jV) \\ \sinh \gamma &= 2\sqrt{U + jV}\sqrt{1 + U + jV} \end{aligned}\right\} \ldots\ldots \; (29\,\mathrm{a})$$

In practice, when doing filter calculations, one generally assumes the impedances forming the series- and shunt-arms to be non-dissipative. On this assumption, z_1 and z_2 will be purely imaginary, and the ratio $\frac{z_1}{z_2}$ will then be real; and the formulæ (29 a) may consequently be simplified to

$$\left.\begin{aligned} \mathfrak{Z}_T &= \sqrt{z_1 z_2}\sqrt{1 + U} \\ \mathfrak{Z}_\pi &= \frac{\sqrt{z_1 z_2}}{\sqrt{1 + U}} \\ \cosh \gamma &= 1 + 2U \\ \sinh \gamma &= 2\sqrt{U} \cdot \sqrt{1 + U} \end{aligned}\right\} \ldots\ldots \; (29\,\mathrm{b})$$

Above (p. 8) we have pointed out that half T- or Π-sections are identical. The filter propagation constant for such a half section—what is called an L-section—can be shown to be $\frac{\gamma}{2}$, if γ be the corresponding constant for a whole T- or Π-section. For the L-section (fig. 7) the following equations are obtained

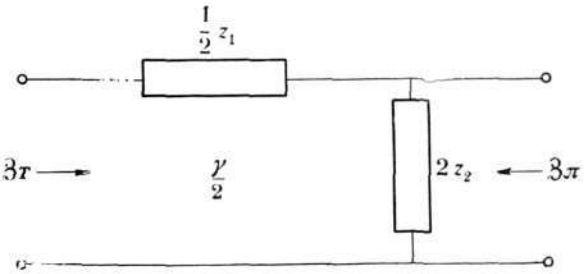


Fig. 7.

$$\left.\begin{aligned} \mathfrak{Z}_T &= \sqrt{z_1 z_2}\sqrt{1 + U} \\ \mathfrak{Z}_\pi &= \frac{\sqrt{z_1 z_2}}{\sqrt{1 + U}} \\ \cosh \frac{\gamma}{2} &= \sqrt{1 + U} \\ \sinh \frac{\gamma}{2} &= \sqrt{U} \end{aligned}\right\} \ldots\ldots \; (30)$$

As we have already mentioned (p. 7) there is a certain group of filters called the "constant k" group. In this, $\sqrt{z_1 z_2}$ is a constant (whence the name) independent of the frequency

$$\sqrt{z_1 z_2} = k \quad \ldots\ldots \; (31)$$

If this value is substituted in (29 b) or (30), we find that the mid-series and mid-shunt image impedances, as well as the filter propagation constant, are determined exclusively by the value of U or U_k in this case. Later we will show that other properties, such as load impedances, effective attenuation, and others, may also be expressed as functions of U_k. The calculation of U or U_k as a function of the frequency is therefore of fundamental importance to calculations of filters.

4. Two fundamental filter conceptions.

4 A. Load impedance.

In chapter 2 A, we found a way of computing the load impedance of a quadripole. We will now examine a little more closely the calculation of the same quantity for a filter. When filters are used in practical electrotechnics, it often happens that their impedances have to be matched to one another or to a generator or load having a given impedance. The reflection coefficient

$$\varepsilon = \left| \frac{\mathfrak{Z}_B - \mathfrak{Z}}{\mathfrak{Z}_B + \mathfrak{Z}} \right| \quad \ldots\ldots\ldots\ldots\ldots \quad (32)$$

is a measure of the accuracy of this matching and in this equation $\mathfrak{Z}_B$ is the impedance of the loaded filter—the load impedance—and $\mathfrak{Z}$ the arbitrary impedance to be matched. To calculate this reflection coefficient it is therefore necessary to know $\mathfrak{Z}_B$, the load impedance of the filter.

In other instances also it is useful to have an expression for the load impedance, particularly in series or parallel connexion of filters passing different frequencies, when by judicious designing of the filters, they can be exellently matched with a line, i. e. the resultant impedance of the connected filters be made practically ohmic at all frequencies.

The load impedance of a filter is defined as the ratio of voltage to current at the input, or output, side of the filter, when its output, or input, side is loaded with a certain known impedance. With the aid of Breisig's formulæ (p. 9) we have already found an expression for the load impedance of a quadripole at the $'$-end.

$$\mathfrak{Z}_B' = \frac{V_1}{J_1} = \frac{a_1 R'' + b}{a_2 + cR''} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

By exchanging input and output sides, we find instead the load impedance at the $''$-end.

$$\mathfrak{Z}_B'' = \frac{V_2}{-J_2} = \frac{a_2 R' + b}{a_1 + cR'}, \text{ since } \frac{V_1}{-J_1} = R' \quad (33)$$

The values of the load impedances $\mathfrak{Z}_B'$, $\mathfrak{Z}_B''$ expressed by the Swedish and Anglo-Saxon methods respectively, may be obtained in an analogous way, or from Table I, and will be:

$$\left.\begin{aligned} \mathfrak{Z}_B' &= \frac{I_1 (R'' + R_2)}{I_2 + R''} \\ \mathfrak{Z}_B'' &= \frac{I_2 (R' + R_1)}{I_1 + R'} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (34)$$

respectively

$$\left.\begin{aligned} \mathfrak{Z}_B' &= \mathfrak{Z}' \frac{R'' + \mathfrak{Z}'' \operatorname{tgh} \Theta}{\mathfrak{Z}'' + R'' \operatorname{tgh} \Theta} = \mathfrak{Z}' \operatorname{tgh} (\Theta + \sigma'') \\ \mathfrak{Z}_B'' &= \mathfrak{Z}'' \frac{R' + \mathfrak{Z}' \operatorname{tgh} \Theta}{R' \operatorname{tgh} \Theta + \mathfrak{Z}'} = \mathfrak{Z}'' \operatorname{tgh} (\Theta + \sigma') \end{aligned}\right\} (35)$$

in which - - - - - -

$$\left.\begin{aligned} \operatorname{tgh} \sigma' &= \frac{R'}{\mathfrak{Z}'} \\ \operatorname{tgh} \sigma'' &= \frac{R''}{\mathfrak{Z}''} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (36)$$

according to the definitions introduced by Kennelly. (35) affords the most suitable means for calculating the load impedance of an arbitrary filter, as it will only be necessary to find the angles σ' and σ'' and the total angle Θ for the filter, to get the ratio of the desired load impedance $\mathfrak{Z}_B$ to the image impedance $\mathfrak{Z}$. As an example we will work out the load impedance of an *L*-section as in fig. 8, which we assume to be of the "con-

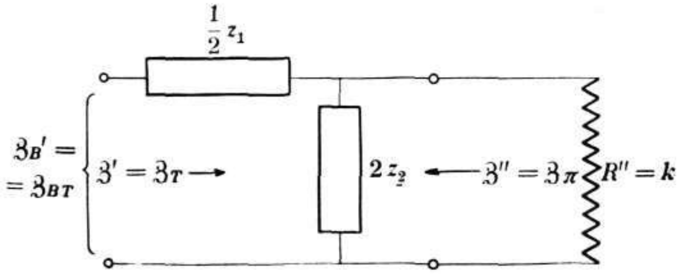


Fig. 8.

stant k" type—for which $k = \sqrt{z_1 z_2}$—and loaded with an ohmic resistance $R'' = k$. According to (30) we then have:

$$\mathfrak{Z}' = \mathfrak{Z}_T = k \sqrt{1 + U_k}$$

$$\mathfrak{Z}'' = \mathfrak{Z}_\pi = \frac{k}{\sqrt{1 + U_k}}$$

$$\operatorname{tgh} \frac{\gamma}{2} = \sqrt{\frac{U_k}{1 + U_k}}$$

whence:

$$\operatorname{tgh} \sigma'' = \frac{R''}{\mathfrak{Z}''} = \sqrt{1 + U_k}$$

$$\mathfrak{Z}_B' = \mathfrak{Z}_{BT} = \mathfrak{Z}_T \frac{\operatorname{tgh} \frac{\gamma}{2} + \operatorname{tgh} \sigma''}{1 + \operatorname{tgh} \frac{\gamma}{2} \operatorname{tgh} \sigma''}$$

and thus

$$\mathfrak{Z}_{BT} = k \sqrt{1 + U_k} \frac{\sqrt{\frac{U_k}{1 + U_k}} + \sqrt{1 + U_k}}{1 + \frac{\sqrt{U_k} \sqrt{1 + U_k}}{\sqrt{1 + U_k}}}$$

$$\mathfrak{Z}_{BT} = k \frac{1 + U_k + \sqrt{U_k}}{1 + \sqrt{U_k}} \quad \ldots\ldots (37)$$

In the same way we obtain the mid-shunt load impedance $\mathfrak{Z}_{B\pi}$.

$$\mathfrak{Z}_{B\pi} = k \frac{1 + \sqrt{U_k}}{1 + U_k + \sqrt{U_k}} \quad \ldots\ldots (38)$$

(37) and (38) show that the load impedances of the "constant k" class can also be expressed solely as functions of the variable U_k, which thus again proves to be of fundamental importance. We also see that $\mathfrak{Z}_{BT}$ and $\mathfrak{Z}_{B\pi}$ vary inversely when there is variation of the frequency or U_k, i. e. we have $\mathfrak{Z}_{BT} \cdot \mathfrak{Z}_{B\pi} = k^2$. This is a very important property of this group of filter and is just the reason for its having been so excessively used, for instance in two wire repeaters with differential transformer.[1]

4 B. Effective attenuation.

Above we have learnt how to define the propagation constant γ of a filter section. Knowing the value of this, it is possible to work out the ratio of the outgoing to the incoming voltage or current in a filter section, for

$$V_2 = V_1 e^{-\gamma} \qquad J_2 = J_1 e^{-\gamma} \quad \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

These equations for the voltage and current are, however, only valid on the assumption that the filter section forms part of an infinite chain of identical sections or, in other words, for a single section, that this is connected between a generator impedance and a terminal load impedance each of which is at all frequencies equal to the image impedances of the section on its corresponding sides. Such an ideal condition never occurs in practice, as generally both the internal impedance of the generator and the terminal load impedance are constant ohmic resistances. In the general case, it will only be possible at one, or in certain cases two—of which more in Ch. 6—to design the filter section so that its image impedances will be identical with generator and terminal load impedances respectively. At all other frequencies the matching will not be ideal, and reflections will therefore occur. These will cause additional attenuations, which in certain cases impair, in others improve, the frequency-selective properties of the filter. In practice it is of great importance to have an unambiguous definition of the effective attenuation of the filter, taking just these reflections into account. For this effective attenuation we will henceforth use the symbol b_D.

The effective attenuation b_D of a given filter is defined as the natural logarithm of the ratio of the square root of the maximum power, P_{2fmax}, which a given E. M. F. E can supply to the terminal load impedance before the filter is connected in, to the root of the power, P_2, obtained in the terminal load impedance after the filter is connected in (see fig. 9).

$$b_D = \log_e \sqrt{\frac{P_{2fmax}}{P_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

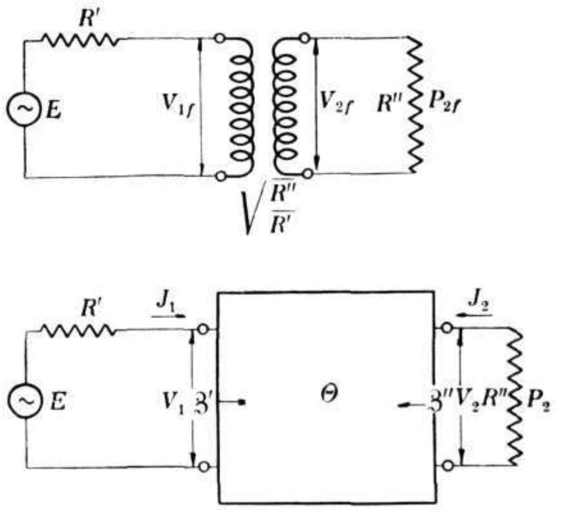


Fig. 9.

By expressing the effective attenuation like this in terms of the maximum power that a given terminal load impedance R'' can draw from a given generator of internal impedance R', we get a definition which can also be used for calculating the effective attenuation of a filter with transformation or for a filter connected in cascade to a transformer. In that case, the filter is assumed to consist of an ideal transformer, i. e. a transformer with no magnetic leakage, infinite induc-

[1] See Swedish patent applications by M. Vos and T. Laurent 5107/29, 1835/30, and M. Vos 4509/30.

tance, no losses and a ratio $\sqrt{\frac{R''}{R'}}$, and the right number of impedance elements to give the frequency-selective property of the filter. These elements may or may not form part of the ideal transformer. Defined in this way, the effective attenuation will exclusively determine the function of the filter as such, and is therefore independent of how the external impedances can be matched by transformation. This is of special importance in measurements, when the adjustment of and losses in the inductances and capacities of the filter elements have to be checked. Calculations too are helped by this definition, as in the general case they can easily be brought back to calculations of the effective attenuation of a filter without transformation.

The powers are calculated on the assumption that the impedances R' and R'' are ohmic, which is usually the case in actual practice (cf. Chap. 6). We get:

$$P_{2f} = \left(\frac{E}{R' + \frac{R'}{R''}R''}\right)^2 \frac{R'}{R''}R''.$$

and hence $P_{2f\max} = \frac{E^2}{4\,R'}$

In addition, $P_2 = \frac{V_2^2}{R''}$,

and hence

$$b_D = \log_e \left|\frac{E}{2\,V_2}\right| \sqrt{\frac{R''}{R'}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (41)$$

According to (4):

$$\left|\frac{E}{V_2}\right| = \left|a_1 + a_2\frac{R'}{R''} + \frac{b}{R''} + cR'\right|, \quad \ldots\ldots\ldots \ (4)$$

whence finally

$$b_D = \log_e \frac{1}{2}\left\{\left[a_1\sqrt{\frac{R''}{R'}} + a_2\sqrt{\frac{R'}{R''}} + \frac{b}{\sqrt{R'R''}} + c\sqrt{R'R''}\right]\right\} \ldots \ (42)$$

Using hyperbolic functions, the expression for the effective attenuation given in (42) can be written (cf. Chap. 2 D)

$$b_D = \log_e \frac{1}{2}\left\{\left[\left(\sqrt{\frac{R''}{R'}}\sqrt{\frac{\mathfrak{Z}'}{\mathfrak{Z}''}} + \sqrt{\frac{R'}{R''}}\sqrt{\frac{\mathfrak{Z}''}{\mathfrak{Z}'}}\right)\cosh\Theta + \left(\frac{\sqrt{\mathfrak{Z}'\,\mathfrak{Z}''}}{\sqrt{R'R''}} + \frac{\sqrt{R'R''}}{\sqrt{\mathfrak{Z}'\,\mathfrak{Z}''}}\right)\sinh\Theta\right]\right\} \quad (43)$$

On certain assumptions, this expression can be simplified still more.

We will discuss two cases, namely *symmetrical filters*, in which, when the frequency $\mathfrak{Z}'$ and $\mathfrak{Z}''$ varies proportionally, and *asymmetrical filters*, in which $\mathfrak{Z}'$ and $\mathfrak{Z}''$ vary inversely, i. e. their product is a constant real quantity k^2.

In the general case, when the filter includes *transformation*, the image impedances $\mathfrak{Z}'$ and $\mathfrak{Z}''$ may be of quite different orders of magnitude in both symmetrical and asymmetrical filters. If, for example, we represent the values of $\mathfrak{Z}'$ and $\mathfrak{Z}''$ at the frequency f_{oo} by $\mathfrak{Z}_{oo}'$ and $\mathfrak{Z}_{oo}''$, the transformation in the filter is given by the transformation coefficient $\mu^2 = \frac{\mathfrak{Z}_{oo}''}{\mathfrak{Z}_{oo}'}$.

4 Ba. Symmetrical filters.

In a symmetrical filter $\mathfrak{Z}'$ and $\mathfrak{Z}''$ vary proportionally when the frequency varies, and we may therefore put

$$\mathfrak{Z}' = n^2\,\mathfrak{Z}'' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (44\text{a})$$

where n is a real number independent of the frequency.

The matching of the terminal load impedances R' and R'' at the generator side and loaded side with the image impedances $\mathfrak{Z}'$ and $\mathfrak{Z}''$ may be selected arbitrarily. Filters are however, usually designed by making R' and R'' equal to the values of $\mathfrak{Z}'$ and $\mathfrak{Z}''$ respectively at a given frequency, in most cases the geometric mean frequency f_{oo} in a band filter.

The problem of matching will be discussed in greater detail in a later chapter. Here we will only show that the general expression for the effective attenuation (43) can under certain circumstances be simplified. These circumstances involve the assumption of a constant ratio of the terminal load impedance R', on the generator side, to the input image impedance $\mathfrak{Z}'$ of the filter, and of the terminal load impedance R'' to the output image impedance $\mathfrak{Z}''$ of the filter i. e.

$$\frac{R'}{\mathfrak{Z}'} = \frac{R''}{\mathfrak{Z}''}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (45)$$

or according to (44 a)

$$R' = n^2\,R'' \ \ldots\ldots\ldots\ldots\ldots \ (44\text{b})$$

If in addition, like Kennelly, we introduce as in (36) a position angle σ defined as follows

$$\text{tgh}\ \sigma = \frac{R''}{\mathfrak{Z}''},$$

we get, according to (45):

$$\text{tgh}\ \sigma = \frac{R''}{\mathfrak{Z}''} = \frac{R'}{\mathfrak{Z}'} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (46)$$

On substituting this in (43), we get

$$b_D = \log_e \frac{1}{2}\left|\left[2\cosh\Theta + \left(\frac{1}{\text{tgh}\ \sigma} + \text{tgh}\ \sigma\right)\sinh\Theta\right]\right|$$

or

$$\boxed{b_D = \log_e \left|\frac{\sinh(\Theta + 2\sigma)}{\sinh 2\sigma}\right|} \quad \ldots\ldots \quad (47)$$

4 Bb. **Asymmetrical filters with $\mathfrak{Z}'\,\mathfrak{Z}'' = k^2$.**

In certain asymmetrical filters the input image impedance $\mathfrak{Z}'$ varies inversely as the output image impedance $\mathfrak{Z}''$, i. e. the product of these impedances is a constant, real quantity, independent of the frequency, that is

$$\mathfrak{Z}'\,\mathfrak{Z}'' = k^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (48\text{a})$$

This property is not uncommon in filters composed of a number of whole or half sections. For a certain group, viz. the "constant k" filters mentioned above, the two image impedances $\mathfrak{Z}'$ and $\mathfrak{Z}''$ vary inversely even in a half or *L*-section, as is indicated for instance by (30) and (31).

The general expression for the effective attenuation (43) can be simplified for asymmetrical filters satisfying condition (48 a), if we presume the generator impedance R' and the terminal load impedance R'' to be selected so that we also have

$$R'\,R'' = k^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (48)$$

We introduce the position angle σ here too, and with the help of (48 a) and (48 b) we get

$$\text{tgh}\ \sigma = \frac{R''}{\mathfrak{Z}''} = \frac{\mathfrak{Z}'}{R'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (49)$$

Finally, on substituting in (43) we get

$$\boxed{b_D = \log_e \left|\frac{\cosh(\Theta + 2\sigma)}{\sinh 2\sigma}\right|} \quad \ldots\ldots \quad (50)$$

These two equations, (47) and (50), show that the definition chosen for the effective attenuation is particularly suitable, for even if the filters do have transformation, this will disappear from the formulæ for the effective attenuation. Only the complex angle Θ and an auxiliary angle σ, as defined in (46) or (49), will enter into the calculations. We shall see later how these calculations are worked out in practice.

The effective attenuation formulæ (47) and (50) are of the same general type as Kennelly used in calculations with position angles. *We have thus proved that position angles may with advantage be used for calculating effective attenuations also.*

Before going on to the calculation of certain much used filter types, we will make a little more detailed comparison between the various definitions of the effective attenuation b_D used in electrotechnics up to now.

In "Telefonledningarnas elektriska egenskaper", p. 53, Pleijel has given a definition of the effective attenuation slightly different from that given above. According to that author, the effective attenuation is the logarithm of the ratio in which the voltage or current in a given terminal load impedance is reduced by the insertion of an arbitrary quadripole, i. e., in this case, a filter in

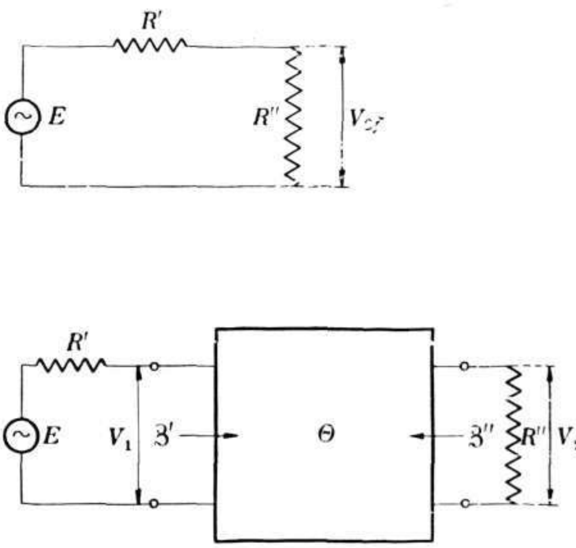


Fig. 10.

front of the terminal load impedance. Referring to fig. 10 the definition will thus be

$$\overline{b}_D = \log_e \left|\frac{V_{2f}}{V_2}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (51)$$

V_{2f} is obtained direct from the figure:

$$V_{2f} = \frac{R''}{R' + R''}\,E \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (52)$$

and V_2 as before from (4):

$$\left|\frac{E}{V_2}\right| = \left|a_1 + a_2\frac{R'}{R''} + \frac{b}{R''} + cR'\right| \quad \text{(4)}$$

and hence

$$b_D = \log_e \left|\frac{a_1 R'' + a_2 R' + b + cR'R''}{R' + R''}\right|,$$

which can also, according to Table I, be written:

$$b_D = \log_e \left| \frac{1}{\sqrt{\frac{R'}{R''}} + \sqrt{\frac{R''}{R'}}} \cdot \left[\left(\sqrt{\frac{R''}{R'}}\sqrt{\frac{\mathfrak{Z}'}{\mathfrak{Z}''}} + \sqrt{\frac{R'}{R''}}\sqrt{\frac{\mathfrak{Z}''}{\mathfrak{Z}'}}\right) \cdot \cosh\Theta + \left(\frac{\sqrt{\mathfrak{Z}'\mathfrak{Z}''}}{\sqrt{R'R''}} + \frac{\sqrt{R'R''}}{\sqrt{\mathfrak{Z}'\mathfrak{Z}''}}\right)\sinh\Theta\right]\right| \quad (54)$$

This is identical with (43) except for the factor $\frac{2}{\sqrt{\frac{R'}{R''}} + \sqrt{\frac{R''}{R'}}}$, representing the transformation that can be introduced in the filter. The other factor within the brackets is, on the assumptions made for symmetrical or asymmetrical filters—see (46) or (49)—independent of the ratio R'/R''. Only when $R' = R''$ does the factor

$$\frac{2}{\sqrt{\frac{R'}{R''}} + \sqrt{\frac{R''}{R'}}} = 1,$$

and (54) becomes (43)

For all other values of R'/R'', this factor will be less than unity, and the effective attenuation therefore smaller than in the first case, in other words, an apparent gain has been introduced. The reason for this is that we have assumed that a certain transformation is included in the filter. On account of this transformation, the transmission of energy from the E. M. F. to R'' will be better when the filter is inserted than before this is done. This is equivalent to a lower effective attenuation.—To avoid any misunderstanding, we wish here to point out that the formula for the effective attenuation given in (54) is generally applicable, and that it will always give a correct value for b_D.

Besides Pleijel's definition of the effective attenuation given above, another definition is also used, by the CCI among others, according to which the effective attenuation of a quadripole is the logarithm of the ratio of the square root of the apparent power which a given generator can supply to an impedance equal to its internal impedance, to the root of the apparent power generated in a terminal load impedance connected behind the quadripole in question. This definition leads to exactly same formulæ as in (43).

The difference between, on the one hand, the definition given here or the CCI definition, and on the other, Pleijel's definition, is only that in the former the power output is compared with the maximum output from a given E. M. F., while in the latter no maximum output of power is required. In the former case it is often a great help in the calculations to introduce an ideal transformer of ratio $\sqrt{\frac{R''}{R'}}$, while in the latter case this is not needed (cf. figs. 9 and 10).

Which definition is to be preferred is to some extent a matter of taste, as long as due consideration is given to the factor $\frac{2}{\sqrt{\frac{R''}{R'}} + \sqrt{\frac{R'}{R''}}}$, when using the Pleijel definition. According to the definition here given and used, we obtain in the two cases examined—symmetrical filters satisfying the conditions of (46), and asymmetrical filters satisfying those of (49)—the same formulæ for and value of the effective attenuation, whatever the variation in the ratio of the external impedances R' and R''. This will not be the case according to Pleijel's definition.

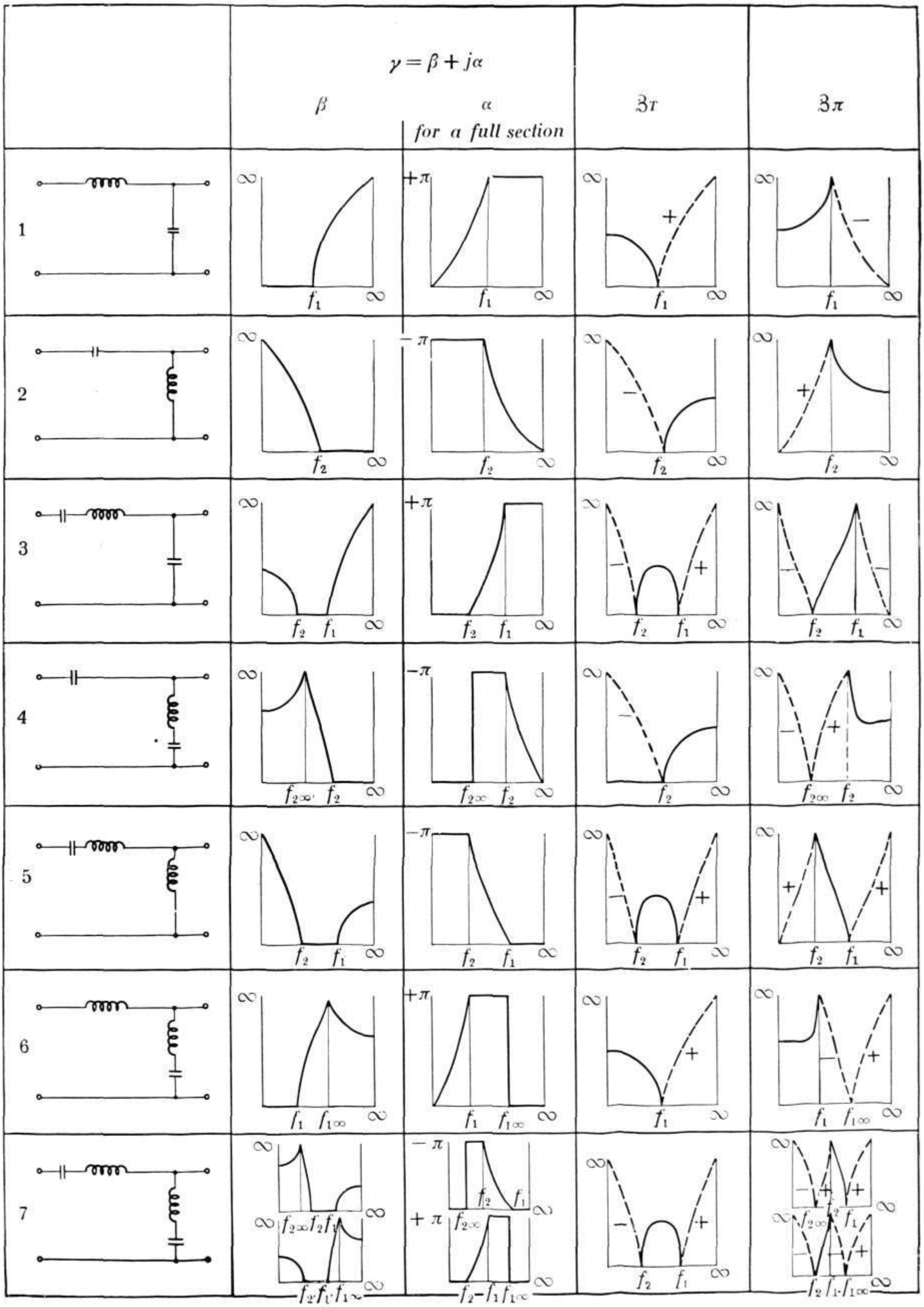
5. Various kinds of simple filter sections.

5 A. Diagrams and curves.

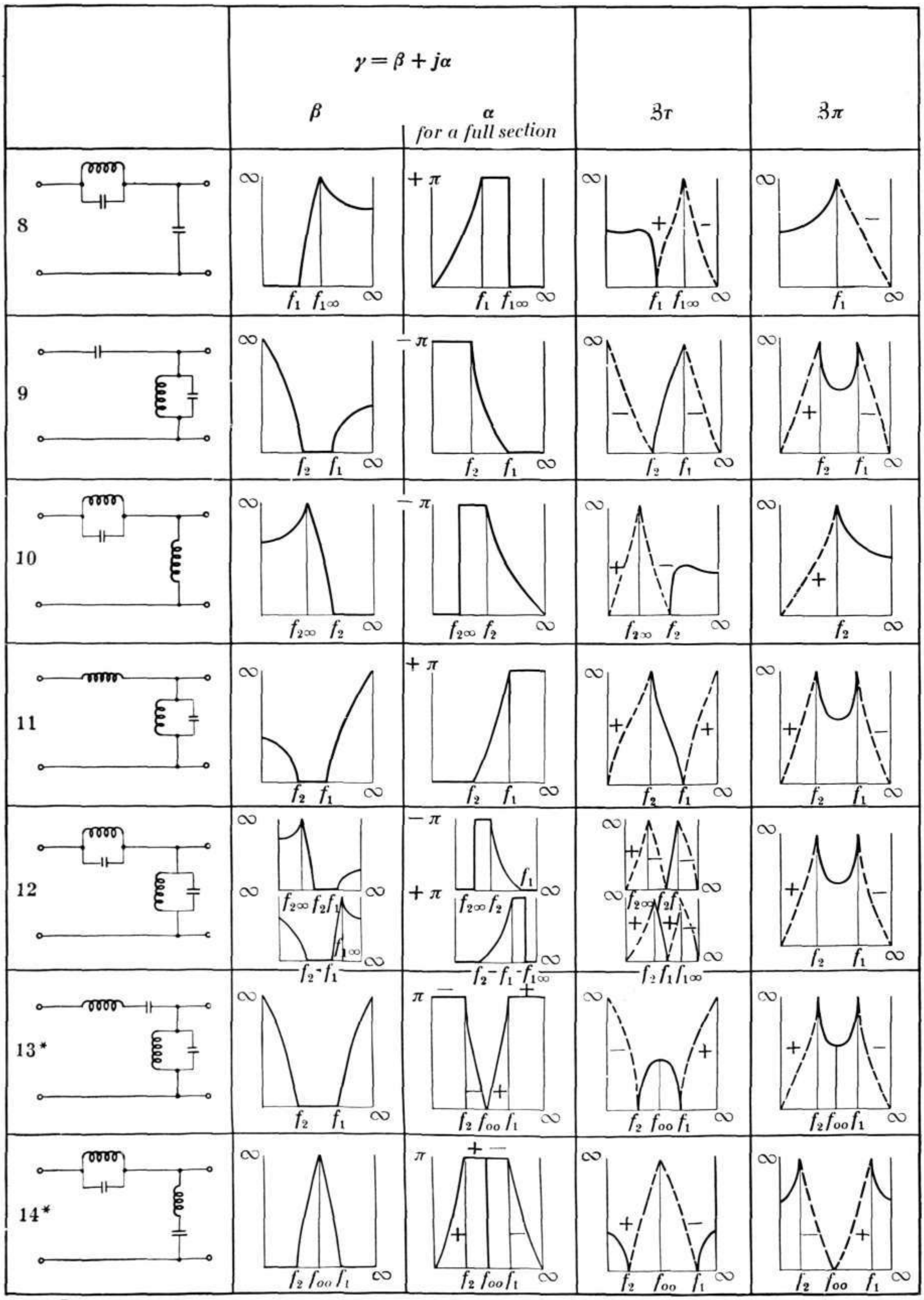
The filter propagation constant $\gamma = \beta + j\alpha$ and the two image impedances $\mathfrak{Z}'$ and $\mathfrak{Z}''$, or $\mathfrak{Z}_T$ or $\mathfrak{Z}_\pi$, are the quantities which characterize a certain type of filter. These quantities are determined exclusively by the composition of the impedances z_1 and z_2 forming the series and shunt arms respectively of the filter section.

As we have already mentioned, a filter is generally calculated on the simplifying assumption that there is no energy dissipation in either inductances or capacities, i. e. that no ohmic resistances enter into the filter arms. On this assumption, z_1 and z_2 consist of pure inductances or capacities, or else a series, a parallel or a series-parallel connexion of inductances and capacities. In Table II (p. 19 and 20) various different filter sections are plotted and the attenuation constant β, phase constant α, mid-series image impedance $\mathfrak{Z}_T$ and mid-shunt image impedance $\mathfrak{Z}_\pi$ are given.

TABLE II.



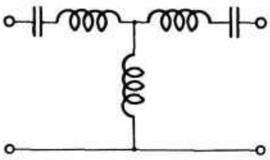
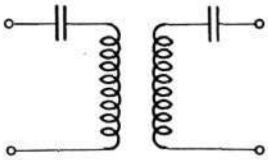
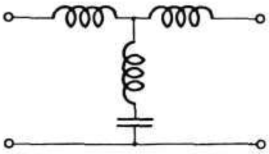
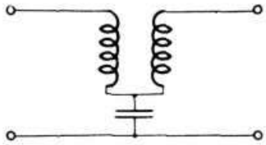
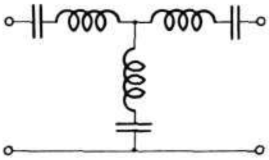
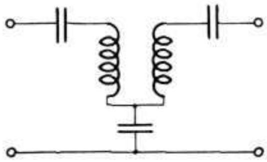
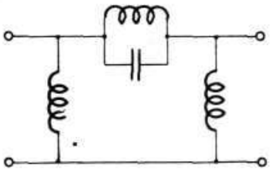
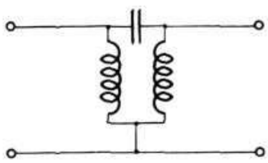
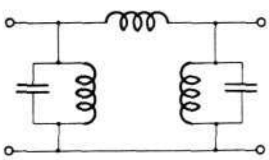
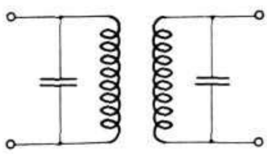
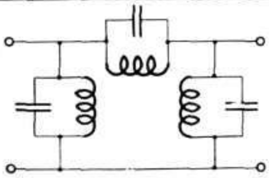
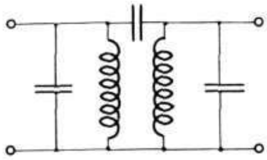
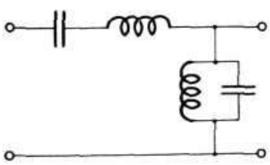
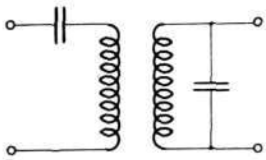
$\gamma = \beta + j\alpha$
β
α
for a full section
$\mathfrak{Z}_T$
$\mathfrak{Z}_\pi$
1
2
3
4
5
6
7



Tabell II b

* These curves only apply if the series and parallel circuits are tuned to the same frequency f_{oo}. Otherwise these filters will act as double band pass and double band suppress filters respectively.

TABLE III.

No.	*Fundamental type*	No.	*Filter type with mutual inductance*	*Remarks*
5		5 a		*Band pass filter*
6		6 a		*Low pass filter*
7		7 a		*Band pass filter*
10		10 a		*High pass filter*
11		11 a		*Band pass filter*
12		12 a		*Band pass filter*
13		13 a		*Band pass filter*

Tabell III

The symbolic method used for plotting these quantities is given by Zobel. This involves a shortening of the abscissa axis for the frequency and the ordinate axis for the attenuation constant and image impedance respectively, so that infinity will be at a finite distance from the origin. Real values of the image impedances are represented by solid lines, and imaginary values by dotted lines, while the signs + and − indicate positive and negative reactances respectively.

The *L*-sections included in the table contain at most four elements. Obviously there is nothing to prevent the use of more, though in that case the filters will be more complicated and costly. Those who desire full information on the properties of these filters are referred to a paper by Johnson and Shea.[1]

If we examine the table, we see that nos. 1, 6, and 8 are low pass filters, i. e. filters which pass low, but cut off high frequencies. Nos. 2, 4, and 10 are high pass filters, with properties directly opposite to those of the low pass filters. The others, with the exception of no. 14, are band pass filters, passing a band of frequencies and cutting off all the rest. Finally, No. 14 is a so-called band-suppress filter, i. e. a filter suppressing a certain band of frequencies while allowing all others to pass.

Table II further shows that filters nos. 1, 2, 13, and 14 have mid-series and mid-shunt image impedances which always make the product of $\mathfrak{Z}_T$ and $\mathfrak{Z}_\pi$ constant and $= k^2$. The proof of this will be given in the next chapter. These are so-called "constant k" filters, and are as we have already mentioned particularly widely used in telephony and telegraphy; we will therefore now discuss their properties in detail. We will, however, first deal with certain modified forms which are equivalent to the filters given above, which provide possibilities both of circumventing in certain cases the difficulties of design, and of introducing an impedance transformation.

As we know, it is always possible to substitute for a certain *T*- or star-network of inductances or capacities a corresponding *Π*- or *Δ*-network, also of inductances or capacities. We know further that the equivalent diagram for a transformer is a *T*- or *Π*-combination of inductances, and that therefore, conversely, a *T*- or *Π*-combination of inductances is equivalent to a transformer.

When connecting some of the filters given in Table II to whole *T*- or *Π*-sections, we find that in some cases the capacities, in others the inductances, will form *T*- or *Π*-combinations, which can therefore, according to what has been said above, be converted. Here we will consider primarily the cases which lead to the introduction of a transformer in the filter. This case is of great practical importance, as the introduction of a transformer, beyond the advantage in respect of transformation, in most cases also means a saving in coils. The coils being usually the most expensive parts of a filter, it will be a greater saving to combine a number of coils into a transformer than to change a condenser *T* into a condenser *Π*, or vice versa. Table III gives in the third column a number of such transformer-filters and the fundamental types from which these filters are deduced.

The filter type 6 a is identical with Campbell's frequency meter, which from a theoretical point of view is based on the mutual inductance being adjusted so as to make the peak of the filter attenuation curve coincide with the measured frequency (cf. Table II 6). Types 5 a, 11 a and 13 a[2] are filters of great practical importance. They are equivalent to connecting in cascade a given filter of the fundamental type without transformation, with a transformer designed to match as desired. In this case, however, the desired matching is obtained in the filter itself, and the transformer in the filter therefore serves a double purpose: it makes transformation possible, and its inductances form part of the filter, which gives a frequency-selective effect. The costs of attaining the desired result will therefore be considerably reduced. In Sweden such filters with transformation have been widely used, but not much as yet in other countries.

5 B. Computation of U_k for "constant k" filters.

We will now pass on to the calculation of the parameter U_k for so-called "constant k" filters, or filters of types 1, 2, 13, and 14 in table II, and 13 a in table III. Among these, we will dwell

[1] See Bibliography 16.

[2] Swedish patent application (M. Vos and H. Sterky) no. 965/28.

mainly on types 13 and 13 a, as nos. 1, 2, 14 are special cases of this band filter.

In fig. 11 a half section of this band filter is drawn with the accepted symbols for inductances and capacities. To compute U_k, we will first find simple expressions for the impedances z_1 and z_2 of the series and shunt arms.

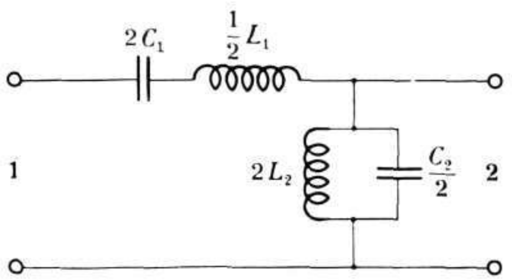


Fig. 11.

We get:

$$\left.\begin{aligned}\frac{1}{2}z_1 &= j\omega\frac{L_1}{2}+\frac{1}{j\omega 2 C_1}\\ \frac{1}{2z_2} &= \frac{1}{j\omega 2L_2}+j\omega\frac{C_2}{2}\end{aligned}\right\}$$

or

$$\left.\begin{aligned}z_1 &= j\omega L_1\left(1-\frac{1}{\omega^2 L_1 C_1}\right)\\ \frac{1}{z_2} &= j\omega C_2\left(1-\frac{1}{\omega^2 L_2 C_2}\right)\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (55\text{ a})$$

To satisfy (31)

$$\sqrt{z_1 z_2}=k$$

we must obviously have

$$L_1 C_1 = L_2 C_2 = \frac{1}{\omega_{oo}^2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (56)$$

which, physically, means that the series resonance circuit of the series arm and the parallel resonance circuit of the shunt arm must be tuned to the same angular frequency ω_{oo}.

(55 a) will then become

$$\left.\begin{aligned}z_1 &= j\omega L_1\left[1-\left(\frac{\omega_{oo}}{\omega}\right)^2\right]\\ \frac{1}{z_2} &= j\omega C_2\left[1-\left(\frac{\omega_{oo}}{\omega}\right)^2\right]\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (55\text{ b})$$

and we obtain

$$z_1 z_2 = \frac{L_1}{C_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

According to (30) we get further

$$\mathfrak{Z}_T\,\mathfrak{Z}_\pi = z_1 z_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

and the following properties of filters of the "constant k" group, can thus be given:

The product of the series and shunt arm impedances is equal to the product of the mid-series and mid-shunt image impedances of the filter and these products are constant and independent of the frequency, or

$$z_1 z_2 = \mathfrak{Z}_T\,\mathfrak{Z}_\pi = k^2 = \frac{L_1}{C_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (59)$$

We will now find the value of U_k from (28)

$$U_k = \frac{z_1}{4z_2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

whence, by substituting the values of z_1 and z_2 from (55 b)

$$U_k = -\frac{\omega^2 L_1 C_2}{4}\left[1-\left(\frac{\omega_{oo}}{\omega}\right)^2\right]^2 \quad \ldots\ldots\ldots\ldots (60)$$

To obtain U_k as a function of the angular frequency alone, we must obviously determine the product $L_1 C_2$. This is done by determining the cut-off frequencies ω_1 and ω_2 from (29 b).

$$\cosh\gamma = \cosh(\beta + j a) = 1 + 2U_k \quad \ldots\ldots (29\text{ b})$$

The position of the filter band is obtained by determining the limits of U_k corresponding to purely imaginary values of the angle γ, for if γ is purely imaginary, β, or the attenuation constant, must be zero. These limits are obtained for

$\cosh\gamma = \pm 1$ corresponding to

$U_k = 0$ and $U_k = -1$ respectively.

According to (60) $U_k = 0$ will give us

$$\omega = \omega_{oo}$$

and $U_k = -1$

$$\omega = \frac{1}{\sqrt{L_1 C_2}}\left\{\mp 1 \pm \sqrt{1+\omega_{oo}^2 L_1 C_2}\right\}$$

As ω must be >0, only two roots will be obtained, which will give us the two cut-off frequencies represented by ω_1 and ω_2 respectively.

We thus get:

$$\left.\begin{aligned}\omega_1 &= \frac{1}{\sqrt{L_1 C_2}}\left\{\sqrt{1+\omega_{oo}^2 L_1 C_2}+1\right\}\\ \omega_2 &= \frac{1}{\sqrt{L_1 C_2}}\left\{\sqrt{1+\omega_{oo}^2 L_1 C_2}-1\right\}\end{aligned}\right\} \quad \ldots\ldots (61\text{ a})$$

If we now introduce what is called the relative band width x, as the ratio of the opper cut-off frequency to the lower

$$x = \frac{\omega_1}{\omega_2}, \quad \ldots\ldots (62)$$

we obtain from (61 a)

$$\frac{L_1 C_2}{4} = \frac{x}{(x-1)^2}\frac{1}{\omega_{oo}^2}, \quad \ldots\ldots (61\,b)$$

which finally, on substitution in (60), gives

$$U_k = -\frac{x}{(x-1)^2}\left[\frac{\omega}{\omega_{oo}} - \frac{\omega_{oo}}{\omega}\right]^2 \ldots\ldots (63)$$

Wi find further from (61 a) that the resonance frequency ω_{oo} is the geometric mean of the two cut-off frequencies ω_1 and ω_2.

$$\omega_1 \omega_2 = \omega_{oo}^2 \quad \ldots\ldots (64)$$

If frequencies are substituted for angular frequencies in (63), this equation can be given another form, which is found in, among others, Zobel's work quoted above:

$$U_k = \frac{f_1 f_2}{(f_1 - f_2)^2}\left[\frac{f^2}{f_1 f_2} + \frac{f_1 f_2}{f^2} - 2\right] \ldots\ldots (63\,a)$$

This form is not, however, the most suitable for calculating U_k in practice. We will now give an account of the simplifications that can be made in the above expression. It should be noted that no approximations at all have been made in the deduction of the following formulæ.

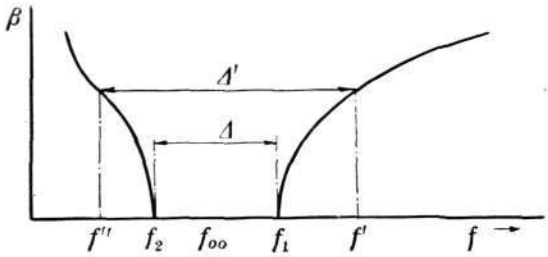


Fig. 12.

We introduce the symbols employed in fig. 12, where a curve has been plotted of the attenuation constant β against the frequency f. It should be pointed out that the following deduction of a simple formula for U_k would be equally valid if say, the effective attenuation, or the load impedance, or any other quantity depending on U_k, had been plotted as a function of the frequency. The following symbols are also introduced.

$$y = \frac{f'}{f''} \quad x = \frac{f_1}{f_2} \quad \ldots\ldots (65)$$
$$\triangle' = f' - f'' \quad \triangle = f_1 - f_2$$

The frequencies f' and f'' are so selected that

$$f' f'' = f_{oo}^2, \quad \ldots\ldots (66)$$

but the position of f' or f'' is arbitrary. Also, according to (65) and (66),

$$\left.\begin{aligned} \frac{f'}{f_{oo}} = \frac{f_{oo}}{f''} = \sqrt{y} \text{ and } \frac{f_{oo}}{f'} = \frac{f''}{f_{oo}} = \frac{1}{\sqrt{y}} \\ \frac{f_1}{f_{oo}} = \frac{f_{oo}}{f_2} = \sqrt{x} \text{ and } \frac{f_{oo}}{f_1} = \frac{f_2}{f_{oo}} = \frac{1}{\sqrt{x}} \end{aligned}\right\}$$

Now the expression

$$U_k = -\frac{x}{(x-1)^2}\left[\frac{f}{f_{oo}} - \frac{f_{oo}}{f}\right]^2 \quad \ldots\ldots (63)$$

is symmetrical with respect to f_{oo}, and according to it two frequencies, f' and f'', correspond to a given U_k. We can therefore write

$$U_k = -\frac{x}{y}\left(\frac{y-1}{x-1}\right)^2 = -\left(\frac{\sqrt{y} - \frac{1}{\sqrt{y}}}{\sqrt{x} - \frac{1}{\sqrt{x}}}\right)^2$$

or, multiplying top and bottom by f_{oo}^2 and with the help of (65),

$$\boxed{U_k = -\left(\frac{f' - f''}{f_1 - f_2}\right)^2 = -\left(\frac{\triangle'}{\triangle}\right)} \quad \ldots\ldots (63\,b)$$

The relation between the absolute band width Δ' and the quantity $\sqrt{y}$ which determines the position of the unknown frequencies f' and f'', we obtain from (65):

$$\frac{\triangle'}{f_{oo}} = \sqrt{y} - \frac{1}{\sqrt{y}} \quad \ldots\ldots (67)$$

If we replace $\frac{\triangle'}{f_{oo}}$ by p', we can solve $\sqrt{y}$ in the above equation and so get

$$\sqrt{y} = \frac{p'}{2} + \sqrt{1 + \frac{p'^2}{4}} \quad \ldots\ldots (68)$$

(the minus sign is discarded, $\sqrt{y}$ being > 1).

As we have shown above (Ch. 4), all calculations for "onstant k" filters are reduced to a calculation of U_k. By means of (63 b), (67), and (68) this quantity in its turn can be very easily determined for any "constant k" filter for which the cut-off frequencies f_1 and f_2, and with them the absolute band width $\Delta = f_1 - f_2$, and their geometric mean $f_{oo} = \sqrt{f_1 f_2}$ are given. These calculations will be even simpler if (68) is plotted as a curve (see fig. 13). A couple of examples showing the calculation of U_k for given band filters will illustrate the simplicity of this method.

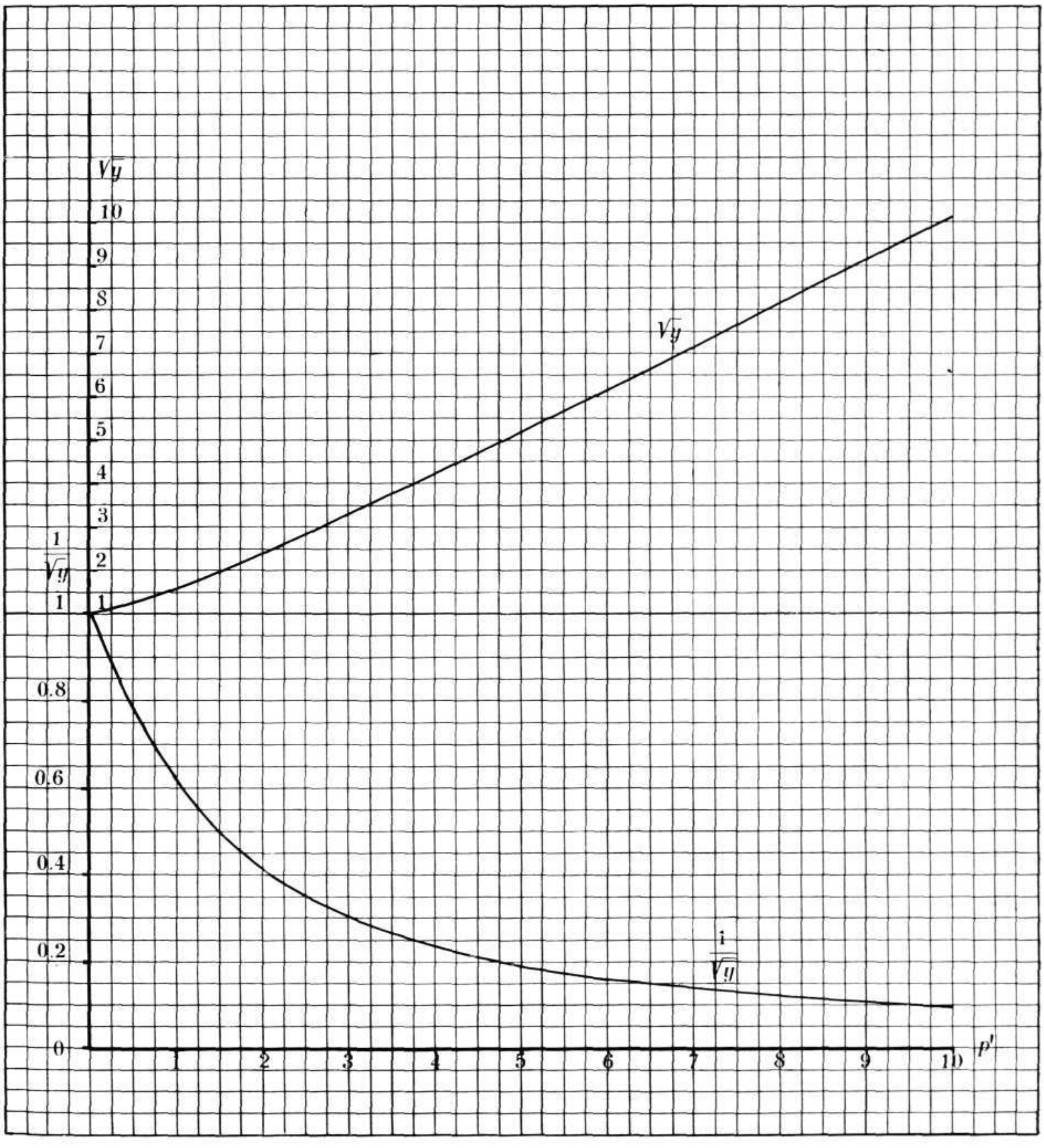


Fig. 13.

Example 1. *Given:* A band filter with the cut-off frequencies 12 750 and 9 750 cycles/sec.

Wanted: U_k for the frequencies 6 000, 11 500 and 15 000 cycles/sec. We first calculate $\Delta = f_1 - f_2 =$ $= 12\,750 - 9\,750 = 3\,000$ cycles/sec. and $f_{00} = \sqrt{f_1\,f_2} =$ $= \sqrt{12\,750 \cdot 9\,750} = 11\,150$ cycles/sec. after which the calculation follows the procedure given in Table IV below:

U_k is obtained not only for the frequencies given but also for the frequencies on the other side of their geometric mean f_{00} according to the formula $f'\,f'' = f_{00}^2$.

TABLE IV.

f'	f''	$f' = \frac{f_{oo}^2}{f''}$	$f'' = \frac{f_{oo}^2}{f'}$	$\triangle' = f' - f''$	$\frac{\triangle'}{\triangle}$	$U_k = -\left(\frac{\triangle'}{\triangle}\right)^2$
—	6000	20719	—	14719	4.906	— 24
11500	—	—	10810	690	0.230	— 0.0529
15000	—	—	8288	6712	2.237	— 5
Given.		Computed.				

Example 2. *Given:* A band filter of absolute width $\varDelta = 200$ cycles/sec. and geometric mean frequency $f_{oo} = 4\,600$ cycles/sec.

Wanted: U_k for varying band width $\varDelta'$.

TABLE V.

$\triangle'$	$p' = \frac{\triangle'}{f_{oo}}$	$\sqrt{y}$	$\frac{1}{\sqrt{y}}$	$f' = f_{oo}\sqrt{y}$	$f'' = \frac{f_{oo}}{\sqrt{y}}$	$\frac{\triangle'}{\triangle}$	$U_k = -\left(\frac{\triangle'}{\triangle}\right)^2$
100	0.0217	1.0104	0.9897	4650	4550	0.50	— 0.25
300	0.0652	1.033	0.9681	4752	4452	1.5	— 2.25
1200	0.261	1.139	0.8780	5239	4039	6.0	— 36.0
Assumed.	Computed.	From curve fig. 13.		Computed.			

As we see, the procedure is very simple. The accuracy will also be sufficient for all practical purposes, especially if the curve $\sqrt{y} = f(p')$ in fig. 13 is plotted on a large scale on mm. paper. When the values of U_k have been found these can be used for calculating all the quantities characteristic of a "constant k" filter, e. g. the filter propagation constant, the mid-series and mid-shunt impedances of the loaded filter, the effective attenuation, etc., from formulæ of which some have already been given and some will be deduced in the next chapter.

The filter types nos. 1, 2, and 14 in Table II and nos. 13 a in Table III are also "constant k" filters. To calculate the value of U_k for these filters we can either proceed in the same way as with the type already discussed (fig. 11) or else find it mathematically from (63 b) by taking the limits. By either of these methods the following expression will then be obtained for calculating U_k. (Table VI.)

TABLE VI.

Filter type no.	of table	$U_k =$
1	II	$-\left(\frac{f'}{f_1}\right)^2$
2	II	$-\left(\frac{f_2}{f''}\right)^2$
13	II	$\left.\right\} -\left(\frac{\triangle'}{\triangle}\right)^2$
13 a	III	
14	II	$-\left(\frac{\triangle}{\triangle'}\right)^2$.

6. Matching of filter image impedances.

In Ch. 4 A and B, "Load impedance and effective attenuation", the question of matching the *image impedance of the filter to external impedances* has been briefly touched upon. In this chapter it will be examined more closely and formulæ deduced for a new method called α-matching for matching filter image impedances with external impedances.

When matching image impedances, the general rule is that one ought to try to make the reflection losses as small as possible, as every reflection causes increased attenuation, and may besides give rise to disturbing subsidiary phenomena, e. g. increased crosstalk etc. When a number of filters of different types are to be connected in cascade, the rule is that only those can be connected together which have the same image impedances at the junctions. Thus, when connecting in cascade different types of filters from Table II or III, one must see that only such types are selected as have at least from one side an image impedance corresponding to the output image impedance of the previous section. The side having the right image impedance should then be connected to the previous filter. If there are several types with such image impedances, it will be the form of the attenuation curve and the consideration that it must be technically possible to manufacture the various inductances and capacities that will determine the choice of type.

The matching of image impedances of different types of filters when connecting them in cascade is thus a fairly simple matter. The problem of matching the image impedance of a particular type with the internal impedance of a generator or with a certain terminal load impedance is, however, considerably more difficult. If these impedances are complex, it is practically impossible to give any general rule for the matching. Fortunately, however, the filters used in telephony and wireless are generally connected between generators with purely ohmic internal impedances and loads consisting of ohmic resistances. Such, for instance, is the case of a filter connected between the anode circuit of an amplifying valve and a resistance in the grid circuit of the succeeding valve, as for example in fig. 14, which shows a band filter with the propagation constant Θ and image impedances $\mathfrak{Z}'$ and $\mathfrak{Z}''$ connected between a generator E of internal resistance R' and a load R''.

The two curves in fig. 14 shows how the image impedances $\mathfrak{Z}'$ and $\mathfrak{Z}''$ vary with the frequency for a "constant k" filter consisting of an odd number of half sections of type 13, Table II, according to the diagrammatic plan given for the same table. These curves indicate that the image im-

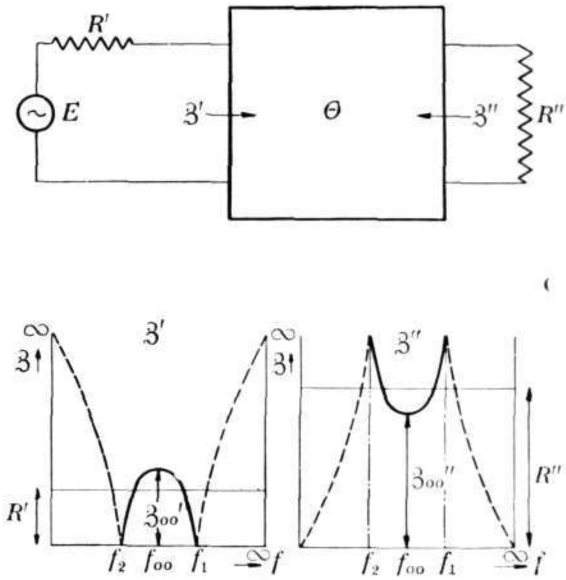
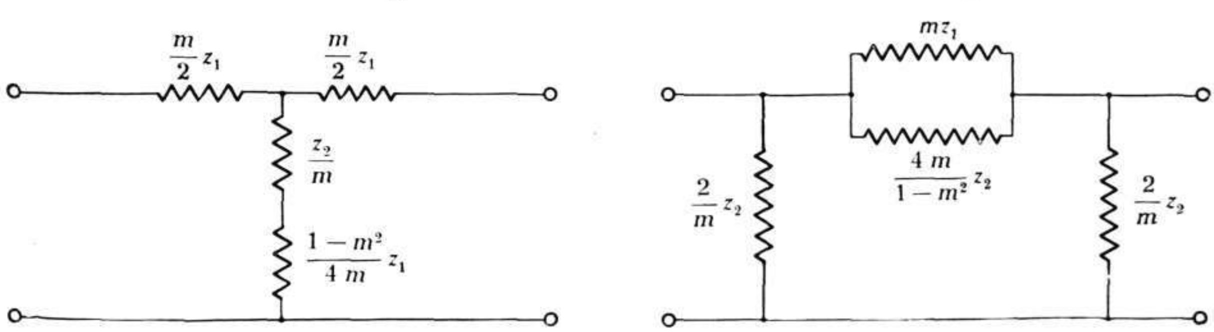


Fig. 14.

pedances within the band are real and thus of the same character as the external resistance. The image impedances within the band are, however, not constant resistances, but vary according to fig. 14 from infinity or zero at one of the cut-of frequencies to a constant value $\mathfrak{Z}_{oo}'$ or $\mathfrak{Z}_{oo}''$ at the geometric mean frequency and back to infinity or zero at the other cut-off frequency. Zobel gives a method of improving the matching within the band for "constant k" filters so that the reflection losses are reduced. This method, called "m-derivation", involves a change of the normal series and shunt arm impedances, z_1 and z_2 respectively, to values according to fig. 15. If the T- and Π-sections in fig. 15 are used as L-



T-section (m-type). *Π-section (m-type).*

Fig. 15.

sections, the mid-shunt image impedance of the m-derived T-section and the mid-series image impedance of the m-derived π-section will have favourable properties for matching to ohmic external resistances. The attenuation curves of filters m-derived in this way will in addition show high peaks for certain frequencies outside the filter bands—properties which are of great value, particularly, for instance, when certain frequencies have to be attenuated more thoroughly than other. To obtain these advantages, however, several new elements have to be introduced into the filters, e. g. in a band filter as in fig. 13, Table II, two inductances and two capacities. For more detailed information of the design and use of m-derived filters, the reader is referred to Zobel's works.

6 A. Effective Attenuation in a-Matching.

Another method will now be discussed for improving the matching in a filter band. As we have already mentioned (Ch. 4 Ba), filters are generally calculated so that $\mathfrak{Z}'$ and $\mathfrak{Z}''$ for *one* particular frequency—usually the geometric mean frecuency f_{oo}—coincide with the external resistances, or so that

$$\mathfrak{Z}_{oo}' = R' \text{ and } \mathfrak{Z}_{oo}'' = R''.$$

Formulæ for the effective attenuation with such matching have been given in ch. 4 B.

If now we look at fig. 14, we find that it would not be far-fetched to calculate the filters instead so that $\mathfrak{Z}'$ and $\mathfrak{Z}''$ for *two* frequencies coinside with the external resistances R' and R'', i. e. so that the straight lines R' and R'' resp. cut the curves of the filter's image impedances. At the geometric mean frecuency f_{oo} we then get

$$R' < \mathfrak{Z}_{oo}' \text{ and } R'' > \mathfrak{Z}_{oo}'',$$

if the filter is asymmetrical, which we assume, as being the most complicated case. We then put

$$\left.\begin{aligned} R' &= \frac{1}{a}\,\mathfrak{Z}_{oo}' \\ R'' &= a\,\mathfrak{Z}_{oo}'' \end{aligned}\right\} \quad (a > 1) \quad \dots\dots\dots\dots\dots\dots \text{(69)}$$

and have to determine the most favourable value of a in some typical cases. Such a-matching has been found to improve the form of the effective attenuation curve not only *inside* but also *outside* the band, while at the same time the reflection losses can be kept within reasonable limits. *We gain these advantages without having to introduce any new elements into the filters.*

In Ch. 4 B we have deduced the formula for the effective attenuation of an arbitrary filter having a propagation constant Θ

$$b_D = \log_e \frac{1}{2} \left| \left[\left(\sqrt{\frac{R''}{R'}} \sqrt{\frac{\mathfrak{Z}'}{\mathfrak{Z}''}} + \sqrt{\frac{R'}{R''}} \sqrt{\frac{\mathfrak{Z}''}{\mathfrak{Z}'}} \right) \cosh \Theta + \left(\frac{\sqrt{\mathfrak{Z}'\mathfrak{Z}''}}{\sqrt{R'R''}} + \frac{\sqrt{R'R''}}{\sqrt{\mathfrak{Z}'\mathfrak{Z}''}} \right) \sinh \Theta \right] \right| \quad (43)$$

This formula can be simplified for a "constant k"-filter, as according to (59)

$$\mathfrak{Z}' \cdot \mathfrak{Z}'' = k^2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(59)}$$

if we turn the filter so that $\mathfrak{Z}' = \mathfrak{Z}_T$ and $\mathfrak{Z}'' = \mathfrak{Z}_\pi$.

$\mathfrak{Z}_{oo}'$ and $\mathfrak{Z}_{oo}''$ being only particular values of $\mathfrak{Z}'$ and $\mathfrak{Z}''$ this must also be true for

$$\mathfrak{Z}_{oo}' \cdot \mathfrak{Z}_{oo}'' = k^2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(70)}$$

If we substitute in (43), (69) and (70) we get the effective attenuation for an *a-matched asymmetrical "constant k" filter:*

$$b_D = \log_e \frac{1}{2} \left| \left[\left(a \sqrt{\frac{\mathfrak{Z}_{oo}''}{\mathfrak{Z}_{oo}'}} \sqrt{\frac{\mathfrak{Z}'}{\mathfrak{Z}''}} + \frac{1}{a} \sqrt{\frac{\mathfrak{Z}_{oo}'}{\mathfrak{Z}_{oo}''}} \sqrt{\frac{\mathfrak{Z}''}{\mathfrak{Z}'}} \right) \cosh \Theta + 2 \sinh \Theta \right] \right|$$

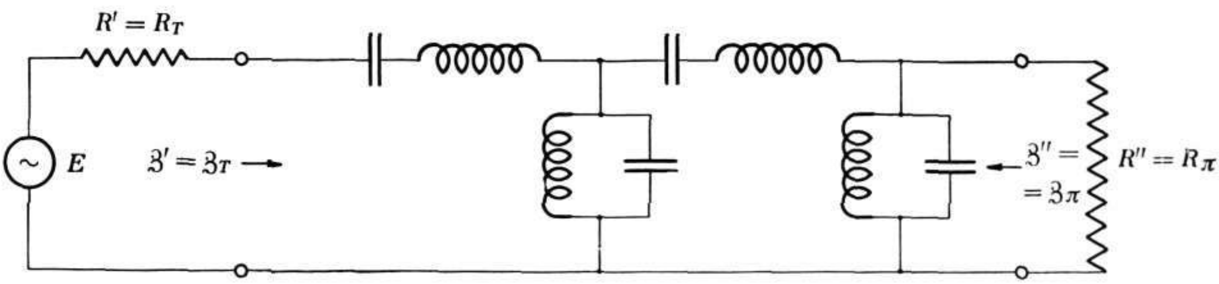
or if we introduce T and π image impedances:

$$\boxed{b_D = \log_e \frac{1}{2} \left| \left[\left(a \frac{\mathfrak{Z}_T}{\mathfrak{Z}_{Too}} + \frac{1}{a} \frac{\mathfrak{Z}_\pi}{\mathfrak{Z}_{oo\pi}} \right) \cosh \Theta + 2 \sinh \Theta \right] \right|} \quad (71)$$

6 Aa. Example of a "constant k" filter composed of three half-sections.

We will now continue the calculation of the effective attenuation for a special filter, and for that purpose will choose a "constant k" filter composed of three half sections, as in fig. 16.

If the propagation constant for a whole section "constant k" filter is γ, Θ will in this case be $\frac{3}{2}\gamma$, so that, leaving out some steps, we get, from (29 b) and (30)



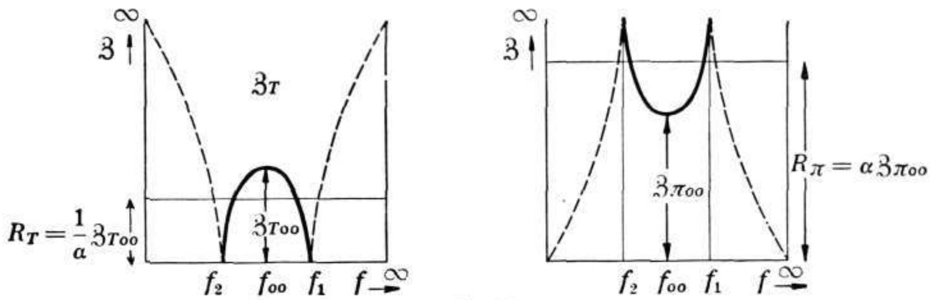


Fig. 16.

$$\left.\begin{aligned}
\sinh\Theta &= \sinh\frac{3}{2}\gamma = (3+4\,U_k)\sqrt{U_k}\\
\cosh\Theta &= \cosh\frac{3}{2}\gamma = (1+4\,U_k)\sqrt{1+U_k}\\
\frac{\mathfrak{Z}_T}{\mathfrak{Z}_{Too}} &= \sqrt{1+U_k}\\
\frac{\mathfrak{Z}_\pi}{\mathfrak{Z}_{\pi oo}} &= \frac{1}{\sqrt{1+U_k}},
\end{aligned}\right\}\quad(72)$$

which if substituted in (71) gives us

$$b_D = \log_e \frac{1}{2}\left|\left[\alpha(1+U_k)+\frac{1}{\alpha}\right]\left[1+4\,U_k\right]+2\left[3+4\,U_k\right]\sqrt{U_k}\right| \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (73\,\mathrm{a})$$

or if we want the absolute value, remembering that U_k is negative

$$b_D = \log_e \frac{1}{2}\sqrt{\left(\alpha^2+2+\frac{1}{\alpha^2}\right)+\left(10\,\alpha^2-18+\frac{8}{\alpha^2}\right)U_k+\left(33\,\alpha^2-48+\frac{16}{\alpha^2}\right)U_k^2+(40\,\alpha^2-32)\,U_k^3+16\,\alpha^2\,U_k^4} \quad (73\,\mathrm{b})$$

$$= \log_e \frac{1}{2}\sqrt{y}$$

When $\alpha=1$ this becomes

$$b_D = \log_e\sqrt{1+\frac{U_k^2}{4}+2\,U_k^3+4\,U_k^4} \quad\ldots\ldots\ (74)$$

It will now be of interest to examine the value of the expression γ for various values of U_k and find the positions of any maxima and minima there may be. The calculations required for this take rather a long time and are not very interesting, so the results alone are given here (Table VII).

If we see whether there is any value at which all the maxima and minima coincide, we find there is one at $\alpha = \frac{2}{\sqrt{3}}$. We also want a value of α, for which b_D and therefore also y will be the same for $U_k=0$ (f_{oo}) as for $U_k=-1$ (f_1 and f_2 respectively). This will occur when $\alpha=2$. Table VIII below gives values of U_k and b_D for various values of α and for the more important points.

The results are given graphically in figs. 17 and 18, of which the former shows the effective attenuation curves mainly outside the cut-off frequencies ($U_k<-1$) and the latter shows on a large scale a small part of the same curve within the band ($-1<U_k<0$). For the correct interpretation of the curves it should here be pointed out that a fairly good idea of the form of the effective attenuation curve of an α-matched band fil-

TABLE VII.

U_k	y	Remarks
0	$\left(a+\frac{1}{a}\right)^2$	Geometric mean frequency f_{oo}
-1	$4+\frac{9}{a^2}$	Cut-off frequencies f_1 and f_2
$\frac{1}{a^2}-1$	4	Minima
$-\frac{1}{4}$	4	Absolute minima
$\frac{1}{2a^2}-\frac{5}{8}$	$\frac{37}{16}+\frac{81}{256}a^2+\frac{27}{8a^2}-\frac{3}{a^4}+\frac{1}{a^6}$	Maxima

TABLE VIII.

For special values of U_k (independent of a)					
U_k	$b_D=\log_e\frac{1}{2}\sqrt{y}$ for $a=$				
	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$2\sqrt{2}$
0	0	0.010	0.060	0.223	0.46
$-\frac{1}{4}$	0	0	0	0	0
-1	0.589	0.492	0.378	0.223	0.132
For minima $U_k=\frac{1}{a^2}-1$					
$a=$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$2\sqrt{2}$
U_k	0	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{3}{4}$	$-\frac{7}{8}$
$b_D=\log_e\frac{1}{2}\sqrt{y}$	0	0	0	0	0
For maxima $U_k=\frac{1}{2a^2}-\frac{5}{8}$					
$a=$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$2\sqrt{2}$
U_k	$-\frac{1}{8}$	$-\frac{1}{4}$	$-\frac{3}{8}$	$-\frac{1}{2}$	$-\frac{9}{16}$
$b_D=\log_e\frac{1}{2}\sqrt{y}$	0.0005	0	0.001	0.03	0.108

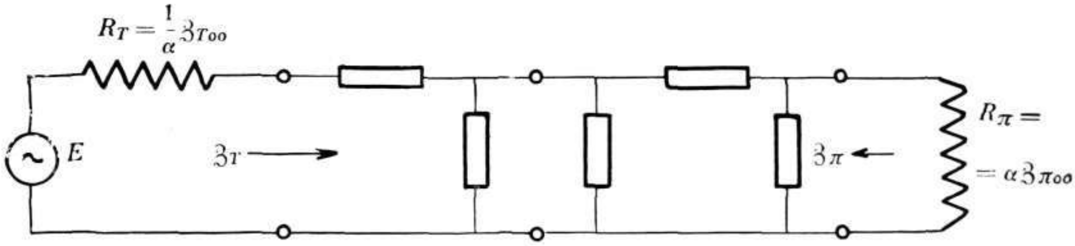
ter can be obtained by imagining the curves in figs. 17 and 18 extended to the right by their images. The cut off frequencies f_2 and f_1 will then be to the left and right of $U_k=0$ at the points where $U_k=-1$. The abscissa axis must then be conceived as having a frequency scale related to the U_k scale according to (68) or fig. 13. Also the curves for the effective attenuation are only applicable on the assumption that the filters have no losses.

From fig. 17 we first find that below a certain value of U_k all a-matched filters ($a>1$) will be better from the point of view of the attenuation than the normal filter where $a=1$. *The effective attenuation of an a-matched filter ($a>1$) slightly outside the cut-off frequencies is thus greater than in the corresponding normally matched filter ($a=1$).* The reason for this is that the reflections due to the a-matching will cause an increase in the effective attenuation: the greater a, the greater the reflections.

Within the band, for $-1<U_k<0$ the effective attenuation depends very much on the value of a. Fig. 18 shows that the effective attenuation in a filter where $a=1$ is relatively large at the edges of the band. *The greater a is, the lower will be the effective attenuation at the cut-off frequencies, at the same time increasing at the geometric mean frequency.* This fact gives us a chance of obtaining a more rectangular effective attenuation curve for a filter of this kind.

How large then should a be made? To answer this we will consider fig. 19 which gives the effective attenuation curves for a three-half-sections "constant k"-filter with $a=1$ (curve 1) and $a>1$ (curve 2), in both cases without losses, and for $a=1$ with losses (curve 3).

If allowance is made for the losses in coils and condensers the theoretically effective attenuation curve 1 for $a=1$ will become curve 3. As we see, the losses cause an increase of attenuation within the band, but are less important outside. Both curves 1 and 3 show that, on account of the reflections for which we have just allowed in calculating the effective attenuation, the band will be narrower than that determined by the cut-off frequencies f_1 and f_2. If a is made greater than


$R_T = \frac{1}{\alpha} Z_{T\infty}$
E
Z_T
Z_π
$R_\pi =$
$= \alpha Z_{\pi\infty}$

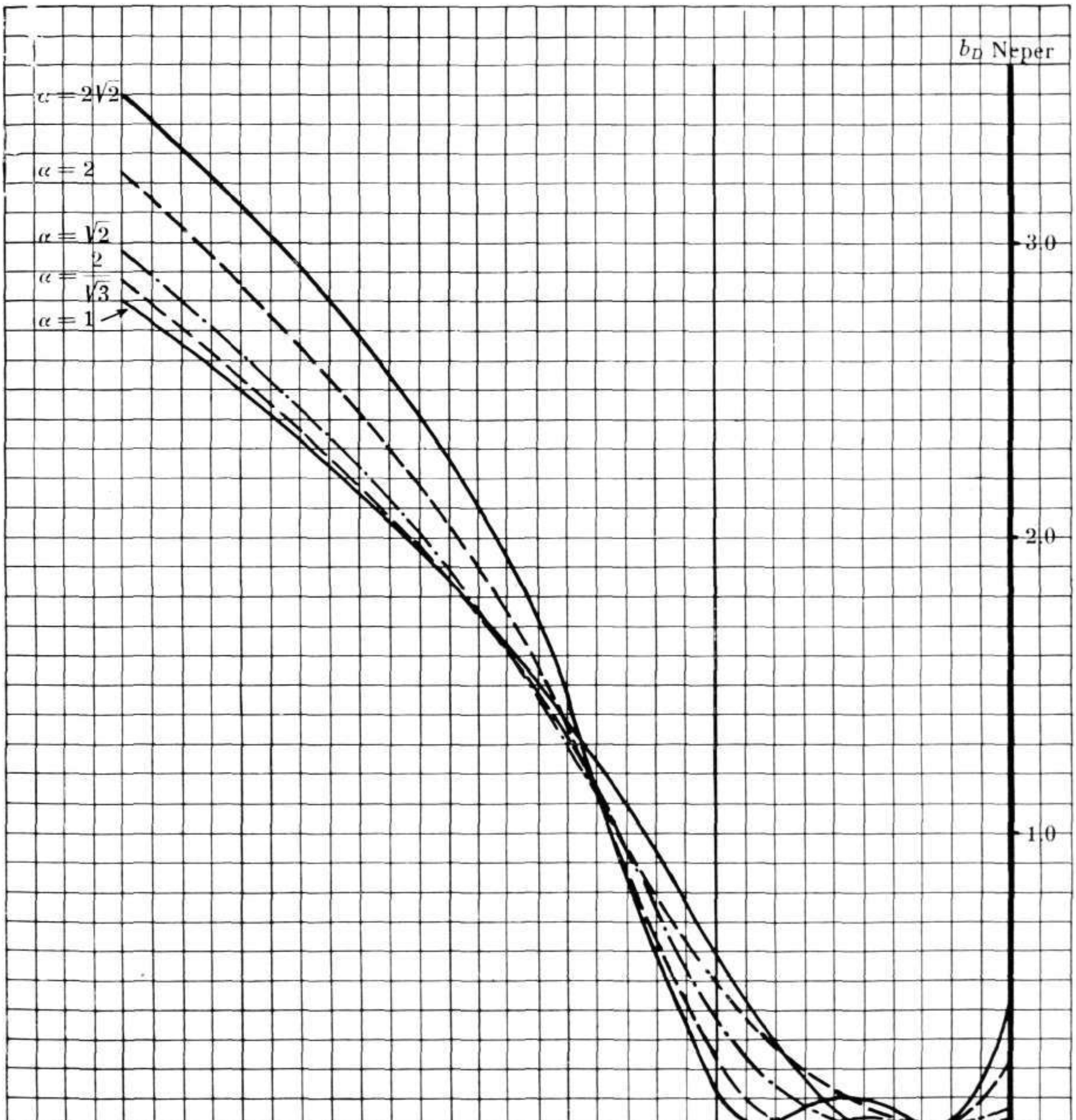

b_D Neper
$\alpha = 2\sqrt{2}$
$\alpha = 2$
$\alpha = \sqrt{2}$
$\alpha = \frac{2}{\sqrt{3}}$
$\alpha = 1$
3.0
2.0
1.0
U_k
— 3
— 2
— 1
— 0.5
0

Fig. 17.

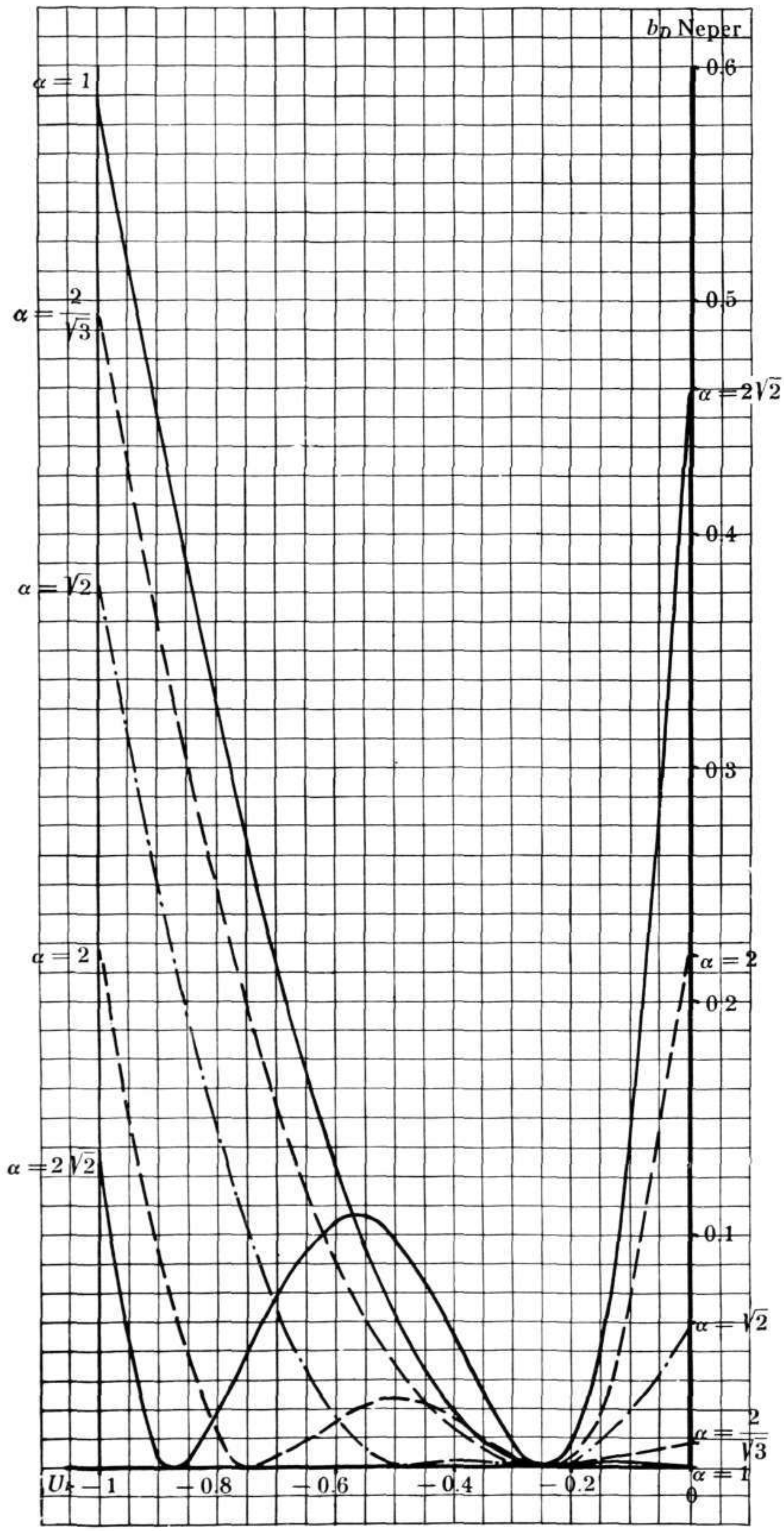


Fig. 18.

1, the band will be widened, as can be seen by comparing curves 1 and 2. If allowance is made for the losses in a filter where $a>1$ also, an effective attenuation will be obtained as in curve 4, fig. 19. This shows an apparently increased band width, owing to the attenuation being reduced at the cut-off frequencies and increased in the middle.

The ideal shape for the effective attenuation curve is a rectangle open at the top. To obtain a curve approaching this, a should be determined for the filter *with losses,* so that b_D will be the same for f_1, f_{oo} and f_2. Such a calculation will, however, be very complicated, and before that is done the following procedure will serve the purpose with sufficient accuracy. a is determined for the filter *without losses* so that the effective attenuation at f_{oo} ($U_k=0$) will be equal to that at the cut-off frequencies ($U_k=-1$). For the filter in question this calculation will give us $a=2$. Measurements of calculated and manufactured filters also show this value of a to be suitable. If there is any reason for suspecting that reflections *within* the band will cause disturbances, a smaller value of a should, however, be chosen. (cf. ch. 6 C). For $a>1$ an improvement of the effective attenuation is always obtained in the direction of the ideal rectangular shape.

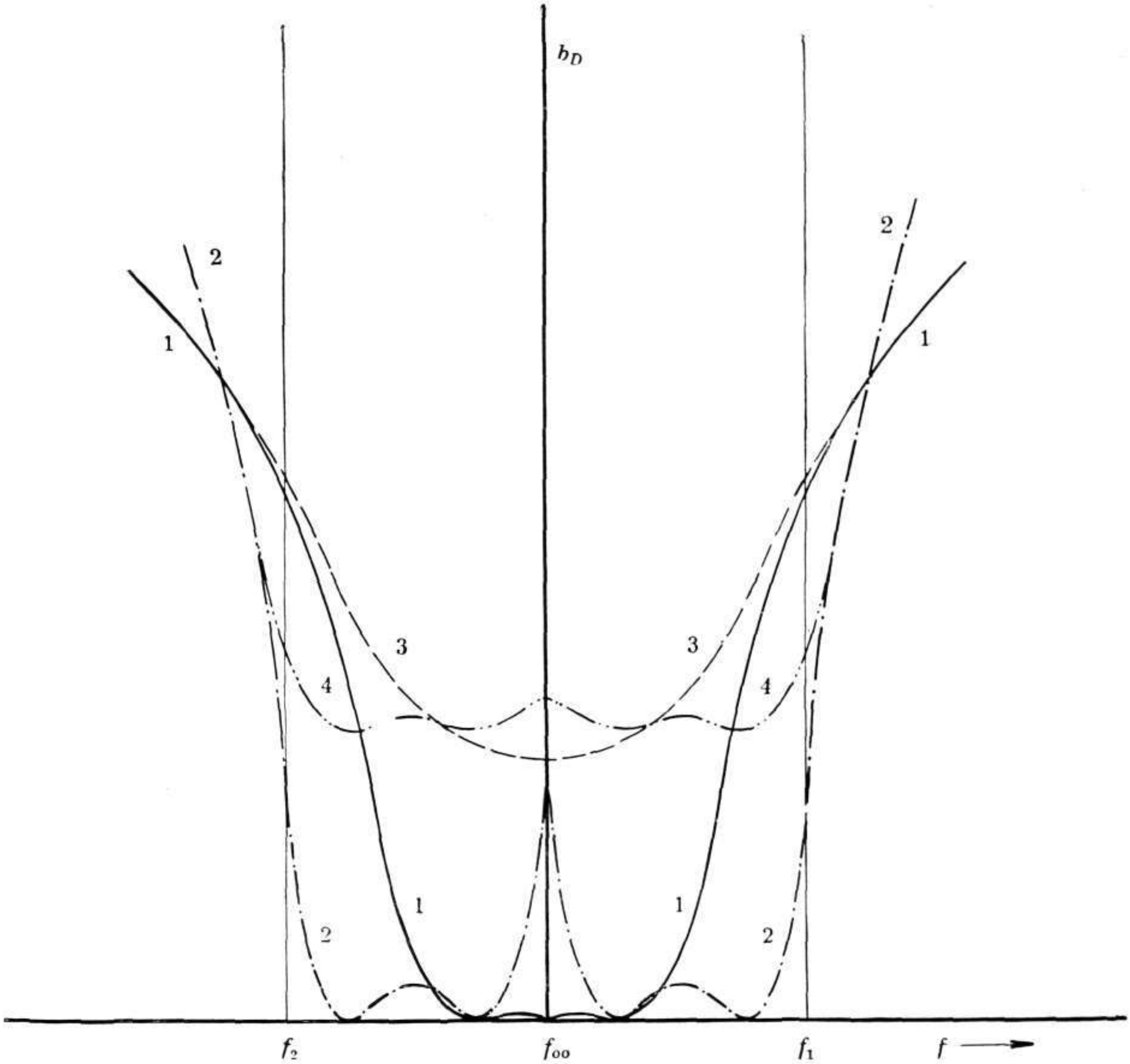


Fig. 19.

6 A b. Effective attenuation in α-matched symmetrical or asymmetrical "constant k" filters.

In the preceding chapter we have deduced a general formula for the effective attenuation in an asymmetrical α-matched "constant k" filter. We will now deduce the corresponding formula for a symmetrical filter of the same kind (fig. 20).

We assume both $\mathfrak{Z}'$ and $\mathfrak{Z}''$ to be of W-type, but that they are differentiated by a constant factor n^2 (cf. 44)

$$\mathfrak{Z}' = n^2 \mathfrak{Z}'' \quad \ldots\ldots (44)$$

In addition we introduce an α-matching

$$\left.\begin{aligned} R' &= \frac{1}{a} \mathfrak{Z}_{oo}' \\ R'' &= \frac{1}{a} \mathfrak{Z}_{oo}'' \end{aligned}\right\} \quad \ldots\ldots (75)$$

According to (44 a) we also have

$$\mathfrak{Z}_{oo}' = n^2 \mathfrak{Z}_{oo}''$$

and from this we get:

$$\left.\begin{aligned} \frac{R'}{\mathfrak{Z}'} &= \frac{R''}{\mathfrak{Z}''} \\ R' R'' &= \frac{1}{a^2} \mathfrak{Z}' \mathfrak{Z}'' \left(\frac{\mathfrak{Z}_{oo}'}{\mathfrak{Z}'}\right)^2 \end{aligned}\right\} \quad \ldots\ldots (76)$$

Substituting (76) in the general expression for the effective attenuation b_D according to (43)

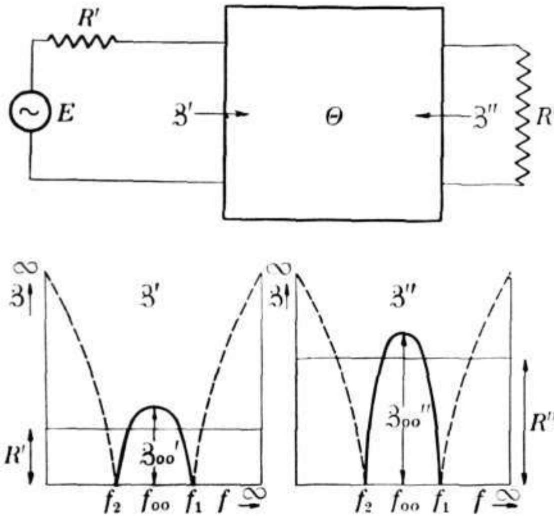


Fig. 20.

gives us

$$b_D = \log_e \frac{1}{2}\left[2\cosh\Theta + \left(a\frac{\mathfrak{Z}'}{\mathfrak{Z}_{oo}'} + \frac{1}{a}\frac{\mathfrak{Z}_{oo}'}{\mathfrak{Z}'}\right)\sinh\Theta\right], \quad (77)$$

or, if we introduce the mid-series impedance $\mathfrak{Z}_T$ instead of $\mathfrak{Z}'$

$$b_D = \log_e \frac{1}{2}\left[2\cosh\Theta + \left(a\frac{\mathfrak{Z}_T}{\mathfrak{Z}_{Too}} + \frac{1}{2}\frac{\mathfrak{Z}_{Too}}{\mathfrak{Z}_T}\right)\sinh\Theta\right] \quad (78)$$

which is the general expression for the effective attenuation in an *a-matched symmetrical "constant k" filter.*

It will be appropriate to point out here that *both (71) and (78) apply even if we exchange the mid-series image impedances for the mid-shunt image impedances,* which can easily be verified if we remember that in a "constant k" filter we have

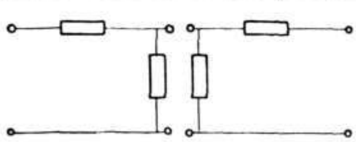
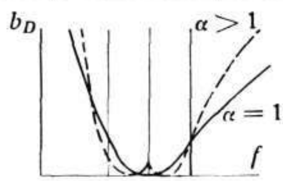
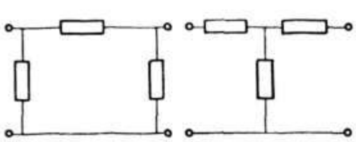
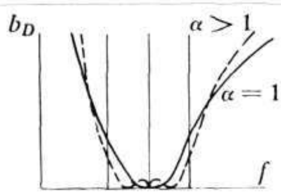
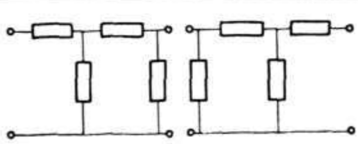
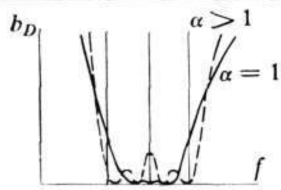
$$\frac{\mathfrak{Z}_T}{\mathfrak{Z}_{Too}} = \frac{\mathfrak{Z}_{\pi oo}}{\mathfrak{Z}_\pi} \quad \ldots\ldots \quad (79)$$

and that, when exchanging image impedances, a must be replaced by $\frac{1}{a}$.

The two formulæ (71) and (78) can be used for calculating the effective attenuation of any a-matched "constant k" filter consisting of an arbitrary number of half sections. In the following table IX are given the results obtained by applying these formulæ to "constant k" band pass filters of one, two, and three half-sections.

The formulæ for calculating the effective attenuation indicate that the degree of the function $y = f(U_k)$ under the root sign rises by one for each half section added. For each degree of $y = f(U_k)$ we find a maximum or minimum in

TABLE IX.

No. of ½ sections in the filter	*Formula for computing the effective attenuation b_D nepers*	*Effective attenuation curve*
 ½-section.	$b_D = \log_e \frac{1}{2}\sqrt{\left(a+\frac{1}{a}\right)^2 + 2\,(a^2-1)\,U_k + a^2 U_k^2}$	
 ²/₂-sections.	$b_D = \log_e \sqrt{1 - \left(a-\frac{1}{a}\right)^2 U_k + 2\,(1-a^2)\,U_k^2 - a^2 U_k^3}$	
 ³/₂-sections.	$b_D = \log_e \frac{1}{2}\sqrt{\left(a+\frac{1}{a}\right)^2 + \left(10\,a^2 - 18 + \frac{8}{a^2}\right)U_k + \left(33\,a^2 - 48 + \frac{16}{a^2}\right)U_k^2 + (40\,a^2 - 32)\,U_k^3 + 16a^2 U_k^4}$	

the effective attenuation curve on either side of the geometric mean frequency.

The formulæ are also applicable to high pass, low pass, and band-suppress filters as long as values for U_k are introduced as in Table VI.

6 B. Load impedance of α-matched filters.

The formulæ deduced in the preceding section have provided us with a basis for calculating the effective attenuation of any α-matched "constant k" filter. The load impedance of such a filter must, however, often be known before one is able to determine how practical and suitable it is. We will therefore find, on the basis of the general formulæ given in Ch. 4 A, general expressions for the load impedance of α-matched "constant k"-filters and work out an example of a half section of such a filter.

According to (35) and (36) we have

$$\left.\begin{aligned} \mathfrak{Z}_B' &= \mathfrak{Z}' \operatorname{tgh}(\Theta + \sigma'') \\ \mathfrak{Z}_B'' &= \mathfrak{Z}'' \operatorname{tgh}(\Theta + \sigma') \end{aligned}\right\} \quad \ldots\ldots\ldots (35)$$

where

$$\left.\begin{aligned} \operatorname{tgh} \sigma' &= \frac{R'}{\mathfrak{Z}'} \\ \operatorname{tgh} \sigma'' &= \frac{R''}{\mathfrak{Z}''} \end{aligned}\right\} \quad \ldots\ldots\ldots (36)$$

As in the foregoing chapters we identify $\mathfrak{Z}'$ with $\mathfrak{Z}_T$ and $\mathfrak{Z}''$ with $\mathfrak{Z}_\pi$ and further introduce according to (69)

$$\left.\begin{aligned} R' &= \frac{1}{\alpha}\mathfrak{Z}'_{oo} = \frac{1}{\alpha}\mathfrak{Z}_{Too} \\ R'' &= \alpha\,\mathfrak{Z}''_{oo} = \alpha\,\mathfrak{Z}_{\pi oo} \end{aligned}\right\} \quad \ldots\ldots\ldots (69)$$

We then get

$$\left.\begin{aligned} \operatorname{tgh} \sigma' &= \frac{1}{\alpha}\frac{\mathfrak{Z}_{Too}}{\mathfrak{Z}_T} \\ \operatorname{tgh} \sigma'' &= \alpha\frac{\mathfrak{Z}_{\pi oo}}{\mathfrak{Z}_\pi} \end{aligned}\right\} \quad \ldots\ldots\ldots (80)$$

and, from (35),

$$\left.\begin{aligned} \mathfrak{Z}_{BT} &= \mathfrak{Z}_T \frac{\operatorname{tgh}\Theta + \alpha\dfrac{\mathfrak{Z}_{\pi oo}}{\mathfrak{Z}_\pi}}{1 + \alpha\dfrac{\mathfrak{Z}_{\pi oo}}{\mathfrak{Z}_\pi}\operatorname{tgh}\Theta} \\ \mathfrak{Z}_{B\pi} &= \mathfrak{Z}_\pi \frac{\operatorname{tgh}\Theta + \dfrac{1}{\alpha}\dfrac{\mathfrak{Z}_{Too}}{\mathfrak{Z}_T}}{1 + \dfrac{1}{\alpha}\dfrac{\mathfrak{Z}_{Too}}{\mathfrak{Z}_T}\operatorname{tgh}\Theta} \end{aligned}\right\} \quad \ldots\ldots\ldots (81\text{ a})$$

and finally, with the aid of (72), the load impedance of an α-matched "constant k"-filter.

$$\left.\begin{aligned} \mathfrak{Z}_{BT} &= \mathfrak{Z}_T \frac{\operatorname{tgh}\Theta + \alpha\sqrt{1+U_k}}{1 + \alpha\sqrt{1+U_k}\operatorname{tgh}\Theta} \\ \mathfrak{Z}_{B\pi} &= \mathfrak{Z}_\pi \frac{\operatorname{tgh}\Theta + \dfrac{1}{\alpha\sqrt{1+U_k}}}{1 + \dfrac{1}{\alpha\sqrt{1+U_k}}\operatorname{tgh}\Theta} \end{aligned}\right\} \quad \ldots\ldots\ldots (81\text{ b})$$

In the above equations tgh Θ can be expressed in terms of U_k if the number of sections forming the filter in question is known. This generally simplifies the expression considerably.

As an example we will apply (81 b) to filter section 13 a, Table III, connected as in fig. 21.

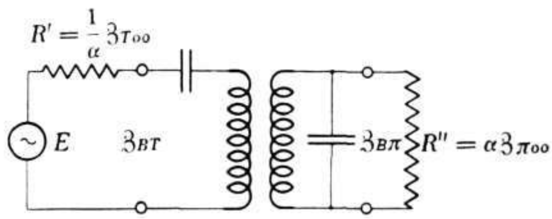


Fig. 21.

This filter section is equivalent to a half section "constant k" filter and according to (30) (cf. also (72)) we thus have

$$\left.\begin{aligned} \mathfrak{Z}_T &= \mathfrak{Z}_{Too}\sqrt{1+U_k} \\ \mathfrak{Z}_\pi &= \frac{\mathfrak{Z}_{\pi oo}}{\sqrt{1+U_k}} \\ \sinh\Theta &= \sinh\frac{\gamma}{2} = \sqrt{U_k} \\ \cosh\Theta &= \cosh\frac{\gamma}{2} = \sqrt{1+U_k} \end{aligned}\right\} \quad \ldots\ldots\ldots (82)$$

from which $\operatorname{tgh}\Theta = \operatorname{tgh}\frac{\gamma}{2}$ can be calculated. The value thus obtained is substituted in (81 b), which when reduced to its simplest terms becomes

$$\left.\begin{aligned} \mathfrak{Z}_{BT} &= \mathfrak{Z}_{Too}\frac{\alpha(1+U_k) + \sqrt{U_k}}{1 + \alpha\sqrt{U_k}} \\ \mathfrak{Z}_{B\pi} &= \mathfrak{Z}_{\pi oo}\frac{1 + \alpha\sqrt{U_k}}{\alpha(1+U_k) + \sqrt{U_k}} \end{aligned}\right\} \quad \ldots\ldots\ldots (83)$$

Here also we find what we expected, namely, that the product of the two load impedances $\mathfrak{Z}_{BT}$ and $\mathfrak{Z}_{B\pi}$ is constant, or that

$$\mathfrak{Z}_{BT} \cdot \mathfrak{Z}_{B\pi} = \mathfrak{Z}_{Too} \cdot \mathfrak{Z}_{\pi oo} = k^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (84)$$

which is always the case in "constant k"-filters.

In (83) U_k is always negative, and hence the load impedance will be complex. We therefore put

$$\frac{\alpha (1 + U_k) + \sqrt{U_k}}{1 + \alpha \sqrt{U_k}} = a + jb \quad \ldots\ldots\ldots\ldots\ldots \quad (85)$$

and solve a and b, which become:

$$\left. \begin{aligned} a &= \frac{\alpha}{1 - \alpha^2 U_k} \\ b &= \pm \sqrt{-U_k}\, \frac{1 - \alpha^2 (1 + U_k)}{1 - \alpha^2 U_k} \end{aligned} \right\} \quad \ldots\ldots\ldots \quad (86\,\text{a})$$

which when $\alpha = 1$, become

$$\left. \begin{aligned} a &= \frac{1}{1 - U_k} \\ b &= \pm \frac{-U_k \sqrt{-U_k}}{1 - U_k} \end{aligned} \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (86\,\text{b})$$

The signs + and − for b mean that the imaginary component may be a positive or negative reactance on different sides of the geometric mean frequency, as will be seen from Tables II and III.

From this we can then calculate the impedance for $\mathfrak{Z}_{BT}$ and the admittance $Y_{B\pi}$ for $\mathfrak{Z}_{B\pi}$ according to the following formulæ:

$$\left. \begin{aligned} \mathfrak{Z}_{BT} &= \mathfrak{Z}_{Too} (a + jb) \\ \mathfrak{Z}_{B\pi} &= \mathfrak{Z}_{\pi oo} \frac{1}{(a + jb)} \\ Y_{B\pi} &= Y_{\pi oo} (a + jb) \end{aligned} \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (87)$$

An examination of (86) with regard to maxima and minima and the limit values when $U_k = 0$ and $U_k = -1$ respectively gives the following results (Table X).

The results are given graphically in fig. 22, where a and b are plotted as functions of $-U_k$ for $\alpha = 1$, $\sqrt{2}$, and 2 respectively. The real component a is always positive, while the imaginary component b, as we have already mentioned, can be either positive or negative, which means that the reactance can be either positive or negative. If, for instance, $\mathfrak{Z}_{BT}$ is plotted as a function of the frequency instead of U_k in a band filter, we find that a varies symmetrically about the geometric mean frequency f_{oo}, while b passes through zero at this frequency and at the two frequencies (fig. 23), determined by the α-matching.

Figs. 22 and 23 show that at normal matching ($\alpha = 1$) the mid-series load impedance $\mathfrak{Z}_{BT}$ is equal to the external resistance, e. g. the internal impedance $\mathfrak{Z}_{Too}$ of the generator where $U_k = 0 (f_{oo})$, and that its real component is continuously approaching zero while its imaginary component is approaching infinity for a falling or rising frequency. When $\alpha > 1$ the load impedance at $U_k = 0$ (f_{oo}) is also real and equal to $\alpha\, \mathfrak{Z}_{Too}$, which is thus larger than in the preceding case. For frequencies lower or higher than f_{oo}, the real part of this load impedance will rapidly diminish, while its imaginary component first rises and becomes positive (or negative), then passes through zero, and subsequently grows continuously, reaching very high negative (or positive) values. The real part becomes $\frac{1}{\alpha} \cdot \mathfrak{Z}_{Too}$, *i. e.* equal to the external

TABLE X.

U_k	a	b	Remarks
0	α	0	Geometric mean frequency f_{oo}
-1	$\frac{\alpha}{1 + \alpha^2}$	$\frac{1}{1 + \alpha^2}$	Cut-off frequency f_1 and f_2
$\frac{1}{\alpha^2} - 1$	$\frac{1}{\alpha}$	0	zero value of b
$\sqrt{\frac{2}{\alpha^2} + \frac{1}{4}} - \left(\frac{1}{\alpha^2} + \frac{1}{2}\right)$	$\frac{1}{2}\sqrt{2 + \frac{\alpha^2}{4}} + \frac{\alpha}{4}$	$\frac{\alpha^2}{2}\sqrt{\sqrt{\frac{2}{\alpha^2} + \frac{1}{4}} - \left(\frac{1}{\alpha^2} + \frac{1}{2}\right) \times} \times \left(\frac{2}{\alpha^2} - \frac{1}{2} - \sqrt{\frac{2}{\alpha^2} + \frac{1}{4}}\right)$	minimum value of b

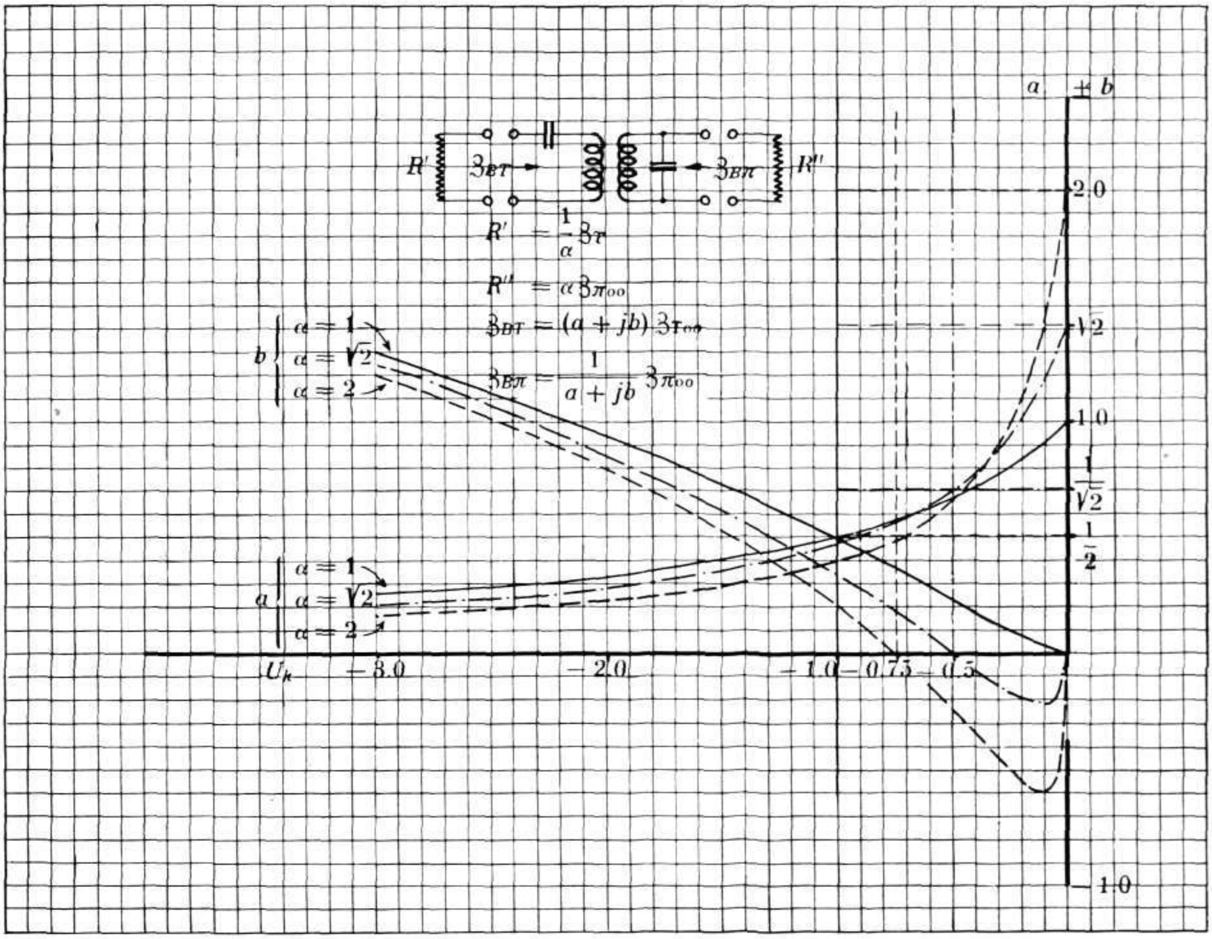


Fig. 22.

resistance, at the same time as the imaginary part becomes zero. Here, then the filter is matched with the external resistance, the reflections disappear and the effective attenuation becomes zero, as can be seen by comparison with Table XI.

It should be observed that the curves in fig. 23 for the load impedance are continuous. In Tables II and III this does not seem to be the case for the filter image impedances, but this is only apparent and due to the symbolic method of plotting (cf. p. 22).

The load impedance of a-matched filters may be calculated from (35) or (81) for various numbers of half sections. In each case we then get curves corresponding to those of fig. 23. A common characteristic of these curves is that the real component of the load impedance is equal to the external resistance for those values of U_k which make the imaginary component zero. Thus here the matching is ideal and the effective attenuation zero.

6 C. Reflections in a-matched filters.

The formulæ deduced in the last chapter for the load impedance may be used for finding the reflection coefficient ε. According to the definition in ch. 4 A.

$$\varepsilon' = \frac{\mathfrak{Z}_B' - R'}{\mathfrak{Z}_B' + R'} \quad \text{or} \quad \varepsilon'' = \frac{\mathfrak{Z}_B'' - R''}{\mathfrak{Z}_B'' + R''}$$

We have now, in the same way as in the foregoing chapter,

$$\left.\begin{aligned}\mathfrak{Z}_{BT} = \mathfrak{Z}_B' = \mathfrak{Z}_T \operatorname{tgh}(\Theta + \sigma'')\\ \mathfrak{Z}_{B\pi} = \mathfrak{Z}_B'' = \mathfrak{Z}_\pi \operatorname{tgh}(\Theta + \sigma')\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (35)$$

and

$$\left.\begin{aligned}R' = \frac{1}{a}\,\mathfrak{Z}_{Too}\\ R'' = a\,\mathfrak{Z}_{\pi oo}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (69)$$

from which we obtain the reflection coefficients ε_T and ε_π respectively

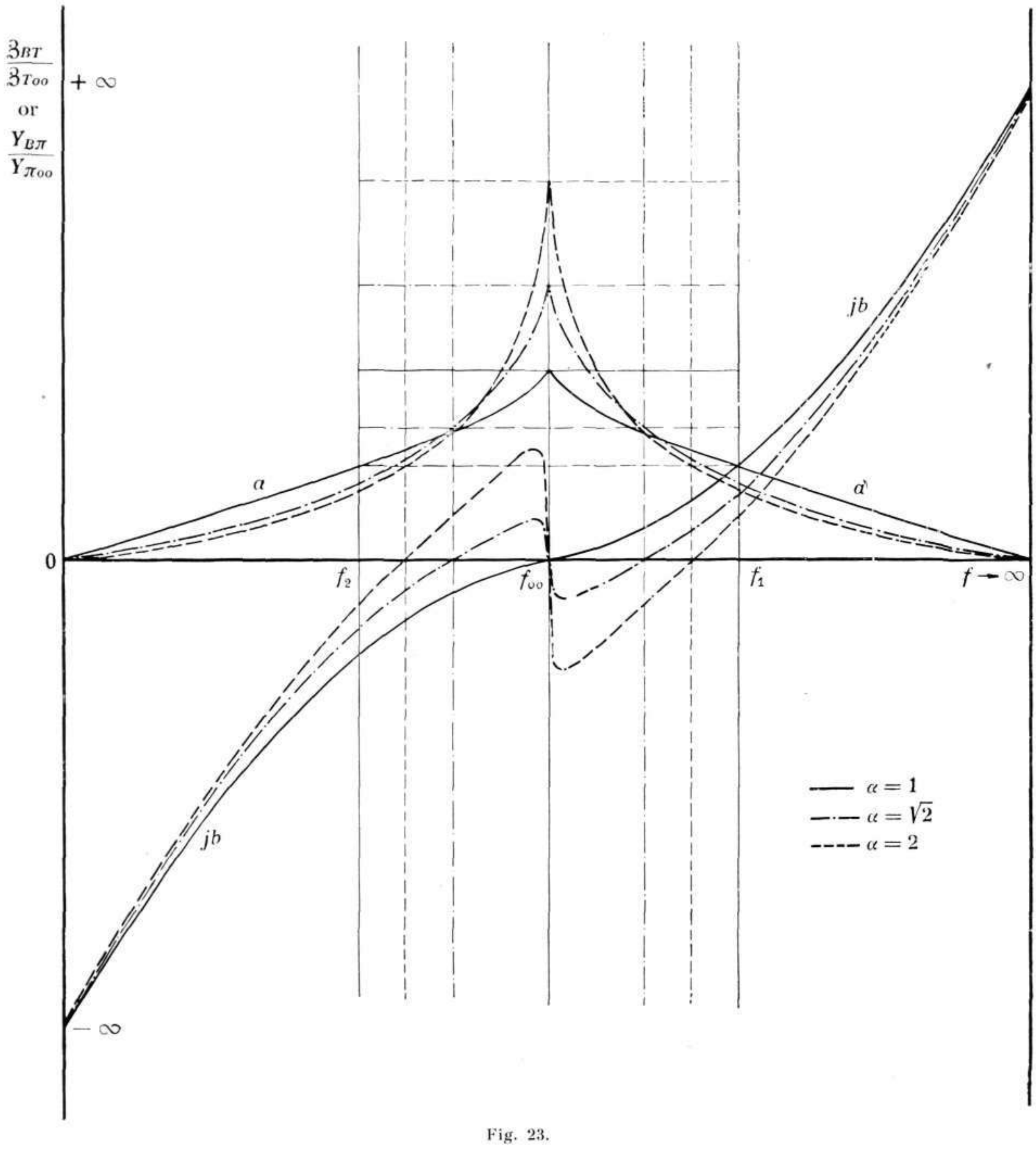


Fig. 23.

$$\left.\begin{aligned}
\varepsilon_T &= \frac{\operatorname{tgh}(\Theta+\sigma'') - \frac{1}{a}\frac{\mathfrak{Z}_{T\infty}}{\mathfrak{Z}_T}}{\operatorname{tgh}(\Theta+\sigma'') + \frac{1}{a}\frac{\mathfrak{Z}_{T\infty}}{\mathfrak{Z}_T}} \\
\varepsilon_\pi &= \frac{\operatorname{tgh}(\Theta+\sigma') - a\frac{\mathfrak{Z}_{\pi\infty}}{\mathfrak{Z}_\pi}}{\operatorname{tgh}(\Theta+\sigma') + a\frac{\mathfrak{Z}_{\pi\infty}}{\mathfrak{Z}_\pi}}
\end{aligned}\right\} \qquad (88)$$

where

$$\left.\begin{aligned}
\operatorname{tgh}\sigma' &= \frac{1}{a}\frac{\mathfrak{Z}_{T\infty}}{\mathfrak{Z}_T} \\
\operatorname{tgh}\sigma'' &= a\frac{\mathfrak{Z}_{\pi\infty}}{\mathfrak{Z}_\pi} \\
&\text{and according to (79)} \\
\operatorname{tgh}\sigma' \operatorname{tgh}\sigma'' &= 1
\end{aligned}\right\} \qquad (89)$$

By reducing (88) and (89) to their simplest terms we finally obtain the reflection coefficients

$$\left.\begin{aligned}\varepsilon_T &= \frac{\text{tgh}\,\sigma'' - \text{tgh}\,\sigma'}{2\,\text{tgh}\,\Theta + \text{tgh}\,\sigma' + \text{tgh}\,\sigma''} \\ \varepsilon_\pi &= \frac{\text{tgh}\,\sigma' - \text{tgh}\,\sigma''}{2\,\text{tgh}\,\Theta + \text{tgh}\,\sigma' + \text{tgh}\,\sigma''}\end{aligned}\right\} \ldots\ldots\ldots\ldots (90)$$

which are thereby shown to be identical at the mid-series and mid-shunt terminations except for the sign. Normally, it is the absolute value of the reflection coefficient that is of interest, and we then have in the general case

$$\varepsilon = \varepsilon_T = \varepsilon_\pi = \left|\frac{\text{tgh}\,\sigma'' - \text{tgh}\,\sigma'}{2\,\text{tgh}\,\Theta + \text{tgh}\,\sigma' + \text{tgh}\,\sigma''}\right| \quad (91)$$

As an example of the calculation of reflections in a filter we will work out ε for the same filter as in the foregoing chapter, namely, a half section "constant k" filter (fig. 21). For this filter we have according to the above

$$\text{tgh}\,\Theta = \text{tgh}\,\frac{\gamma}{2} = \frac{\sinh\frac{\gamma}{2}}{\cosh\frac{\gamma}{2}} = \frac{\sqrt{U_k}}{\sqrt{1+U_k}}$$

$$\text{tgh}\,\sigma' = \frac{1}{a}\,\frac{\mathfrak{Z}_{T00}}{\mathfrak{Z}_T} = \frac{1}{a\sqrt{1+U_k}}$$

$$\text{tgh}\,\sigma'' = a\,\frac{\mathfrak{Z}_{\pi 00}}{\mathfrak{Z}_\pi} = a\sqrt{1+U_k}$$

By substituting in (91) we get

$$\varepsilon = \left|\frac{a^2\,(1+U_k) - 1}{2\,a\,\sqrt{U_k} + a^2\,(1+U_k) + 1}\right| \ldots\ldots\ldots (92\text{ a})$$

or

$$\varepsilon = \frac{|\,a^2\,(1\;U_k) - 1\,|}{\sqrt{[a^2\,(1+U_k)+1]^2 - 4\,a^2\,U_k}} \ldots\ldots (92\text{ b})$$

The reflection coefficient ε for this half section is plotted graphically in fig. 24 for various values of a viz. 1, $\sqrt{2}$, and 2. It will be zero when

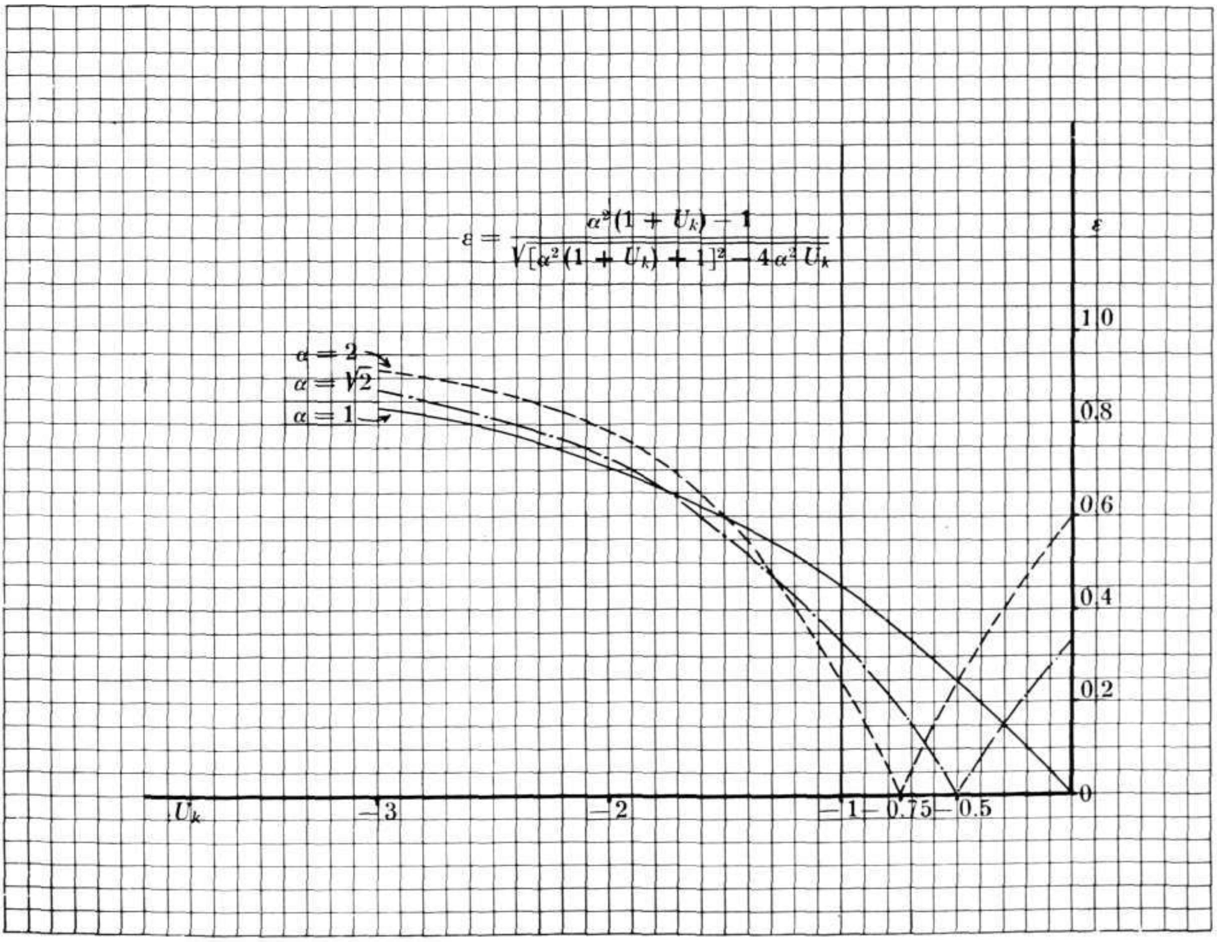


Fig. 24.

$$U_k = \frac{1}{a^2} - 1, \qquad (93)$$

which agrees with the statements in Table X, where it was shown that the imaginary part of the load impedance became zero and the real part exactly equal to the external resistance at this value of U_k.

(92 b) can be simplified still further if instead of the variable U_k—reckoned from the origin along the negative abscissa axis—we introduce the variable u_k, which is reckoned positive or negative from the value of U_k given above in (93) at which ε becomes zero—in other words we move the coordinates. The expression for the reflection coefficient ε then takes the form

$$\varepsilon = \frac{1}{\sqrt{1 + \left(\frac{2}{a u_k}\right)^2}} \qquad (94)$$

and the actual value of U_k is determined by

$$U_k = \frac{1}{a^2} - 1 \pm u_k \qquad (95)$$

The curve for the reflection coefficient ε is symmetrical about an axis determined by $U_k = \frac{1}{a^2} - 1$.

Fig. 24 shows that the reflections are least in the middle of the filter band ($U_k = 0$) in a filter where $a = 1$, while they rise to 45 per cent. at the cut-off frequency ($U_k = -1$) for the same value of a. In a filter where $a = \sqrt{2}$ — which has for other reasons been found a suitable value — the reflection coefficient at the centre of the band and at the cut-off frequencies will be 33.3 per cent, and will be reduced to zero between these points at $U_k = -0.5$. The reflections are thus considerable, even in a-matched filters, but a-matching does make it possible to shift the ranges of minimum reflections to frequencies which are on either side of the centre of the band. As we have already mentioned, not only the effective attenuation but also the reflections must be considered when selecting the value of a. The points of view given above will then form a good guide.

6 D. Discussion of advantages and drawbacks in a-matching.

Proceeding from general formulæ for effective attenuation, load impedance and reflection coefficient, we have above deduced special formulæ for a-matched "constant k" filters. These special formulæ have become relatively simple and suitable for algebraic treatment on account of the favourable properties of these filters. Again, in "constant k" filters it is very simple to make the conversion from expressions having U_k as parameter to other forms with the frequency as parameter, as has been shown in Ch. 5. The universal applicability already attained by this class of filter will have been even further extended by this work. In conclusion we give below a summary of the advantages of a-matching in the calculation of filters.

1. The effective attenuation curve approaches the ideal, a rectangle open at the top. This property is specially marked in narrow band filters of the kind used for carrier-current telegraphy.

2. By choosing a suitable value of a the point on the effective attenuation curve where the effective attenuation is a minimum may be fixed at a frequency which has to be transmitted with particularly low attenuation, e. g. the carrier frequency in carrier current telephony plants working with one of the side bands suppressed. Another example of such a shifting of the minimum on the effective attenuation curve is the case in which it is desired to transmit a certain frequency, e. g. 800 cycles/sec., or certain frequencies within a band of voice frequencies, with the least possible attenuation.

3. In a-matching of filters ($a > 1$) greater effective attenuation is always obtained far outside the filter band than in the normal case where $a = 1$.

Certain drawbacks naturally accompany the use of a-matched filters. The most important of these has already been pointed out above, viz. increased reflections for certain frequencies in the filter band. But these reflections are the very cause of the favourable appearance of the effective attenuation curve, and the drawback of reflections is often balanced by improved attenuation.

In the cases of asymmetrical "constant k" filters discussed above, we have assumed that the filters are to be a-matched both ways, so that the external impedances have been made smaller (or greater) than the image impedances at mid-series (or mid-shunt) terminations. This is of course not essential. One can also sometimes with advantage a-match the filter on one side only, or select different values of a for the two sides.

Methods of designing electric wave filters and a contribution to the theory of matching filter quadripoles. — R. D. D.

Zobel's method, called the "m-derivation" of "constant k" filters, which gives a level image impedance within the filter band, can in certain cases be used in conjunction with α-matching, e. g. by making the filter on the one side of m-type while it is α-matched on the other side. A filter of m-type in itself shows good matching with an external resistance within the filter band, and, by a suitable choice of magnitude for the parameter m, this matching can so be done that the image impedance becomes equal to the external resistance at *three* frequencies.

By α-matching such a m-type filter also, the image impedance can be made to coincide with the external resistance for *four* frequencies, of which two will be very close to the cut-off frequencies. It has not yet been investigated which values should in such a case be given to m and which to α, but what has been proved in this paper makes it possible to say that further improvements in matching are possible by this means.

It has already been pointed out that calculations for the choice of the best value of α to obtain the most rectangular effective attenuation curve possible, ought really to have been performed with due allowance for energy dissipation in coils and condensers. This calculation is, however, very complicated, though it might be done in some special cases. A new general method for calculating the attenuation in filters with energy dissipation, which has just been published by Pleijel,[1] is very promising in this respect.

It now only remains to be pointed out that α-matching can naturally be used not only in "constant k" filters but also in other types of filters, e. g. as in Tables II and III. In these cases, however, the calculations can not be so easily surveyed and will take more time, for in them the product z_1 z_2 is not independent of the frequency. Their image impedances and load impedances can therefore not be expressed as functions only of the parameter U. At the same time it seems very likely that in these cases too α-matching will have advantages, especially with regard to effective attenuation.

1 See Bibliography 21.

Bibliography.

1 A. E. Kennelly: "Articificial Electric Lines; their theory mode of construction and uses." Mc. Graw-Hill Book Company, New York. 1917.

2 H. Pleijel: "Telefonledningars elektriska egenskaper". Tekniska Handböcker utgivna av Kungl. Telegrafstyrelsen. 1923.

3 F. Breisig: "Theoretische Telegraphie." Friedr. Vieweg & Sohn Akt. Ges., Braunschweig. 1924.

4 K. S. Johnson: "Transmission circuits for telephonic communication; methods of analysis and design." D. van Nostrand Company, New York. 1925.

5 Pierre David: "Les filtres électriques, théorie, construction, applications." Gauthier-Villars et C:ie, Paris. 1926.

6 T. E. Shea: "Transmission Networks and Wave-Filters." D. Van Nostrand Company, New York. 1929.

7 A. E. Kennelly: "The Equivalence of Triangles and Three-Pointed Stars in Conducting Networks", Electrical World and Engineer, New York, Vol. XXXIV, pp. 413—414, Sept. 16, 1899.

8 G. A. Campbell: "Cisoidal Oscillations", Trans. A. I. E. E., Vol. XXX, Part II, pp. 873—909. 1911.

9 K. W. Wagner: Arch. für Elektrotechnik, Vol. 8, p. 61. 1919, och E. T. Z. Aug. 7, 1919.

10 G. A. Campbell: "Physical Theory of the Electric Wave-Filter", Bell Sys. Techn. Jour., Nov. 1922.

11 O. J. Zobel: "Theory and Design of Uniform and Composite Electric Wave-Filters", Bell Sys. Techn. Jour., Jan. 1923.

12 L. J. Peters: "Theory of Electric Wave-Filters Built up of Coupled Circuit Elements", Jour. A. I. E. E. May 1923.

13 J. R. Carson and O. J. Zobel: "Transient Oscillations in Electric Wave-Filters", Bell Sys. Tech. Jour., July 1923.

14 R. S. Hoyt: "Impedance of Loaded Lines and Design of Simulating and Compensating Networks", Bell Sys. Techn. Jour., July 1924.

15 O. J. Zobel: "Transmission Characteristics of Electric Wave-Filters", Bell Sys. Tech. Jour., Oct 1924.

16 K. S. Johnson and T. E. Shea: "Mutual Inductance in wave filters with an introduction on filter design." Bell Sys. Techn. Jour. Vol. IV. No. 1 January 1925.

17 R. Feldtkeller: "Über einige Endnetzwerke von Kettenleitern", E. N. T. Bd. 4, Heft. 6, S. 253, 1927.

18 H. W. Bode: "A Method of Impedance Correction", Bell Sys. Techn. Jour., October 1930.

19 E. B. Payne: "Impedance Correction of Wave-Filters", Bell Sys. Techn. Jour., October 1930.

20 O. J. Zobel: "Extensions to the Theory and Design of Electric Wave Filters." Bell Sys. Techn. Jour., April 1931.

21 H. Pleijel: "A method of computing the attenuation in a band pass filter arbitrarily composed of resistances, inductances, capacities and transformers." L. M. Ericsson Review No. 7—9, 1931.

Method of Installation of Subscriber's Automatic Telephones when Changing over from LB to the Automatic System.

Communication from the Concession Department.

By *Hugo Blomberg.*

When a telephone plant is automatized it is often found possible to retain the same lines in the automatic system—although perhaps a few alterations and repairs may be necessary. The new automatic exchange is connected by transfer joints to the primary cables coming from the old manual exchange, and at a fixed moment all the subscribers, or in large exchanges a group of subscribers, are switched over to the automatic exchange. If the manual exchange is designed on the CB system, the subscribers' stations have been fitted with dials or exchanged for automatic telephones before the switch-over, so that they do not complicate that process.

But if the manual exchange is arranged on the LB-system, the old subscribers' LB telephones obviously cannot be exchanged for an automatic one before the switch-over; instead it must be left, and the new automatic telephone installed as well. The LB telephones have therefore to be used right up to the actual time of switch-over, and not until then can the automatic ones be used.

The usual plan for this has been to connect the subscriber's incoming line to a special switch with which the subscriber can turn his line over from one set to the other at a definite time. In this case, however, it has been found necessary to put in, besides the switch, a condenser in series with the LB telephone, as otherwise every telephone having its bell connected direct to the subscriber line would, when the switch-over is done (usually at night), make call at the exchange until the subscriber had thrown over the switch and so changed the line to the automatic set.

Apart from considerations of cost, this arrangement is not very convenient, for the subscriber's incoming line cannot be fixed finally to the line terminals of the automatic telephone before the switch-over is done, because the switch, condenser and LB telephone have to be removed afterwards.

In order to avoid this, the following method has been worked out, and recently used in practice with good results.

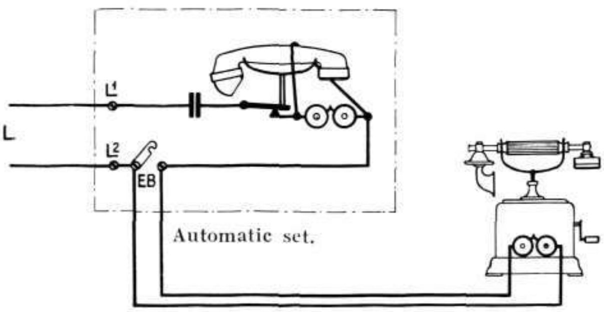


Fig. 1.

Fig. 1 shows how the connexions are made when the automatic telephone is installed before the switch-over. The incoming subscriber's line *L* is connected to the two line terminals, L_1 and L_2, of the automatic telephone, which is therefore connected at once in its permanent position. The latter's terminal strip always has two terminals *E B*, for connecting an extra bell, these being normally joined by a metal strip. To these the line terminals of the LB telephone are connected, the metal strip being folded to one side. When, therefore, the hook of the automatic telephone is depressed, i. e. when it is in the signalling position, the LB telephone will be connected to the line in series with the bell and condenser of the automatic telephone. The bell impedance is, however, far too high for speech currents and

R 4068 Fig. 2.

R 4067 Fig. 3.

cannot therefore be connected while the LB telephone is being used. It is therefore short circuited by an ordinary connecting wire arranged at the telephone and connected to the bell terminals on the strip of the telephone, taken out through the hole for the cord in the casing,, and laid round the receiver hook or over the microtelephone and dial in such a way as to keep the hook depressed, as in figs. 2 and 3. This connecting wire has two functions, the electrical one of short-circuiting the ringing bell, and the mechanical one of preventing the hook from being moved. By means of it (it might suitably be enamelled and spun over in some colour) the automatic set is sealed in the ringing position and at the same time shows the subscriber that it is not to be used. The LB telephone is connected to the subscriber's line in series with the condenser, and conversation can thus be carried on undisturbed.

The subscriber is instructed to cut this connecting wire at a stated time—subsequent to the switch-over having been completed—thus breaking the mechanical sealing, after which the automatic telephone will be used. By cutting the wire the short circuit across the bell is broken and the automatic telephone is put into normal working order, while the LB telephone remains connected as an extra bell. Incoming signals will thus ring the bells of both sets. After some time a fitter will arrive and remove the LB telephone and the connecting wire, the latter being left hanging till then. He will also connect the two terminals *E* and *B* by the metal strip. No change whatever need be made then in the incoming line.

By this method the new automatic telephone can be installed very simply without either an extra switch or a condenser being necessary, while at the same time the old LB telephone is left in use. Obviously, the method can be used for switching over from a local battery system just as well to a manual as to an automatic CB-system.

Possibilities and Tendencies of Power Transmission.

By Professor *Sten Velander.*

The need of power transmission.

Today, man and his goods are carried at over a hundred miles an hour, and his messages go round the world with the speed of light. A century or two ago, the speed was a few miles an hour and limited to what man and beast could do by muscular effort. Earlier still, speed and transport facilities had remained practically unchanged during the thousands of years of man's existence. Progress has been similar in many spheres of material culture—slow and snail-like at first, then suddenly almost explosive.

The turning point came when humanity began to harness the forces of nature, when the muscles of man and beast were relieved, first by steam power, and later even more by electric power. The internal combustion engine has played a very important part too, particularly in the development of transport during the last decades.

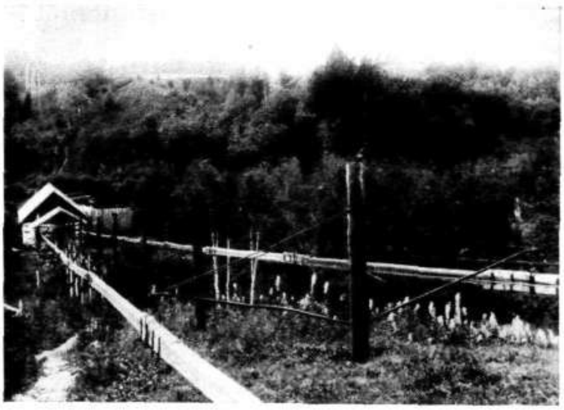
It meant a great deal to progress when man began to fetch raw materials from other countries, from the far corners of the earth. That was done already in ancient times however, though the quantity transported in those days did not go far and consisted mostly of luxuries for the rich, as operations on a large scale were impossible for lack of power. We depend of course on other countries and continents for the raw materials of, say, motor cars, but more than that is required for car manufacture. Even if man had been able to collect these several raw materials, all his efforts to put the modern automobile within the reach of everybody would have been vain if he had not forced the energy contained in coal and water into his service and made it do 99 per cent. of the mechanical work.

For two thousand years man has been very near to subjugating the wild and intractable forces of nature. Suggestions and proposals have been made, but never carried through. The first steam engine was made before the beginning of our era, likewise the first water wheel. The art of making the never-failing energy of running water do one's work was known, but little used. The significance of being able to increase human strength tenfold or a hundredfold was not appreciated and — there was no means of transmitting the power.

Power transmission in ancient times.

Attempts with rods or shafts actually succeeded in transporting the power a few miles. Kristoffer Polhem, our great engineering genius, improved this method considerably, and especially increased its efficiency. These transmissions were greatly admired, and would carry the power of the huge water wheel a whole mile and a half across forest and mountain to the pumps of the mine. At that time wonder and admiration were felt for this transmission of a few horse power, stretching, noisily grating, through the forest. To many it seemed witchcraft, and the legend of a Polhem transmission having frightened ravaging Russian hordes may be true. All things considered, these rod transmissions did good service in their day and odd examples of them have even survived till now, so that the Technical Museum has been able to acquire several. Fig. 1 shows a mechanical power transmission of the 18th century, side by side with an electrical power line of the 20th.

During the 19th century, direct-driving water power, and direct-driving steam power even more, became the dominant source of energy in industry. Our iron works, our textile and paper industries, were lined up along the falls and rapids of the rivers. In the plains the industrial communities were marked by their chimneys. In agriculture, the horse-gear was relieved by the portable engine barely a generation ago. But the small factory, the craftsman, and the home,



R 4022 Fig. 1. Mechanical power transmission by Polhem's rods; in the background an electrical overhead line.

still lacked the untiring helper which large scale industry had procured for the rough work.

Beginning of electric transmission.

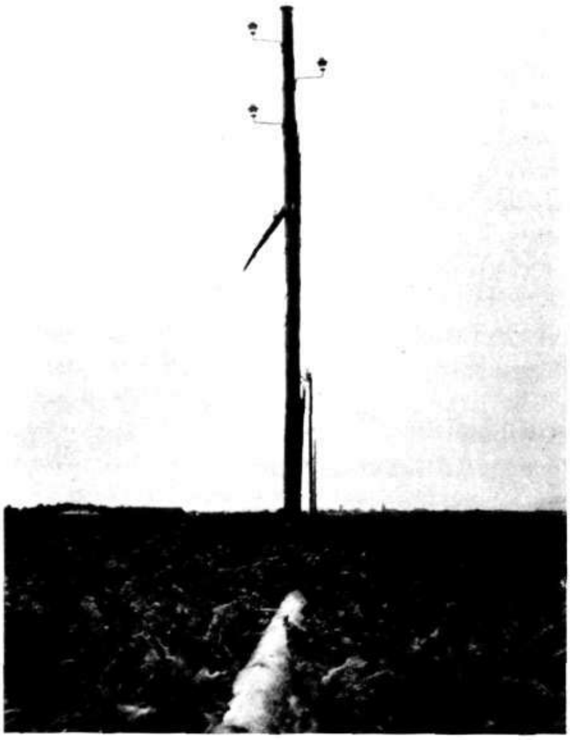
Then, 50 years ago, electrical energy, the new force of nature, began to be heard of. It could be transmitted in any quantity for miles by thin metal wires. It could be used for lighting, for mechanical work, for heating, for chemical processes. It was pure magic.

The first steps in electric power transmission were taken about 1880. Electrical energy was then distributed in towns over short distances for lighting and to some extent for power. The first municipal electricity works were built in the eighties. Even some small Swedish towns built electricity works. The larger towns already possessing gas works as a rule came later. The first municipal electricity works of Stockholm was established in 1892, in Malmö in 1901, and in Norrköping in 1904. Electric power transmission did not seriously begin until the beginning of the nineties, when the 3-phase electric motor was invented. Easily transformable A. C. could then be used for both large and small power requirements in industry. Our iron works were the first to make use of electrical power transmission. The Hellsjön—Grängesberg line, carrying 500 HP at 10 000 volts and completed in 1893, was probably among the first, if not the very first, industrial power line in the world. Its simply equipped power station (fig. 2) is vastly different from modern power stations, and would scarcely suit the increased demands of our days.

These early plants, however, showed that the natural power of the water falls could be successfully transformed into electrical energy, easily carried, and used at incredible distances from the water falls. It was also found that by means of electrical energy power could with great advantage be transmitted and distributed even at as short distances as within a factory. Power was therefore distributed electrically even when the electrical energy was produced by steam power in or in the immediate neighbourhood of the factories. When, at the beginning of this century, the turbine became the dominant type of steam engine and the power requirements at the same time grew to many thousands of horse power, the employment of ropes, countershafts and belting became out of the question. Practically all power transmission and distribution then became electrical, whether the power was produced by water or fuel.

With the close of last century, when technical designs had settled down and become reliable, confidence in electricity grew and its value, particularly as a means of utilizing water power, was exaggerated. The ever-flowing masses of water, producing day after day immense amounts of power, fascinated and dazzled. It was overlooked that the dams, canals, turbines, and generators necessary for harnessing the forces of nature to the engines of industry, cost millions of money. It was forgotten that the power lines had to be carried mile after mile. Even though they were relatively cheap compared to rod or

R 4021 Fig. 2. The old Hellsjön power station, from which the first electric power line was built.



R 4028 Fig. 3. 40 kV line with wooden poles, one shattered by lightning.

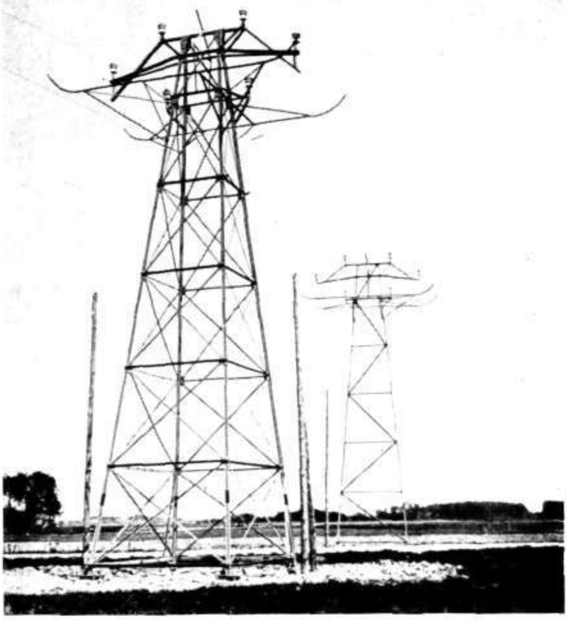
rope transmissions, the large distances made a few poles and a bit of wire swell into thousands of long, heavy poles and tons of valuable copper.

It was found that lines as simply and cheaply built as telephone lines could no longer transmit the increasing power. Their reliability had to be raised. Storm and bad weather must not interfere with the work, lightning must not damage the wires, shatter the poles, or cause any other injury (fig. 3). About 1908—1910 the demand for steel towers (fig. 4) for all more important lines was raised. This further increased the costs. The capital cost of power transmission rose. Even if cheaper maintenance and greater length of life resulted from this, it was nevertheless an additional financial burden. To the economic and financial difficulties which at this time had to be met by the power companies were added political claims for the right of the State to the water, for an extra tax on the power undertakings in the form of concession payments, etc., which all helped to make the money market uncertain and dubious of power plants, even though most of these threats came to nothing.

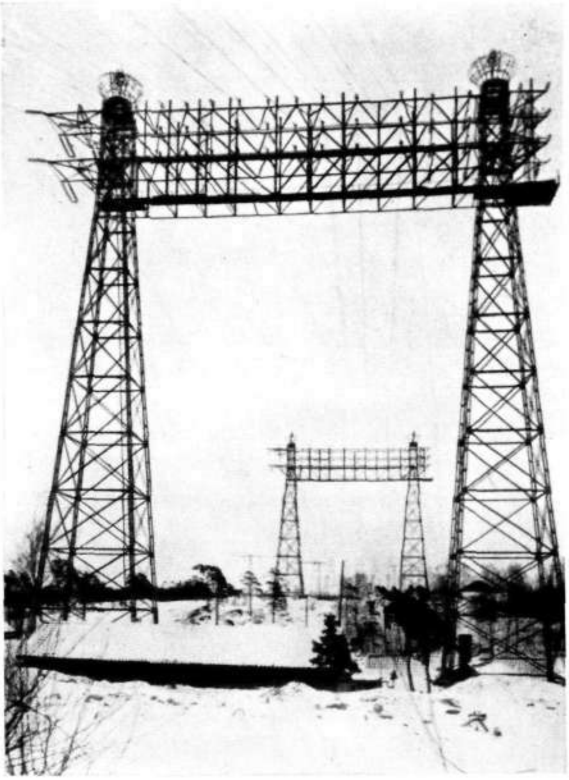
Creation of large power transmission companies.

The beginning was thus none too easy; yet, in spite of everything, the majority of our large power companies were founded at this time. The State built the Trollhättan Power Works, and from this a radial system of power lines was soon built, feeding the towns, villages, and industries all round. On crowned towers, 130 ft. high (fig. 5), an enormous bundle of copper conductors was carried across the canal, to put 100 000 HP at the disposal of every branch of industry. North and south of Trollhättan stretched the lines of the Gullspång and Yngeredsfors Companies. The South Swedish Power Company built a number of magnificent power stations on the Lagan, with the Knäred Works as their centre, whence a heavy bundle of wires ran down the West coast, branching off to the industrial towns there, with Malmö as the natural terminus and centre of a power system which distributed at first 20 000 and later 40 000 HP over the province (fig. 6).

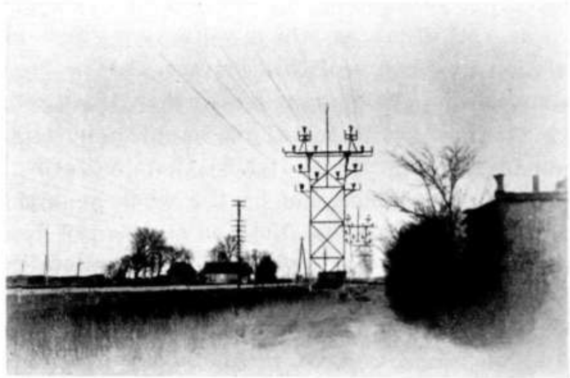
Further East, where prospects of consumption



R 4031 Fig. 4. 50 kV towers with guard brackets at a road crossing.



R 4033 Fig. 5. The 130 ft. towers carrying the Trollhättan lines across the Trollhätte Canal.

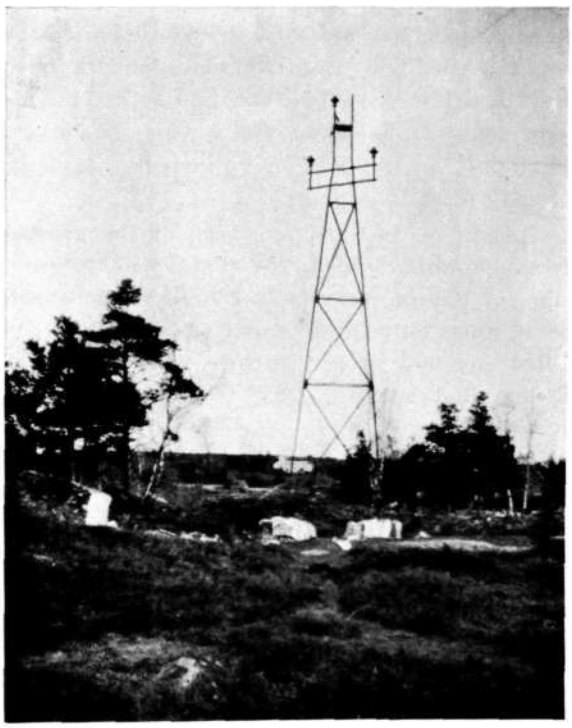


R 4025 Fig. 6. The Sydsvenska Kraftaktiebolaget main line entering Malmö. Old type of tower.

were much more modest, the Hemsjö Power Company built a number of small power stations, with lines crossing Blekinge and North-East Scania (fig. 7). In Östergötland the electrification was also carried out from small power stations in the West by the Motala Ström Power Company, one of the oldest power distributing firms in the country. The urban power supplies of Norrköping, used by the local industries, were in 1910 supplemented from Öjebro. This power was transmitted at the then high voltage of 40 kV in a double line carried on steel towers.

In the Bergslags district of central Sweden also, a few power-distributing companies were formed at an early date, e. g. Örebro Elektriska A.-B., but the transmissions were generally more or less local. As a rule the iron works transmitted power for their own requirements from water falls in the neighbourhood. In Norrland the conditions were similar, though the distances were generally longer and consequently the voltages higher than in Bergslagen. Power distribution systems for serving the public in these districts were as a rule established 5 to 10 years later than in Southern Sweden, whether built by the State or private interests.

At the time, about the end of the first decade of this century, when the majority of public service power distributing concerns were started both in this and other countries, supporting insulators were the only kind available. These restricted the tension used to 50 000 volt or slightly more, which meant a very inconvenient limitation of the possibilities of transmission,

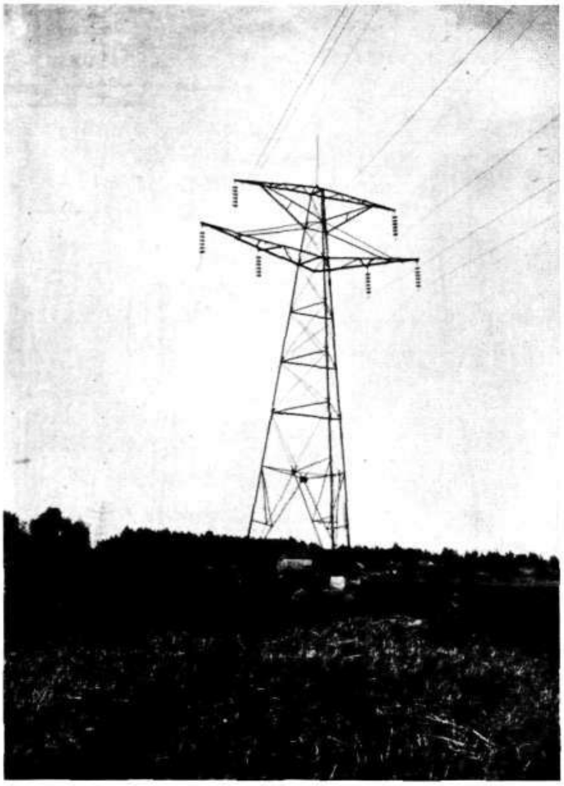


R 4029 Fig. 7. Steel tower on the Hemsjö-Karlshamn line.

noticeable at anything beyond 60 miles, in other words, at distances which were very small in relation to the size of this country.

Suspension insulators were then invented, which theoretically solved the problem of unlimited operating voltages (fig. 8). Certain practical difficulties remained, but on the whole it might be said that from that time on it was not this sort of difficulty, but high costs, that limited the distance at which electric power could be transmitted with advantage.

At the same time as the main cross-country lines spread over large parts of the more densely populated areas of the country, underground cables began to be used for the distribution in towns and villages. At first, technical reasons limited the voltages to 6 to 10 kV, but this was of no great importance, considering the short distances and the moderate amounts of power at first distributed in towns.



R 4030 Fig. 8. 70 kV tower with suspension insulators. Royal Board power station, Älvkarleby.

The increased power consumption, and its significance to transmission methods.

The consumption, however, grew year by year. This universal increase has resulted in the annual output for the whole Sweden, which a little more than 20 years ago stopped at 500 million kWh, having today become 5 000 million kWh, i. e. it has increased tenfold in little more than 20 years. Part of this increase of 200 to 300 million kWh a year comes from new power systems, but a very large part is accounted for by increased loads on already existing systems.

This increased load has been of the greatest economic importance to the methods of transmission. A line to carry 500—1 000 kW a distance of 50 miles is hardly cheaper than one built for 2 000 or 3 000 kW. A further increase to, say, double that amount, i. e. 5 000 kW, raises the cost of the line not by 100 % but only by about 30 %. And it is practically the same for shorter transmission distances and proportionately smaller amounts of power. The total cost of the line, and hence also the total transmission cost, will therefore only rise slightly with increased power. This means that as the power grows the small increase in the total cost can be divided amongst a much increased number of kilowatts. The transmission cost per kW will thus fall as the consumption rises. This applies not only to the large cross-country lines, but even more to the systems in towns, villages, etc., and is the principal reason for the selling price of electric power remaining at the same level in spite of rising wages, cost of materials, etc. It has even been possible to reduce the price of power distributed in towns considerably, while at the same time increasing, both absolutely and relatively, the profit to the towns from this source.

Improving the strength and reliability of the lines.

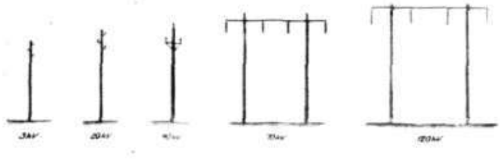
This favourable economic improvement has also had an effect on the purely technical side of power transmission. It was no longer necessary to keep building costs down to a bare minimum to make both ends meet. A fair amount of money could be spent on increasing the strength and reliability in working. What was economically impossible in a line which carried only 500 kW, became an obvious and paying proposition in one of 5 000 kW. As electrical energy gradually became a necessity for an increasing number of purposes, the demands of consumers for greater reliability were also increased, and improvement in quality, i. e. greater freedom from disturbances, was more and more appreciated.

The past 20 years have therefore been marked by incessant efforts to produce better, stronger, and more reliable transmission plants.

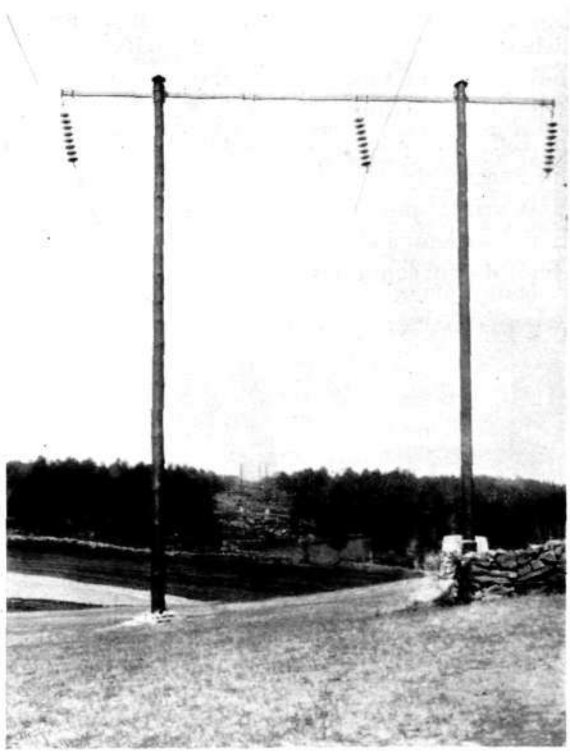
Mechanically, the lines have been improved mainly in the following directions. Experience gained in older systems was collected and analysed. In this way more extensive and certain knowledge was obtained of the strains and stresses to which lines may be exposed in bad weather conditions. Special attention is now given to the stresses to which wires covered by ice may be subjected in a wind or when there is hoar-frost. In the standard specifications for power-lines, which have been in force for some years, values for these extra loads are included. Since, as mentioned above, a line transmitting much power with good economy can, and should, be constructed with greater strength and reliability than one for less power, the reliability requirements of these Swedish specifications are graded according to the importance of the line from a power transmitting point of view — an innovation which is gradually being adopted in other countries. Attempts are also made to allow for the varying weather conditions in different parts of the country.

During the efforts to obtain the strongest and most reliable lines possible, wooden poles have come to the fore in a way undreamt of 20 years ago (figs. 9 and 10). Creosote-impregnated wood has proved as durable as iron. Further, wood needs no painting etc. This is of great importance in these days, when it is becoming more and more imperative that the lines should work incessantly, day and night both Sundays and weekdays, so that there is hardly ever time to switch off the power for inspection and repairs.

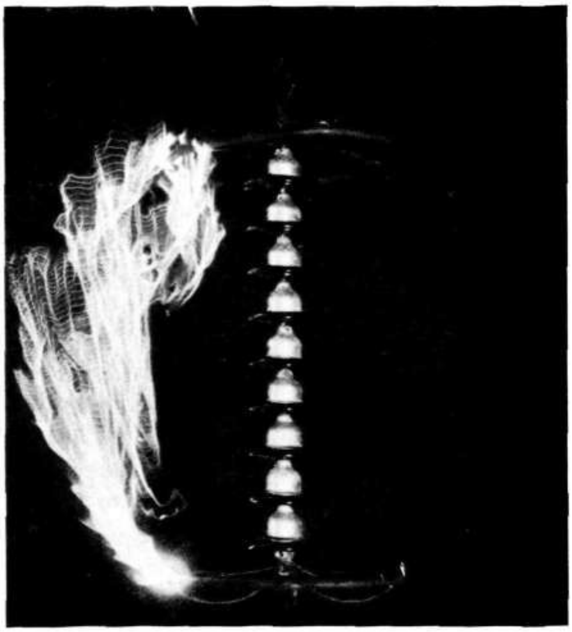
About as important as the mechanical strength of towers and cables is the reliability of the insulators. In past times, the insulators have on occassion left much to be desired in operating reliability. Many expedients have been suggested and tried, and have frequently failed, but the last decade has brought about considerable improvements. It has been realized that insulator faults must be ascribed to mechanical and thermal stresses, and that much can be gained by using a strong, tough porcelain. Extra margin has also been allowed in the size of the insulators. In a suspension insulator, for instance, a couple of extra units may be put in, to give sufficient insulation even when one or two of them are broken, for even the best insulator may be damaged by shots or other causes. The same principle is also applied with good results to pintype insulators, which are made so large that flash-overs will not occur even if one of the shells has been broken clean off. Considerable progress has also been made in pro-



R 4019 *Fig. 9.* Wooden poles of the types used by the Royal Board of Waterfalls.



R 4034 *Fig. 10.* 70 kV wooden pole in the Trollhättan—Alingsås line.



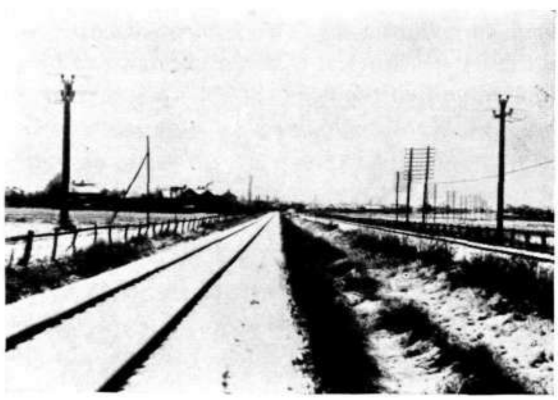
R 4018 Fig. 11. Flash-over on a string of insulators with protective rings.

tecting the strings of insulators from damage by possible flash-overs. The investigations carried out by Sydsvenska Kraftaktiebolaget (South Swedish Power Company) have contributed much to the solution of this problem and have led to the string of insulators with protective rings shown in fig. 11. Such high insulation also makes the lines fairly safe from over-voltages, not only those arising internally from switching operations in the system, but also external ones caused by atmospheric conditions. There are examples of solidly built lines with heavy copper conductors and ample insulation which have worked for years without a single mechanical or electrical disturbance interrupting the service. The time therefore does not seem to be far distant when we will have power transmissions able to work practically 24 hours a day year after year with no interruptions at all.

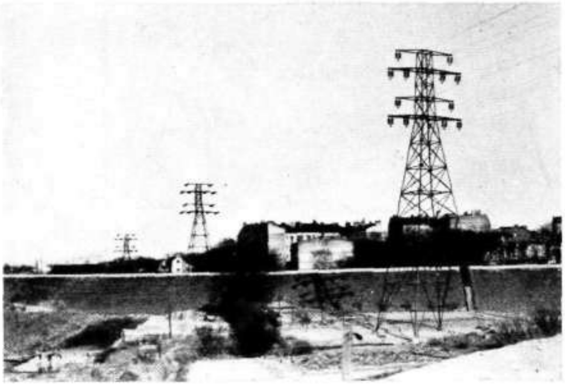
Crossings of power lines and other communication routes.

The strength and reliability attained by modern power lines have made it possible to do away with the protection formerly considered necessary when one of them crossed a railway, a telephone line, or a main road. Not so long ago, at the beginning of the century, it was considered that a power line should be carried across a railway on a viaduct nearly as solid as a road bridge (fig. 12). A sound view, which is rapidly gaining ground, is that it is better to spend more on making the line itself safe against breakages, than to make the line weak and fragile and then put up expensive devices to support and carry the lines when these are once broken and out of action. Stout poles, heavy conductors, and strong insulators — frequently double — will, when properly used, make a line strong enough to allow its crossing railways, telephone lines, etc. in one free span (figs. 13, 14). Such crossings are now beginning to be used regularly in large power transmission systems both in our country and in most others. For minor, weak lines the old guard brackets etc. may possibly be retained.

R 4023 Fig. 12. Old fashioned railway-crossing, with a viaduct for the power line.



R 4020 Fig. 13. Modern fracture-proof crossing, with double insulators and extra stout poles.



R 4024 Fig. 14. The line entering Malmö. Fracture-proof railway crossing.

Underground cables for power transmission.

As mentioned above, underground cables began to be used fairly soon for this purpose, though chiefly in towns and densely populated rural districts, or where it was difficult to find room for overhead lines. Obviously it would often be desirable and convenient to pass directly from a high tension overhead line to an underground cable when bringing the line into a town, crossing some wide water, or the like. In past times the maximum voltages for cables were comparatively low, but the researches and experiments of the last decade have advanced manufacturing methods so far that it is only for the very highest voltages, i. e. above 132 kV, that cables have not yet been designed and used. Underground cables can nowadays be used for all voltages employed in Sweden, and submarine cables for 50 kV have been laid across both the Sound and Kalmar Sound. At the same time voltages in the urban cable systems have risen from 5 or 6 to 30 kV; the latter is now used in, for instance, Stockholm. Cables are already used for taking the overhead line pressure of 40 or 50 kV straight to the centre of the town.

Sieverts Cable Works have contributed in many ways to the development of cable designs for very high voltages. The oil-filled cable boxes, probably the best for this particular purpose, are only one of the valuable contributions of this firm to the design of cable lines.

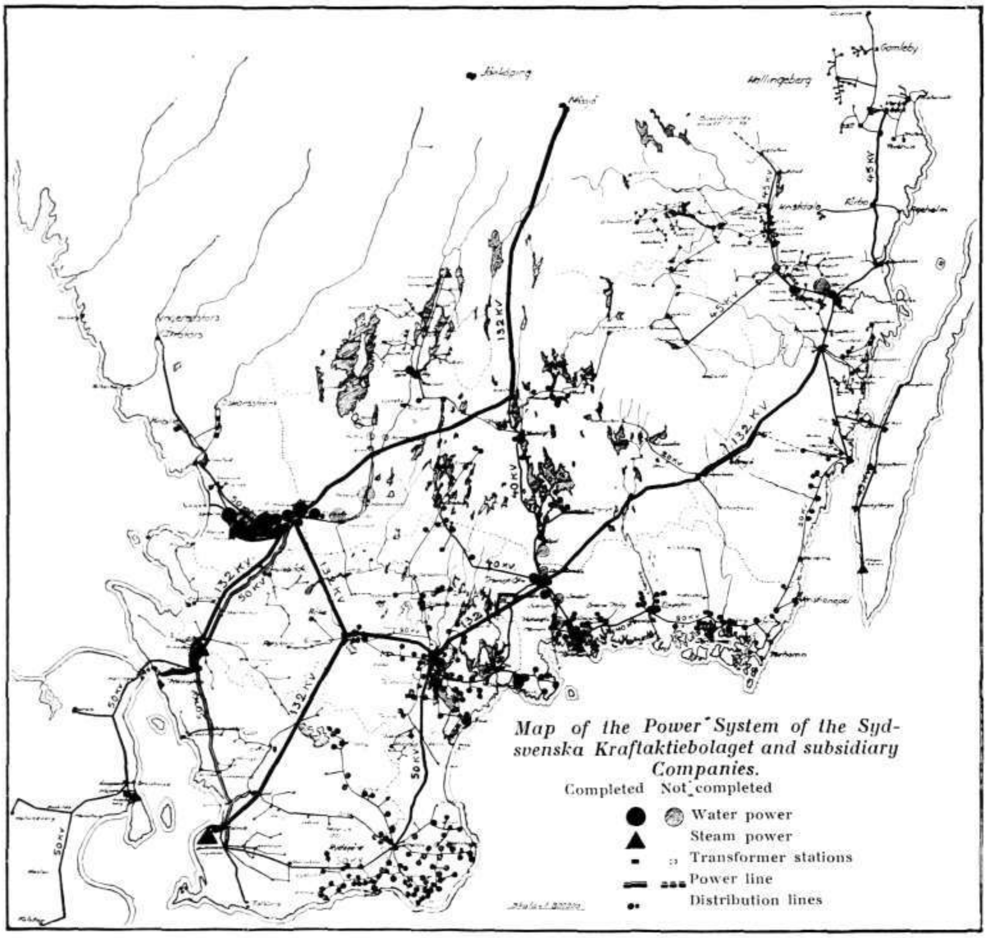
Underground cables, however, are more expensive than overhead lines and are therefore, in spite of their advantages, not used as often as they might be. It is generally the cost of laying which makes the cable so expensive. Attempts have lately been made in Germany to dig the cable trenches by machinery, which, it is hoped, will lead to a more extensive use of cables, for they possess great advantages. An absolutely safe and reliable transmission is often more easily obtained with an underground cable than with an overhead line. The modern cables with rationally designed cable-boxes have proved particularly insensitive to anything in the way of over-voltages. A section of cable before a machine, a transformer, or a switching station, will furnish very effective protection against lightning. It should further be emphasized that even if the initial cost of cables is somewhat higher than of overhead lines, their maintenance costs are considerably less. The annual cost of a cable transmission will therefore frequently be less than that of an air transmission, especially if allowance is made for the above-mentioned technical advantages of the cable. A careful consideration of cables as an alternative to overhead lines is therefore strongly recommended.

Phase compensation in power transmission.

Most electrical machines and appliances used in industry, agriculture etc. unfortunately not only consume electric current in proportion to the work done, but also a certain amount for magnetizing the necessary iron framework. We have the active load and energy which does useful work. We have a frequently equally large reactive load and energy, which is consumed in magnetizing.

This is the much-discussed power factor, the mystic $\cos \varphi$ which is so puzzling to the layman.

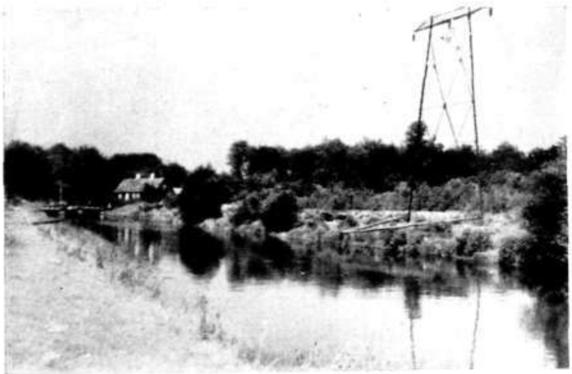
As the electrical transmission plants have been enlarged and come to serve an increasing number of purposes, it has proved more and more unsuitable to produce the magnetizing current, the reactive load, in the power stations and transmit it by the lines. It is generally better to produce it mainly at or near the place of consumption. This is an absolute necessity in long high voltage lines, where it is generally supplied by synchronous light-running machines, so-called synchronous condensers. A very large condenser — of no less than 12 000 kVA — is, for instance, installed at Malmö.



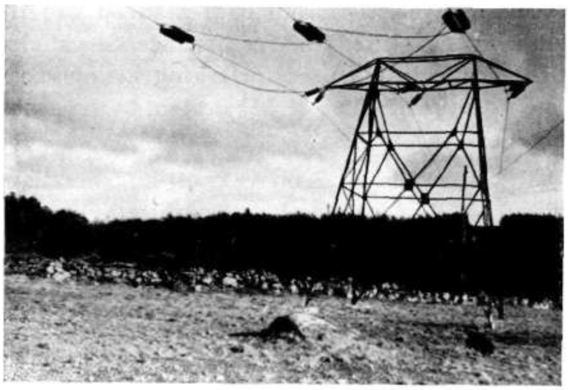
R 4037

Fig. 15. Map of the Sydsvenska Kraftaktiebolaget system.

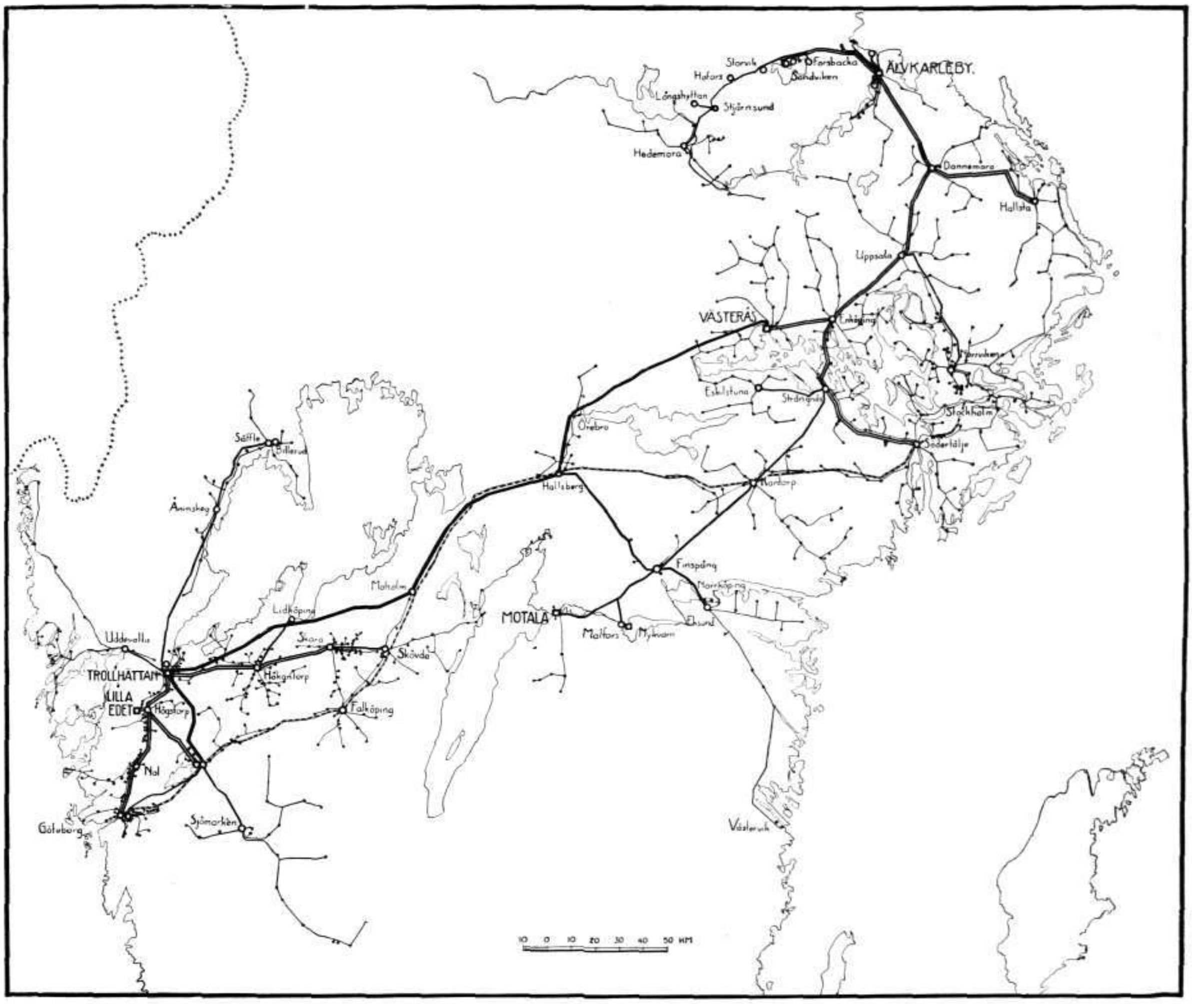
In many other cases, and especially when it is a question of producing a small amount of reactive power close to the place of consumption, static condensers are most practical. Sieverts Cable Works have improved these condensers enormously, and have made them a rational adjunct of electricity-consuming plants. Here we will only emphasize the extremely small losses



R 4026 Fig. 16. Tower built for 132 kV, present voltage 55 kV.



R 4027 Fig. 17. Tower for 132 kV, present voltage 55 kV. Device for giving the line 1/3 of a turn this side of the tower.



R 4054

Fig. 18. Map of the State Central Block system.

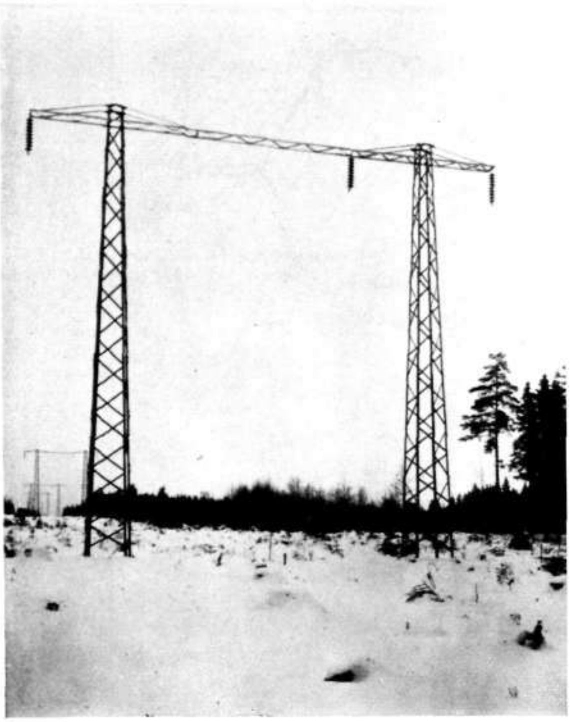
when using static condensers — only about $^{1}/_{10}$ of those in rotary condensers. Static condensers may therefore be used for phase correcting in lines transmitting high powers also, e. g. main transmission lines.

Interconnexion and cooperation.

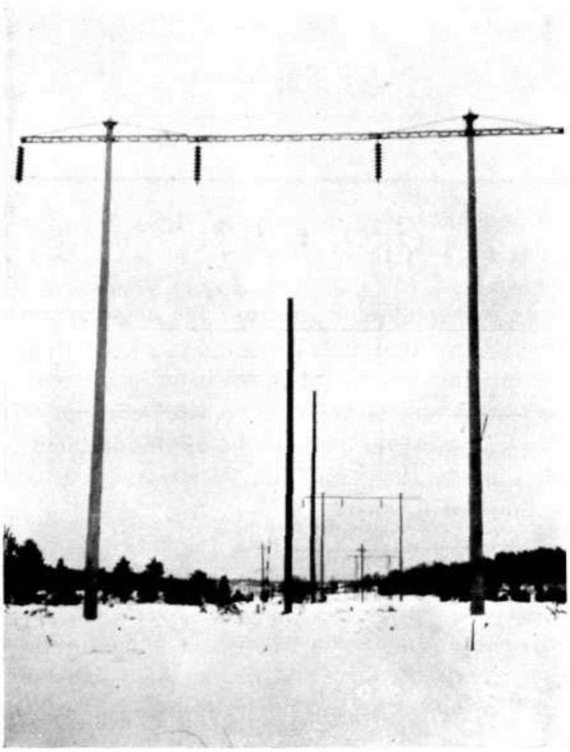
Power transmitting systems are no longer of the early simple type, i. e. a line from power station to consumer, to a factory. The lines are branching out to increasing numbers of consumers, more power stations are connected to the lines and their energy is fed into the joint lines. The advantages of such interconnexions are: both load and supply of power are equalized, many power stations can use the same reserve, when the load is low the number of generators etc. running can be cut down in order to reduce the losses; new power stations will more quickly be working at full load, as the additional load of the whole system, and not only part of it, will be supplied by them.

These reasons have led to the interconnexion and cooperation of an increasing number of power stations.

In Southern Sweden, Hemsjö, and Finsjö have been incorporated in Sydsvenska Kraftaktiebolaget (the South Swedish Power Company). A connecting line is being built to Yngeredsfors Kraftaktiebolag, and another, for the electrification of the State Railway South Main Line, will be con-



R 4036 Fig. 19. Steel tower on the main line from Trollhättan to Västerås, built for 220 kV; present voltage 132 kV.



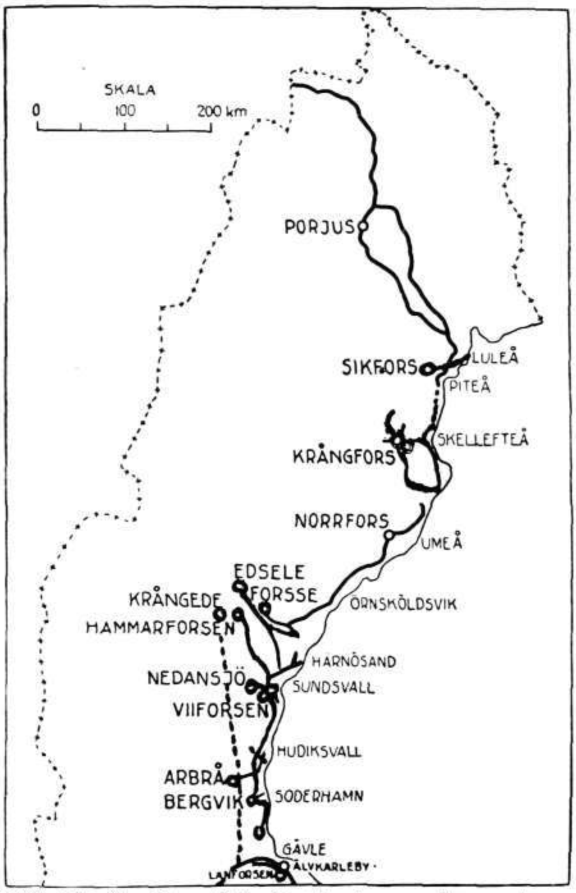
R 4040 *Fig. 20. Concrete pole on the main line from Trollhättan to Västerås, built for 220 kV; present voltage 132 kV.*

nected to the State power system at Nässjö. Practically all the power lines in the seven southernmost counties of Sweden now form a single unit, and gradually the full water-power supply of the district can thus be rationally used (fig. 15). The 132 kV lines built during the last 10 years form the backbone of this system. Steel towers are used in the earlier lines, which are designed for 132 kV, though the present voltage is only 55 kV (figs. 16, 17). A change to 132 kV can be made by simply placing a few more units in the strings of insulators. All the wires are put on the same level, which has proved the most suitable method of coping with the snow and wind of this country. I will return later to the most recent of the 132 kV lines built by Sydsvenska Kraftaktiebolaget.

The Norrköping system has also been gradually connected up with its neighbours, and most of the power stations on the Svartån, Stångån, and Motala Ström are now working in parallel, or at least can do so. Finally, the whole of this system is also connected up with the State power stations.

In central Sweden, the various power stations belonging to the State are connected, so that Trollhättan, Älvkarleö, and Motala work together in a common transmission system (fig. 18). All round the far-flung ramifications of this State Central Block, a large number of cooperating private power stations are connected. The Western and Eastern portions of this system cooperate by means of the main line from Trollhättan to Västerås. This is built for 220 kV, but at present only 132 kV (fig. 19) is employed. The line will be increased by another pylon, making it into a double-circuit 220 kV line. Some sections of this line have been built with poles of reinforced concrete, a material which seems to offer certain advantages (fig. 20). Experiments with concrete poles have been made elsewhere too, but they have never been widely used, chiefly for economic reasons.

Cooperation in Norrland has only begun lately (fig. 21). By enlarging its own lines and buying up others, the Hammarforsen Kraftaktiebolag in Southern Norrland has created a system stretching from Sundsvall almost down to Gävle, where the lines of the Central Block end. Northwards, the Hammarforsen system reaches the State



R 4032 Fig. 21. Map of the Norrland power line systems.

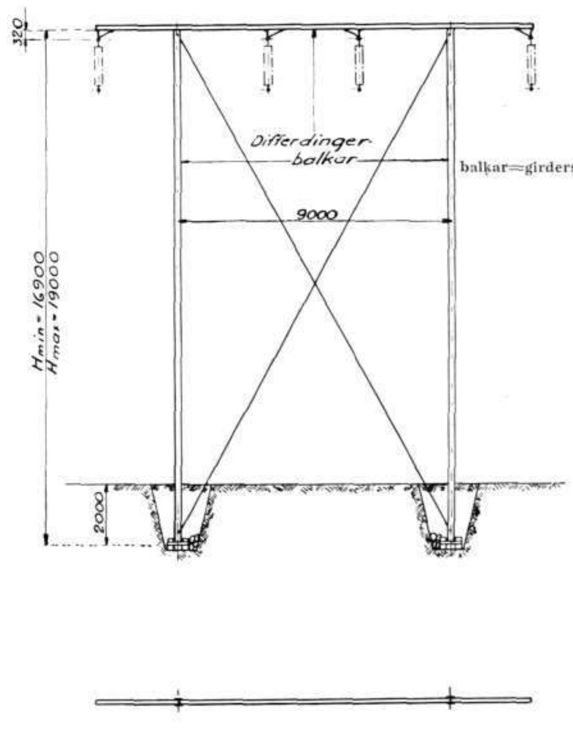
power station at Norrforsen, to which it is connected, and it also communicates with the largest of the wood industry power plants in the Ångermanland district.

Further, the Skellefteå system will be connected to the State-owned Porjus system in the North, and probably in the South to the Norrfors system. We shall thus have one connected system from Lappland to Gästrikland—i. e. covering the whole of Norrland. The present lines, however, are only designed for rather local cooperation, and to supply about 5 000 or possibly 10 000 kW, and cannot be excepted to transmit power from Porjus to Central Sweden. For this purpose lines of very different size and calibre will be required. Certain parts, however, are strongly built, for instance the 132 kV Porjus—Boden line (fig. 22). It is also worth noticing that in this line the towers are of a new type, built up of two galvanized Differdinger girders and anchored by another method than the formerly used concrete foundations.

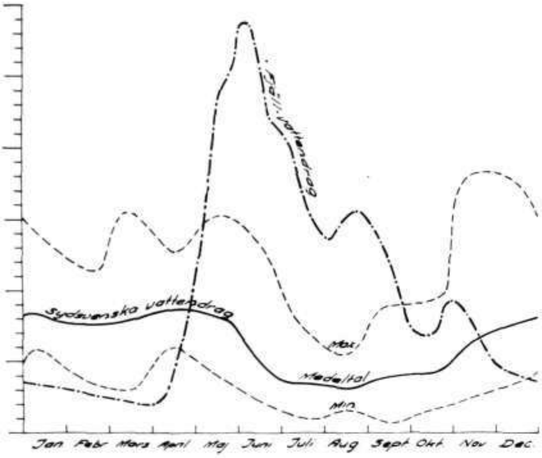
Distribution lines for national cooperation in Sweden.

Such rather local connexions are nevertheless of great importance, and will prepare the way for cooperation over longer distances and involving more power.

This extended cooperation over the whole or greater part of Sweden will require a transmission system of somewhat different form from those previously described. It will here be a question of transmitting power at distances getting on for five or six hundred miles, many times the present distances. The first transmissions to be dealt with will be those from central Norrland, primarily from the Indal river down to the industrial districts of central Sweden, and subsequently further South. The principal object of this transmission is to supplement the power resources of the latter districts from the enormous supply in the Norrland rivers. About 20 years ago certain works in central Sweden acquired between them the great Krångede falls; more than 10 years ago the city of Stockholm purchased Svarthålsforsen, and the Board of Waterfalls



R 4038 Fig. 22. Modern pylon on the Porjus—Boden line, made of galvanized Differdinger girders and anchored by a new method.

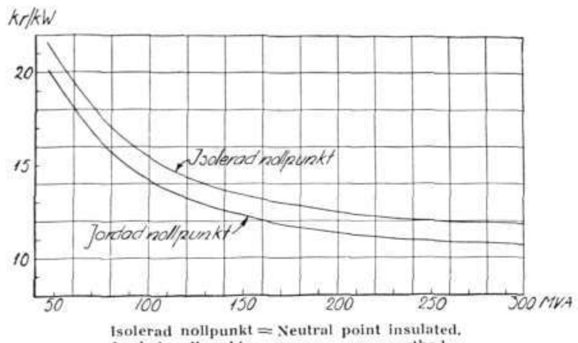


Fjällvattendrag = Norrland Rivers.
Sydsvenska vattendrag = South Sweden Rivers.
Medeltal = Mean.

R 4039 Fig. 23. Curves giving the water-flow in rivers of Norrland and of South of Sweden.

has bought Stadsforsen and others for the State. All these power resources are in the Indal river, and are intended for safeguarding the requirements of their respective owners. This scheme will involve a transmission of power which will gradually approach a million kilowatts.

The immediate task of these Swedish main distribution lines will be to transmit power from Norrland to the South. A distributing system as extensive as this may, however, also serve other important objects. The arguments in favour of and the profits from cooperation will be stronger and greater when power stations and consumers are distributed over as large an area as half Sweden. It will suffice to show how the annual water-flow fluctuations in a Norrland river differ from these in South Sweden (fig. 23). In the winter there is always a shortage of water in the former, and generally a surplus in the latter. The spring flood in Norrland has just begun when the summer low-water period begins to be felt in Southern Sweden. The abundance of summer water in the Norrland rivers often lasts up to the time in the autumn when the South Swedish rivers again begin to fill. Often exceptionally dry years in the South are balanced by good water years in the North, and vice versa.



Isolerad nollpunkt = Neutral point insulated.
Jordad nollpunkt = „ „ earthed.

Fig. 24. Variation of minimum transmission costs (including losses) with the power; distance 300 km.

Clearly the task of main distribution lines between Northern and Southern Sweden—to carry the surplus power from South to North in the winter and from North to South in the summer—will be very important. The load on these lines will therefore fluctuate considerably, and they must be designed to work satisfactorily even under these conditions.

Economic aspects of long-distance transmission.

From the above, it will be obvious that these lines must transmit very large amounts of power if the transmission costs per kW are to be low enough to make the scheme profitable. To give an idea of the economic factors affecting such a transmission, I will give the results arrived at in the recent investigations on transmission costs at a distance of 300 km. (180 miles), which is approximately the distance between the Indal

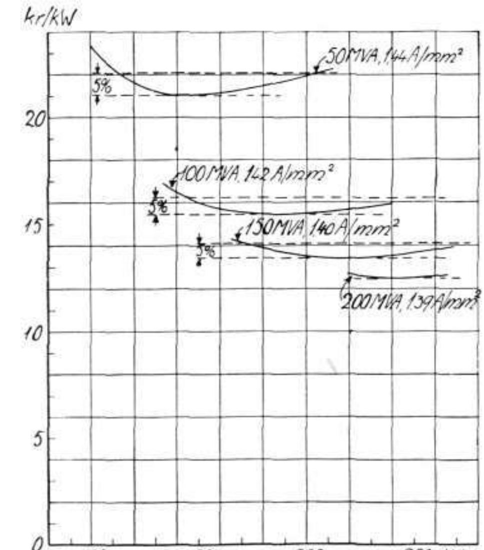


Fig. 25. Variation of transmission costs with voltage, the power constant, 50, 100, 150 and 250 MVA; distance 300 km.

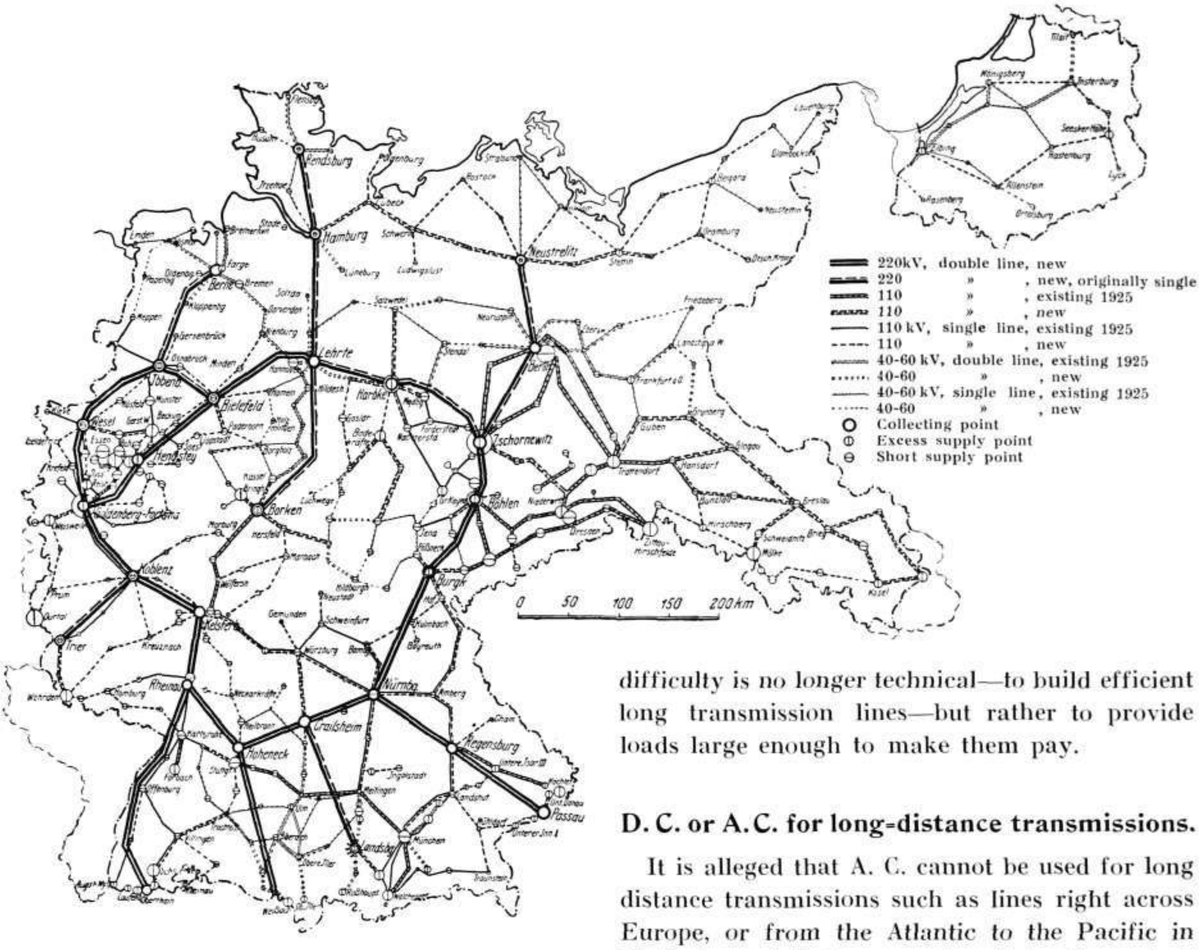


Fig. 26. Map of proposed main transmission lines in Germany.

river and Bergslagen. The curves of fig. 24 show that the transmission cost per kW and year — including costs of line, transformer station, and all losses—decreases rapidly with increasing power and approaches a limit, apparently in the neighbourhood of 10 kr. per kW and year, at about 200 000 kW carried by a single line. Fig. 25 shows that the most economical voltage rises with the amount of power, and to be economically sound a line of this kind should accordingly be built for at least 150 000 kW per line, with a corresponding voltage of 220 kV.

The question will then be: Is there enough power to provide a satisfactory load for one line, or preferably for two? As we have said, the power supply in Norrland is sufficient, which brings us to the factor which in reality sets a limit to our long-distance transmissions. The difficulty is no longer technical—to build efficient long transmission lines—but rather to provide loads large enough to make them pay.

D. C. or A. C. for long-distance transmissions.

It is alleged that A. C. cannot be used for long distance transmissions such as lines right across Europe, or from the Atlantic to the Pacific in U. S. A., and that we must wait for the arrival of high voltage D. C. Self-induction and capacity, always present in A. C., are supposed to make transmission impossible at distances very little greater than those already attained. In lines of such length alternating current is supposed not to give stability in operating.

It is quite true that if the load on a long three-phase transmission is increased above a certain value, the reactive losses from self-induction will be so large that work is inconceivable, and the stability will also be so low that the machines will be thrown out of phase. In very long lines it may also—if there are no intermediate stations —be difficult to deal with the capacity currents. These difficulties are supposed to disappear with direct current.

But the specific load at which these difficulties appear is generally appreciably higher than that which gives the best results economically. The transmission problem being primarily economic, there is consequently no cause to worry about difficulties occurring only in lines which, from an economic point of view, are incorrectly designed.

Extensive systems of main transmission lines are feasible even with alternating current.

Nor need we be intimidated by difficulties which might arise in a line one or two thousand miles long without intermediate stations. If a through line of this length were built, there would seem to be no reason why it should not be connected at several points to existing systems for cooperation and interchange of power. For natural reasons, the distances between transformer stations in, for instance, the Swedish 220 kV main distribution lines are not likely to be more than 200, or at most 250 miles. If such intermediate stations are arranged, the problem of stability and capacity currents will be comparatively easily solved.

In Germany also, the plan for coordinating all the power works includes so many stations, and consequently points of support, that fears of such difficulties in working may be dismissed (fig. 26).

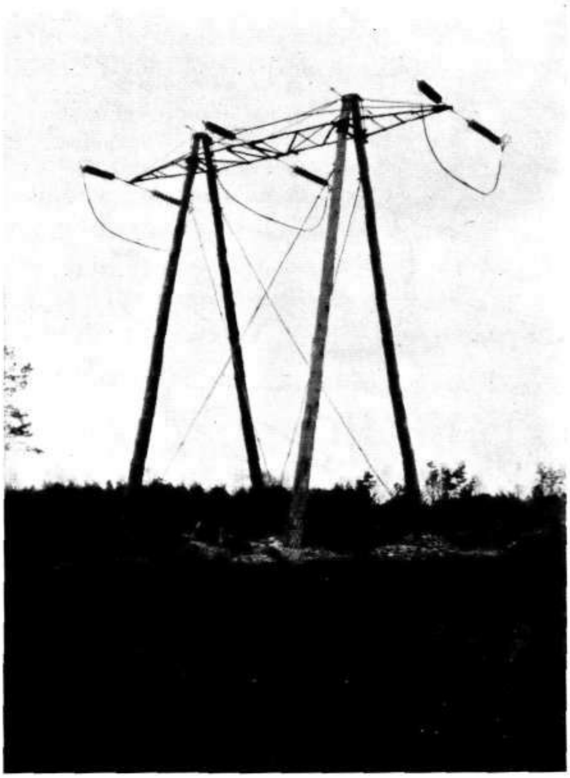
Such a system of large trunk lines will thus, at intervals adapted to the voltage and increasing with this, be provided with transformer stations at which they will be connected for cooperation with the regional power distribution systems. At all points where large power stations are not connected, rotary condensers will be installed, in order not only to stabilize effectively the whole system, but also, by some over-compensation, to regulate the voltage to a constant value throughout the system. Reactance coils, compensating the capacity currents at no load, may also be placed at these points.

Some problems of detail in 220 and 400 kV lines.

In principle, consequently, there seem to be no fundamental difficulties in building very long lines for very high A. C. voltages. But the building of such lines will encounter several important practical difficulties which must be overcome. The corona, for instance, necessitates making conductors for high voltages of very large diameter. It is not certain that the expedients so far used—steel-cored aluminium or hollow cables—will be suitable in their present forms. Vibrations and fatigue-cracks, compelling modifications of design and greater attention to the quality of the material, and its properties under fatigue stresses, have ocurred in these heavy conductors. The ordinary types of insulators may for mechanical reasons be impossible to use. The forces from the conductors, which at 400 kV must have diameters of nearly two inches, and cross-sections of more than one sq. inch, will be so great that it may be necessary to construct new types and designs of suspension insulators. A study of the effect of fatigue stresses on the insulator porcelain is also of the greatest importance.

In any case it is essential that the risk of arcing, which at these high voltages is very destructive, should be reduced. Arcs must be kept away from the porcelain, which is sensitive to great heat, and devices like the protective rings shown in fig. 11 are used. Other parts must also be sufficiently separated. It has, for instance, proved necessary to stiffen and frame the slack loops on the anchor tower, to keep them far enough from cross arms etc. to prevent arcing (fig. 27).

As regards types of towers and general arrangements, it would seem that on the whole the same principles apply when building a line for, say, 220 kV as for 132 kV. We may suitably take as our starting point the latest and most up-to-date constructions, those used by Sydsvenska Kraftaktiebolaget for their new 132 kV lines. The material used there is very largely creosote-impregnated wood (figs. 27, 28). If we are to use this type for 220 kV, the poles must be made higher, and therefore also heavier, than for 132 kV. Instead of heavier single wooden poles, which will be hard to get, it will probably be necessary to use timber constructions or wooden poles with lattice work superstructure, or even to change over to all-steel towers. All these alternatives may have to be tried in practice before any particular type suitable to our country is found. I hardly think, however, that this country need go in for the American types with fine-meshed lattice work of comparatively small-gauge steel. We have good reason to stick to our own simple, neat,



R 4035 Fig. 27. Wooden poles for 132 kV with protective rings and specially shaped loops to reduce the risk of flash-overs. The new Sydsvenska Kraftaktiebolaget main transmission line.

and strong types of pole even for these high voltages.

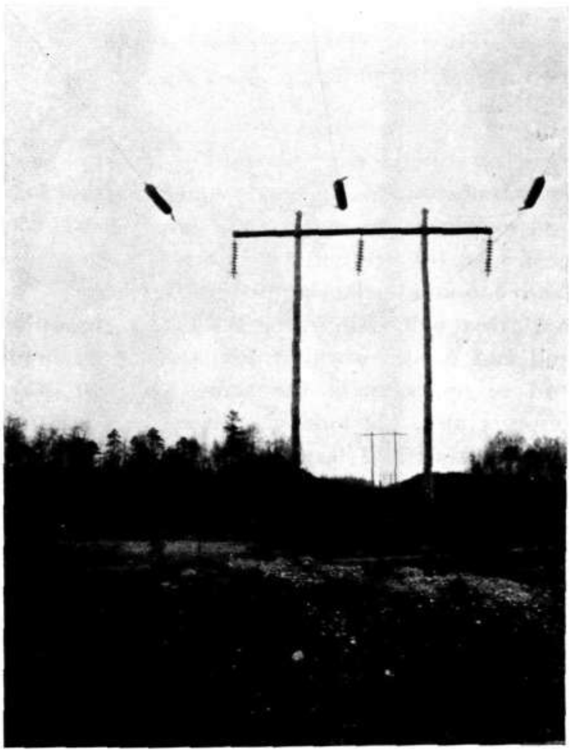
Although practical problems of this kind will have to be solved when building 220 kV lines, and even more if 400 kV lines have to be made, they are not of a nature to prevent the technical achievement of these transmissions while retaining the alternating current.

Aspects of long-distance D. C. transmission.

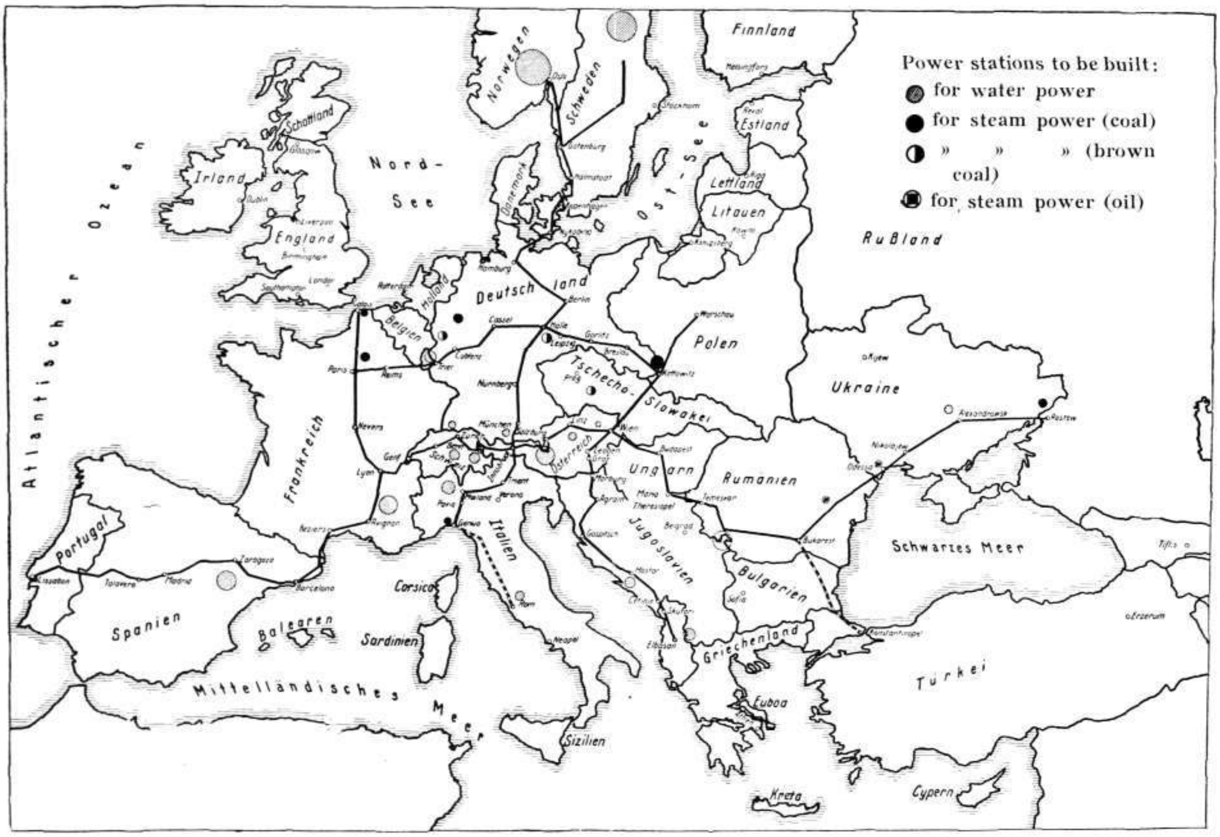
Matters will be different if a cheap and simple machine, able to transform alternating current to high voltage direct current and vice versa is invented. In that case there would be every reason for inquiring into what economic advantages might be gained by using that method. The machines known and used so far, however, have been much too expensive and inconvenient to be of any real benefit in our present transmission systems, but new designs may of course appear. At the World Power Conference in Berlin in the summer of 1930 rumours were current of impending revolutionary inventions. As far as I know, nothing more has been heard of the matter since. I wish to point out that D. C. transmissions suffer from several drawbacks from which A. C. lines are comparatively free. Voltage regulation, for example, is generally far more difficult in D. C. than in A. C. Changing from A. C. to D. C. and vice versa will therefore not be the only difficulty; a good many other problems must also be solved before we can pass to long distance transmission of high voltage direct current.

An European super power system.

As a final illustration of the trend of power transmission, I will give a map of the proposed system of inter-European super power system (fig. 29). This proposal was submitted by Mr. Oskar Oliven, Dr. Ing., in a lecture given to the entire World Power Conference in Berlin. The funda-



R 4041 Fig. 28. Wooden pole for 132 kV with a device for giving the line 1/3 of a turn this side of the pole. The new Sydsvenska Kraftaktiebolaget main transmission line.



R 4042 Fig. 29. Dr. Oliven's proposed super power-system for Europe, submitted at the World Power Conference in Berlin, 1930.

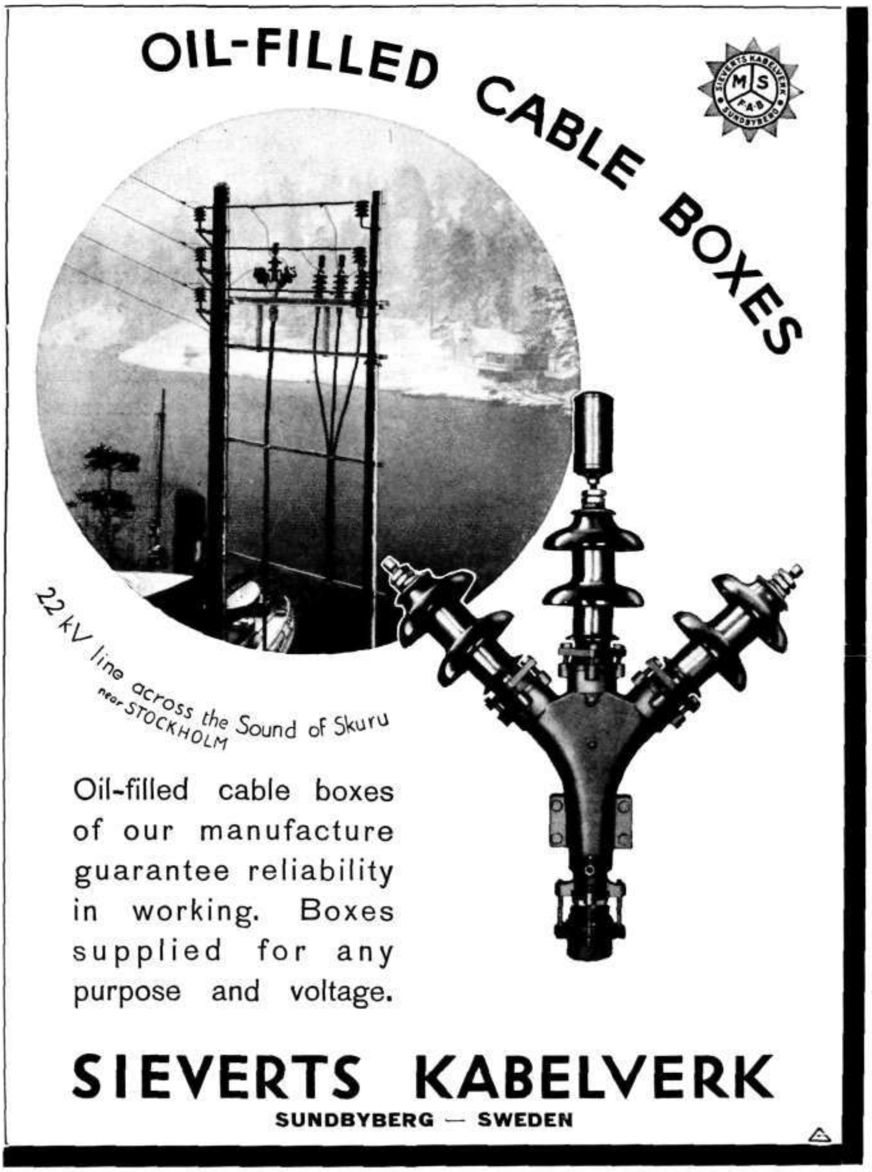
mental idea of this project is equalization of load and exchange of power on a large scale. The peak load for lighting, for instance, occurs more than 3 hours earlier at Rostov, farthest to the East, than at Lisbon in the West. The beginning and end of the working day, the dinner hour, and so on, vary in the same way. In other respects also, the load variations in a network of that kind will largely cancel out. There is no need to fear that a total eclipse of the sun in the forenoon will cause the power stations to break down from over-load, as very nearly happened in New York some years ago. All such difficulties from irregular load peaks will disappear.

The available sources of water power would supplement one another far more than I indicated as possible between North and South Sweden. Long-period variations in particular would certainly be almost completely balanced, as drought and scarcity of water do not appear to occur simultaneously all over Europe. In a system of this kind many waterfalls could also be brought in which would otherwise be inaccessible, and most of the existing power stations could also be utilized to better advantage, whether they employ water or fuel.

These are the advantages. The drawbacks are the £100 million odd estimated to be the cost of the plant, with lines and stations designed for 400 kV.

The quantities of power and energy consumed in Europe today are hardly likely to be enough to make such a system a paying proposition. But in view of the growth of power consumption it seems very probable that the time is rapidly approaching when the financing of such a scheme will be possible. I am therefore able to agree with Dr. Oliven's opinion that the project should be ventilated, principally in order that the various national transmission systems might be planned so as not to hinder, but rather to facilitate, the advent of the international organization. Even in Sweden, and particularly in Scania, through which presumably the large Scandinavian trunk lines will pass, we have every reason to prepare for the future, even if the projects of today will only be realized by the next generation.

In this review I hope to have shown that although the improvements in transmission methods have in a few decades, and not least by Swedish efforts, wrought many changes in our material culture, there are still, in spite of past achievements, many possible and as yet unused ways in which the willing and untiring forces of nature can be made more extensively available for mankind, providing power and light and heat to make life easier and brighter and warmer for us all.



Forest Telephone Lines. — Some Points in their Design.

By *Folke Johansson.*

Speaking generally of forestry, the term "communications" may be taken in two different senses. It may refer to the means of moving people and material from one point to another by road, rail, or the like—that is, exclusively a transport question, wholly outside the range of this paper. But it may also mean the transmission of messages of one kind or another, which need not necessarily be combined with a simultaneous conveyance of substantial objects. Communication is in this case effected by means of telephone, telegraph, wireless, or visual signals, of which there are a number of systems.

General Aspects.

A consideration of the messages usually required in forestry, e. g. in floating operations, fighting forest fires, etc. when speed is essential, and also of the staff employed on such occasions and the distances generally involved, clearly indicates that the telephone is the only efficient means of satisfying all the demands which have to be made on the means of communication. It is perhaps unnecessary, considering how expert readers of this journal are, to point out a few of its advantages—that it is one of the quickest means of communication there is, and particularly suitable for the transmission of long and intricate messages. In addition, practically everyone knows how to use the telephone, and the technique of telephoning is so simple that little instruction or practice is needed. It should also be emphasized that in relation to the costs of construction, operation, and maintenance, the telephone is more efficient and more reliable than any other system. Another great advantage is that a telephone line can be extended comparatively easily and cheaply—within certain limits, naturally—to places with which temporary communication is required for one reason or another. Again, portable field telephones can be used, for tapping the circuit wherever this is wanted. This last feature is invaluable in floating operations, as it enables the lumbermen to keep in constant touch with those who see to the sluice gates and other control arrangements, to give information regarding the formation of jams, and to receive orders regarding the work, thus eliminating as far as possible losses of time and water. Where floating conditions are bad, a telephone line will soon pay for itself, and also in years when it is especially necessary to economize with the water, whether on account of unusual numbers of logs or deficiency of rainfall. Yet again, in fighting a forest fire, the field telephone is exceedingly useful, enabling the commander easily and quickly to direct his various detachments and maintain communication to the rear.

In comparison with the telegraph, the telephone has the advantage of requiring no specially trained staff, and in this it also gains over wireless. Although, theoretically, wireless telephony and telegraphy should be particularly suitable for this purpose, this is far from being the case in practice. Although improvements are made in this sphere almost daily, the wireless set intended for use in the large forests still leaves much to be desired, considering that it should be easy to transport where conditions are primitive, strong, to withstand careless handling and bad external conditions, and should have good properties as regards transmission etc. The establishment of a number of small wireless stations in the wilds would, besides, be comparatively expensive, as would also the maintenance of a staff of trained wireless operators. Here we may, by the way, note that fairly extensive trials have been made for several years under the supervision of the U. S. Forest Service in order to obtain a combined receiver and transmitter suitable for use by sentries against forest fires (in what is called the tower system) in inaccessible districts. These experiments have also included the study of some questions of organization involved in the wireless method. So far, experience seems to indicate that this method may be of advantage in cases where telephone communications can only be established with difficulty and at disproportionately high cost. A set suitable for this special purpose has been designed, which last summer was subjected to very severe tests; with small weight—about 30 kg. altogether—it combines indifference to the most primitive modes of transport (e. g. pack-horses) and unfavourable climatic conditions with ease of handling and good receiving and transmitting properties. According to the P. M. of the tests, the signals from this set could be received at distances of up to 500 km., while the source of power proved sufficient for 24 hours' continuous sending. The general opinion was that the Forest Service set gave exceedingly good service in the field and showed absolute reliability even under extremely exacting conditions. The experiments are not, however, regarded as finished.

What has been briefly mentioned above shows that only the telephone method has the right qualities to enable it to be used as a basis for the special intelligence service of forestry. One of the most striking pieces of evidence in support of this is the telephone network of North America used exclusively for forestry purposes, and at present covering approximately 80 000 km. Unfortunately, no direct statistical information is available as regards Sweden on this point, but there are telephone lines along many of the more important river systems, which are used during the floating season. Considering that there are about 30 000 km. of public floating channels in Sweden—something like twice the aggregate length of our railways—there are evidently good opportunities for development in this sphere. Further, watchmen's lines and other ones for special purposes are not uncommon in forest districts where distances between villages and other habitations are great and the State Telephone network is very wide-meshed. It is therefore in the author's opinion no exaggeration to say that forest telephony has now developed so far that its technical, financial, and organization problems are beginning to call for far more attention than has hitherto been given them—in spite of the fact that they might seem small and insignificant in comparison with the large problems of telephony. This is perhaps most applicable to certain details of the laying of the lines, of which a short and by no means exhaustive account will be given below. For natural reasons it must be based on American conditions, for the simple reason that American engineers and forestry experts have made special efforts to improve the various methods of laying lines on land, and have met with considerable success.

Various Types of Lines.

When putting up telephone lines of the simple type which is generally used in forestry, where the greater part is naturally in more or less wooded areas, it is possible to distinguish two main types of actual line construction, namely, what has received the name of *pole-line*, and what we will by analogy call the *tree-line*. The former is not essentially different from the ordinary type used in country districts; and so poles, insulators, hooks, etc. of the usual models are used for it. The latter, on the other hand, is a type which has been much modified to suit the conditions prevailing in districts with plenty of mature, close-grown forests, in order to simplify and cheapen the work of erection, and at the same time to get a line which will give satisfactory results in practice. The difference between them is considerable, with the practical consequence that, among other things, a staff trained exclusively in putting up pole lines will generally employ methods whose application to the other system will give poor results.

The fundamental principles of the tree line, which are probably not very widely known, are hinted at already in the name, which is due to the use of growing trees instead of special poles for carrying the wire. The trees are selected as far as possible at equal intervals—a span of 30 m. is probably about the best; in no case should the distance exceed 50 to 60 m.—and should be fairly stout, to reduce the sway in high winds. But they must not be so large as to be difficult to climb when the wire is being put up. The branches are lopped off to a height of about 6 m. from the ground, which is the height at which the holders of the hanging insulator are fixed.

These consist of about 50 cm. of iron wire, usually No. 12 B. W. G.; one end is anchored to a 3" bolt in the trunk, the other twisted round a ring-shaped split porcelain insulator as shown in fig. 1. The wire rests freely in the insulator ring, and is not fastened to it.

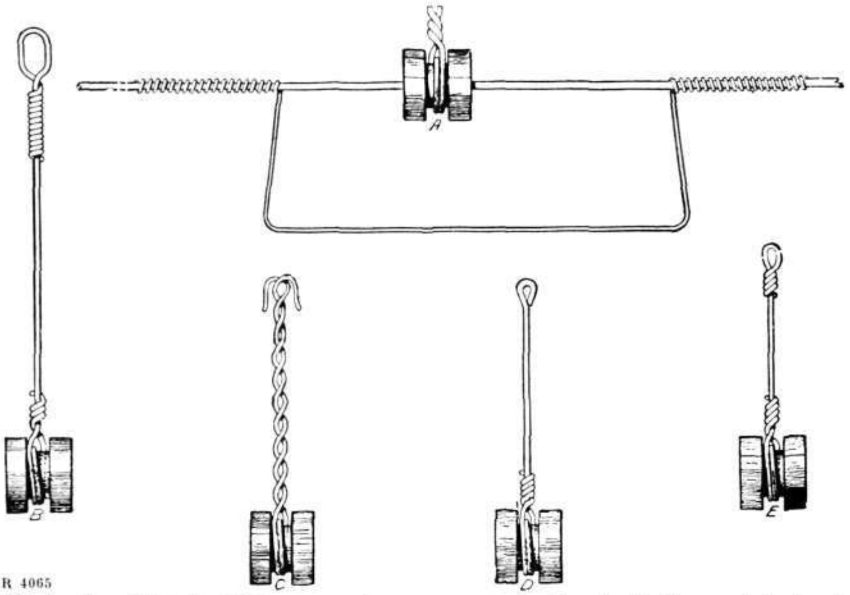


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Fig. 1. Insulator ring into suspension arrangements for elastic lines. A device to prevent a broken wire slipping through the ring. B and E when great steadiness is required, C and D the usual types. All are made from No. 12 B.W.G. except D, for which No. 9 is used.

In this connexion we might mention that two methods are used in selecting suitable trees, the zigzag and the curve methods. In the former trees are chosen so as to form a zigzag line in the right average direction (see fig. 2), and they will then be alternately on either side of the wire, forming an "avenue" about 2 or 3 m. wide. The wire consequently runs a zigzag course through the insulators on the inner sides of the trees and is thereby kept hanging freely in the air out of contact with the tree trunks. In the other method the line is built as a regular series of reversible curves (see fig. 3)—with 6 to 8 trees between the points of inflexion—in which the insulators are fixed on the concave side. This provides a free passage for the wire without having recourse to the zigzag method.

A tree line will accordingly never be as straight as a pole line. It should also be noted that considerably more "sag" is used in the former—about $1^1/_4$ m. in a span of 30 m.—than would usually be needed to compensate the contraction of the wire caused by falls in temperature. The points of support of the wire in this type of line are usually 60 per cent. more in number, and the sag 4 to 6 times greater, than in the pole line.

Between these 2 "pure" types of line there are a number of mixtures, more or less adapted to local conditions and more or less well designed. The so-called "wilderness" lines will probably be well known from the Northern parts of Sweden; in these single lines are put up on growing trees by stripping the branches from that side of the trunk on which the line is, screwing a telephone hook into the wood and stretching the wire over insulators of the usual type. Branches on the other side and those higher than the hook are left untouched,

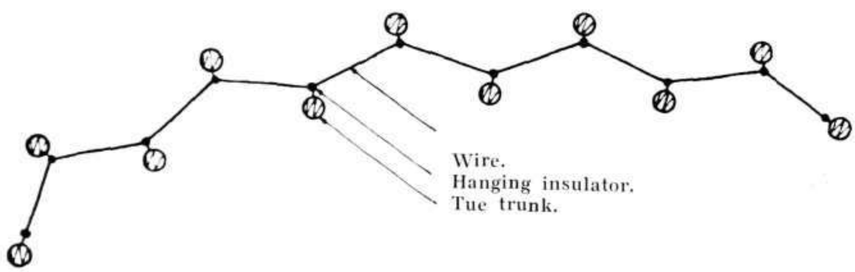

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Fig. 2. Diagram of zig zag method.

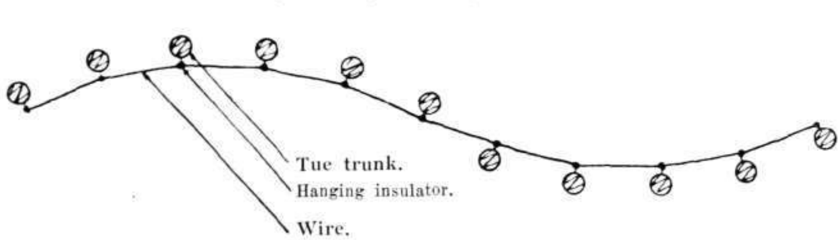

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Fig. 3. Diagram of curved line method.

and the tree is then not considered to suffer any damage.

The Uses and Properties of the Various Line Types.

As regards the use of pole- or tree-lines in various types of country, the former is used mainly in open ground, where there is no wood and not enough growing trees available, as well as along roads and other communication routes. When the line has to be laid through forest it is best to cut a lane, giving the line a free passage and preventing damage from windfalls, etc. A characteristic of the pole line is its "inelasticity", i. e. it is anchored to each insulator. Trees falling across the wire will therefore break it and cause interruptions, a risk which must be eliminated by felling everything near the line on either side.

By its very nature, the tree line can only be used in relatively well stocked, dry forest ground with a good supply of mature trees, but it should be observed that no lane needs to be cut, a point of great importance in practice, both as regards the reduction of building costs and the saving of the growing forests. In this instance the danger from windfalls is obviously increased, but the greater risk is balanced by the line being so arranged that the wire runs freely through the insulators, with ample sag, and is therefore "elastic". The beauty of this arrangement is, that if the line is caught by a falling tree, it will be forced down, giving instead of breaking, on account of the elasticity. If the worst comes to the worst, one or two insulators will be torn off and much of the wire will be lying on the ground, but it will not be broken.

The fundamental idea of this design is that under high stresses it will give without breaking, and that any damage done will be confined to the fastenings of the insulators, which do not form part of the circuit. It might then be asked what the use is of such a method of laying lines in telephony.—The answer is that a broken line means interrupted traffic until the damage is repaired, while a wire will often function moderately well even if large portions of it are lying on the ground, and even buried in dry snow. If the surface of the ground is wet or if the wire falls into water, it will of course stop working, just as if it were broken, but experiments have shown conclusively that a single line may lie for long distances on dry or frozen ground without being rendered completely useless. The amount of undergrowth and how this is made up are two important factors in this connexion.

Some comparisons between the two Types of Line.

The maximum length of a tree line is for various reasons considerably less than that of a pole line. Copper wire cannot of course be used for this type, and ordinary galvanized iron wire must therefore take its place. The smallest dimension that, in view of the large stresses, can be used with safety, is No. 9 B. W. G., and the maximum length of line is then, unless special arrangements are made, 150 to 200 kms., a distance more than ample for our conditions, but which might prove too short, say in the wilds of Northern Canada. When heavier wires are used the range will be longer, but the cost will then be heavier, usually unnecessarily so.

In connexion with the line material it should be noted that, although, the use of growing trees for carrying the line necessarily makes this crooked—either zigzag or sinusoidal in shape—with large sag, yet the consumption of wire is not increased as much as one might expect. The increase over a pole line is, under normal external conditions, an average of about 20 m. per km. If for some reason it is necessary to build a double line, the two branches should not be put up on the same trees, but two parallel lines erected; this causes a large increase in the initial outlay, which will be almost doubled. In this respect, therefore, the pole line method holds a considerable advantage over the tree line method. It should also be pointed out that the insulation will always be worse in the former on account of the simplified construction, and that the line will therefore be exposed to large current losses and leakage; hence its shorter maximum length. This is the same however well the line is built, and must therefore be ascribed to the method.

The Question of Cost.

The most important reason for the introduction of the tree-line method has been the demand for a type in which reduced building costs are combined with very great reliability. But, on the other hand, it is a mistake to assume that this type of line must necessarily be cheaper to build than a pole line, even if the supply of good, well-placed trees to hang the wire on is perfectly satisfactory. As regards efficiency, cost of operating, and maintenance, the pole type is almost always superior.

The higher building costs of the pole line actually only apply to certain parts of the work, i. e. procuring and transporting the poles, making the holes, raising the poles, and clearing a lane for the line. The cost of this may vary considerably and depends largely on the supply of suitable poles in the neighbourhood, and, as regards the digging of the holes, on the nature of the ground. The clearing of the lane depends mainly on what the forest and undergrowth consist of.

In all other respects, however, the tree line must be acknowledged to be practically always more expensive than the pole line. As we have pointed out above, the gauge of wire used for the former cannot be less than No. 9, while for the latter, at least over fairly short distances, No. 12 will do, which only weighs half as much and is therefore cheaper both to buy and to transport. Another feature of the former type of line is that about twice as many points of support for the wire have to be used as for the pole line, and the cost of insulators will therefore be greater too; the fact that the hanging, split-insulator model through which the wire runs is more expensive than the ordinary telephone insulator will make this particular item even more expensive. The greater quantity of the heavier wire and the more expensive insulator material actually make the cost of the tree line nearly 100 per cent. greater than that of the pole line.

Further, it must be pointed out that putting up and stretching the wire is very much harder work when hanging insulators are used. This is so even if specially trained men are employed for this part of the work, and depends partly on the fact that with tree lines more insulators have to be put up and fixed. The laying of the wire also takes more time and trouble, as great attention must always be given to taking the line on the right side of each tree trunk, a detail which may sometimes prove rather troublesome. Also trees are harder to climb with climbing irons than are poles, especially if the trunks are thick and covered with coarse loose bark and the workman has to saw or chop off twigs and branches as he climbs. Finally, putting the wire through the insulators and stretching it to have the proper sag will take more time in the "elastic" system.

One of the most important economic factors in the putting up of a tree line is the organization, as rational planning and methodical execution of the work are essential if the costs are to be kept within reasonable limits. With pole lines various parts of the work are now standardized, and are besides fairly simple when it is a question of one or two wire lines, as is generally the case with forest telephones. The only matter requiring special attention will probably be the proper inclination and strutting of the poles in curves. In the elastic line the work is quite different. Here, practically every single tree that is to be used to support the wire offers a fresh problem to be faced and solved. Experience has shown that, if trained workmen are not available, it is safest even in genuine forest districts to stick to the pole line if good results are to be obtained. The use of this type of line is even more justified when the forest is sparse and the clearing of the lane consequently easier, and where the necessary poles can be prepared along th lines as the work proceeds.

If, however, a comparatively long line has to

R 4057

Fig. 4. Simple pole-line.

R 4059 Fig. 5. Type forest suitable for elastic lines, with straight branchlets trunks of suitable size. Practically no underbrush.

be put up exclusively through thick, superannuated and partly damaged forests, or through other districts, where the risk of windfalls is great and the cost of thoroughly clearing a lane is prohibitive, the tree line method will probably be most suitable both practically and economically, and should then obviously be used. The best type of forest will thus be not too closely grown stands without undergrowth and with straight, branchless trunks of 10″ to 12″ average diameter (see fig. 6). But in districts where the treecrowns meet overhead, or where there are low branches or thick undergrowth, these lines are more difficult to build (see fig. 7).

Outline of Development.

In this connexion a short outline of the development of the "elastic" line method of building might be of interest. Although thousands of km. of provisional lines of this type have been laid during the building of roads, railways and other similar pioneer work in the wilds, the U. S. Forest Service must be given the credit for having so far improved the method that it can be used satisfactorily for more permanent lines as well. Behind it are nearly two decades of practical experience. It should be noted that in the State Forests the telephone network of the Forest Service alone amounts at present to almost 40 000 km., of which a considerable portion consists of tree lines. The network is being extended as available means permit.

The first efforts at using growing trees when building telephone lines were made on the whole on the same principles as in inelastic lines; in other words, it was a kind of pole line where the poles had been replaced by growing trees placed by Providence at suitable intervals. In the line so obtained interruptions were far too common, and this type of line soon proved rather unpractical. To increase the reliability, a more elastic line was wanted, and the line was therefore carried in wire loops fixed to the insulator caps instead of being stretched between the caps. The loops were gradually replaced by fixed porcelain rings, which had in turn to give way to the now generally used suspended and split insulator. In course of time this too has been modified in some respects, and has lately been made oval in shape (see fig. 8).

At the same time the methods of fixing the

R 4058 Fig. 6. In this type of forest, tree-lines should obviously not be used.

hanging insulators in the trees have been improved. At first a piece of ordinary telephone wire was used for this purpose. Nowadays there are—as shown in fig. 1 — a number of different methods of arranging and fixing the wire carrying the insulator ring, each intended for some special purpose.—In the beginning, the importance of ample sag and even spacing of the trees was not recognized, but it was soon found that proper regulation of these factors was of vital importance if the line was to resist various threatening calamities. Although the modern tree line differs widely—in its sag and, from the ordinary telephone builder's point of view, in its peculiar looks—from the usual permanent telephone lines, experience has nevertheless shown that these methods of construction are essential for achieving a method combining relatively low building costs with high efficiency.

As the methods of building have improved, so also have the telephone materials made much progress. But that is another story!

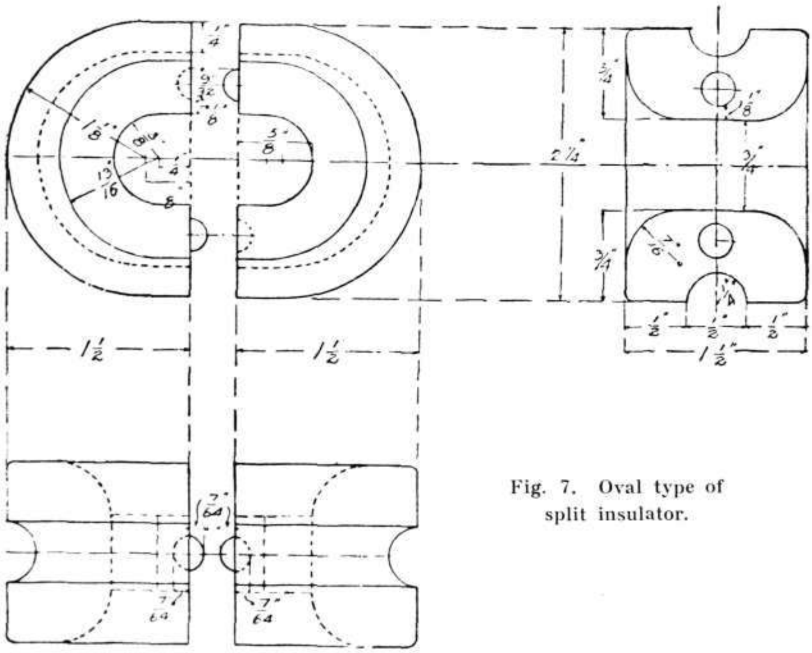


Fig. 7. Oval type of split insulator.

R 4066

Present Tendencies in Electrical Wiring Methods.

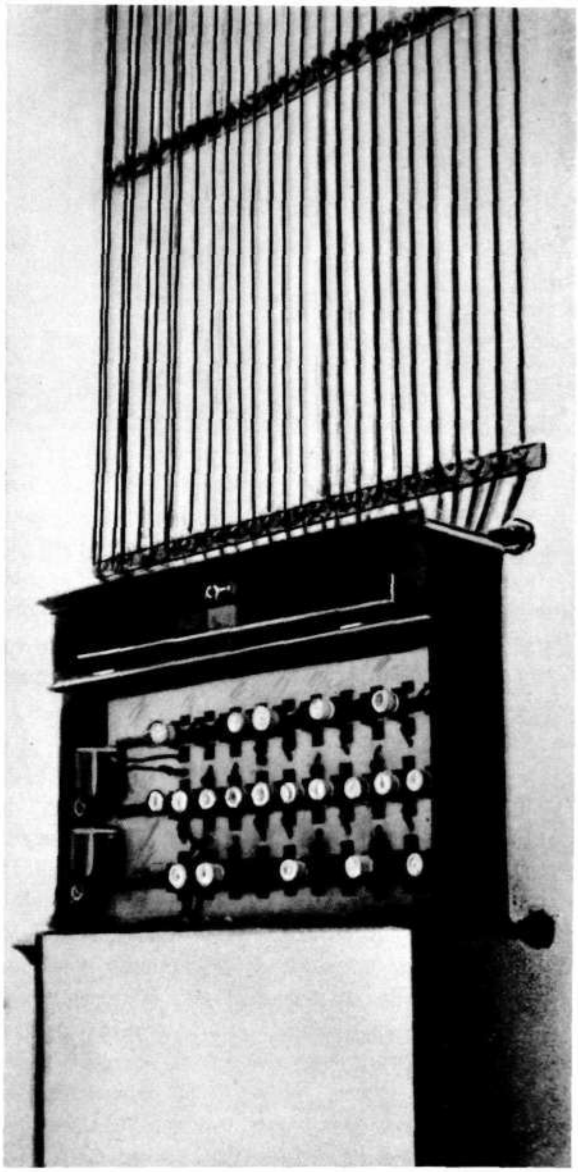
By *T. Husberg.*

In the 1880's, when high tension electric current came into use for lighting supplies and to some extent also for power, the experience gained in low tension installations was, of course, all that was available. It is therefore perfectly natural that mechanical damage to the conductors was primarily considered. It was thought that to let the wires follow the walls, and to provide them with some kind of protective covering, was the best means of safeguarding the working reliability. This led to the use of wood casings, where the wires were put into grooves between two strips of wood screwed to the wall. Where this method was considered too expensive, the wires were fixed to the walls with staples, with some kind of fibrous tape between them and the wall.

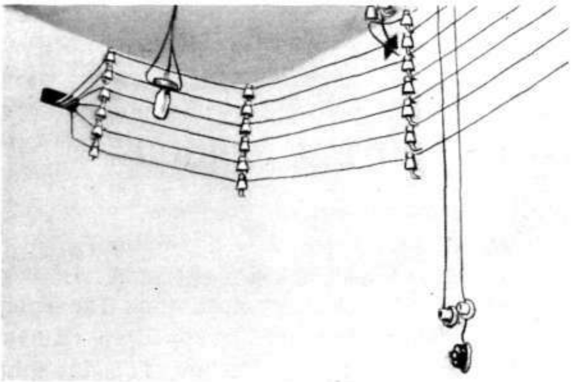
Originally, the conductor was just spun over with cotton, and this insulation was sufficient as long as only 110 volt D. C. was used and the premises to be wired were perfectly dry.

But certain portions of electrical plants were necessarily exposed to chemical action, especially as wiring was soon also needed in other than dry places. The wood casings obviously not only offered insufficient protection against, for inst., moisture, but under certain circumstances they actually involved a risk of fire. The idea of removing the wires from the wall and fixing them on studs made of some insulating material, e. g. porcelain, was then mooted. Wood casings were therefore discarded in favour of the method which, with certain modifications, has been used ever since, and which we call *single conductors on porcelain insulators.*

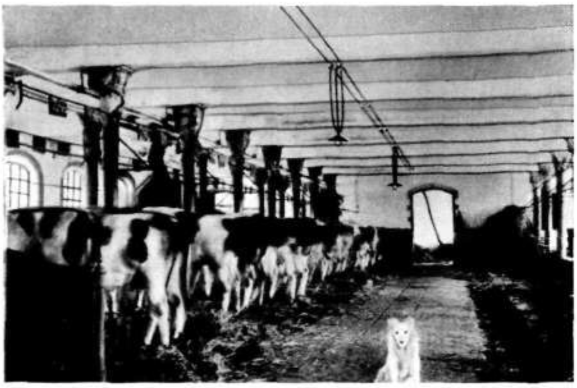
However, conductors fixed on porcelain studs at a certain distance from the wall were not satisfactory, as, when the wires had to be crossed or passed through uninsulated walls etc., occasional short circuits could hardly be avoided. To reduce these risks, the conductors were therefore provided with an insulating cover of considerably greater dielectric strength than the spun cotton, viz. a rubber tape wrapped in one or two layers round the conductor. This brought the *rubber-taped conductor* into being.



R 1886 Fig 1.



R 1884 Fig. 2.



R 1887 Fig. 3.

As electricity became more widely used for illumination purposes, the working conditions for conductors and other wiring material gradually became more exacting. Both the insulating and mechanical strength of the rubber-taped wire fixed as a single conductor on porcelain studs soon proved insufficient to withstand the siresses to which it was exposed. Improvement was sought in both the above directions, i. e. by altering the method of fixing the wires and by strengthening the insulation.

Before the introduction of high tension systems, lines in the open air had furnished valuable experience of how to provide sufficient insulation even for heavy moisture. Certain indoor premises were considered to present working conditions similar to those of the open air, and for damp indoor premises *single conductors on open-air insulators* were therefore resorted to.

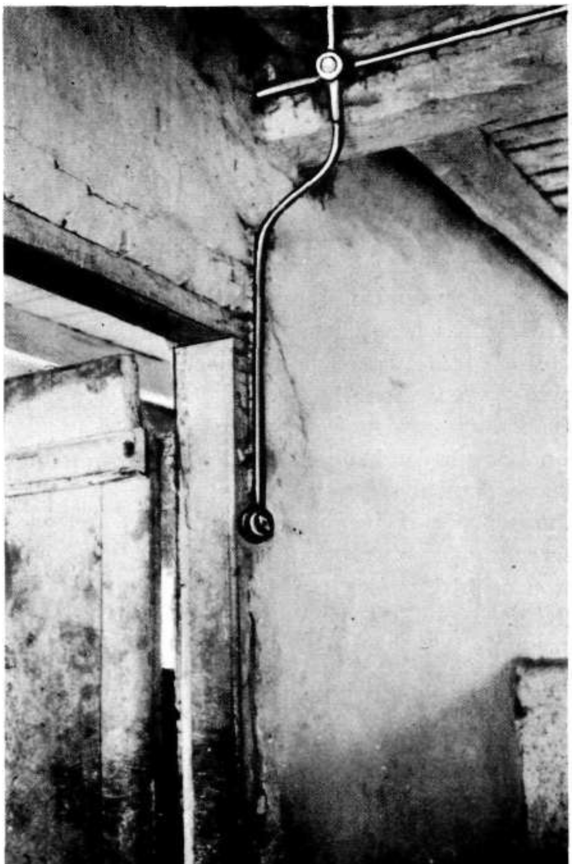
That these wiring methods, if carefully and skilfully applied, gave good and valuable results, is amply proved by the several examples of conductors on outdoor insulators or on small porcelain studs still in use. Some recent photographs from such installations show how the rows of porcelain studs were fixed on strips of wood, (fig. 1) how the outdoor insulators were bolted to the walls in the same vay as to poles in the open air, (fig. 2) how the problem of carrying the conductor through walls was solved, (fig. 2) and how switch wires could in a simple way be led down from the group conductor in the ceiling (figs 2). It is fairly obvious that wires fixed on outdoor insulators in a cowhouse (fig. 3) will offer a considerable resistance to earth. Passages through walls, switches, and lamp-holders will obviously be the weakest points.

Rubber-tape insulation was liable to dry and crack, thereby losing the greater part of its insulating properties. It was therefore necessary to find some jointless and at the same time tough and flexible covering. When the manufacturers succeeded in producing a conductor enclosed in a vulcanized pressed rubber cover

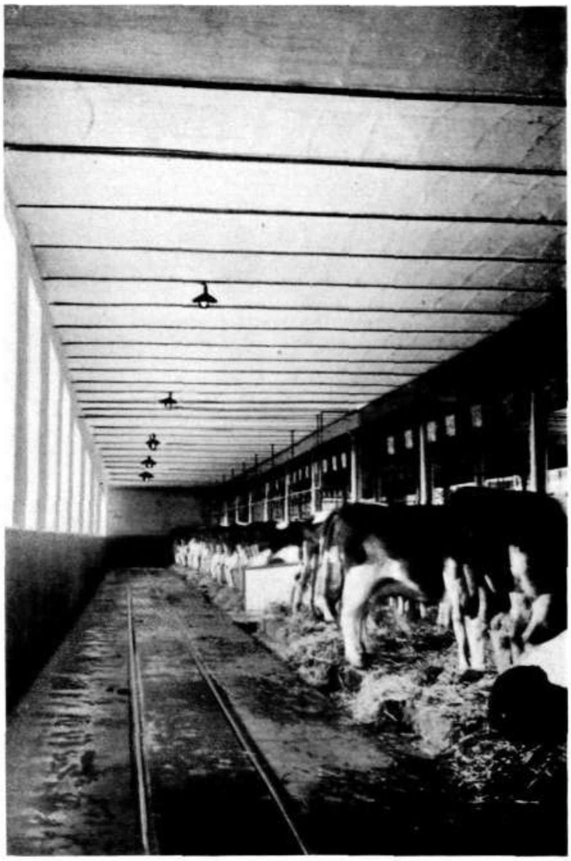
R 1890 Fig. 4.

which fulfilled these conditions, this *vulcanized conductor* proved to be the most valuable means to date for making the wiring of electrical plants safe and reliable. This was particularly convenient, for as electrification became more general, even in rural districts, during the early years of this century, the working voltage was raised and alternating current became more usual for lighting purpose also. The strength of the insulation required for 220-volt A. C. must, as we know, be of an entirely different magnitude than for 110-volt D. C.

Vulcanizing had provided such an excellent insulating material that conductors insulated in this manner could be put up in immediate contact with one another, as for example as *twin-wires on studs,* or enclosed in a common cover of cotton yarn or the like, forming *2-, 3-, and 4-wire conductors,* without the risks run when placing rubber-taped conductors in the same way. One of the indications of the raised demands on the quality of the insulating cover is the well-known increase of the rubber content from 25 to 33 per cent.



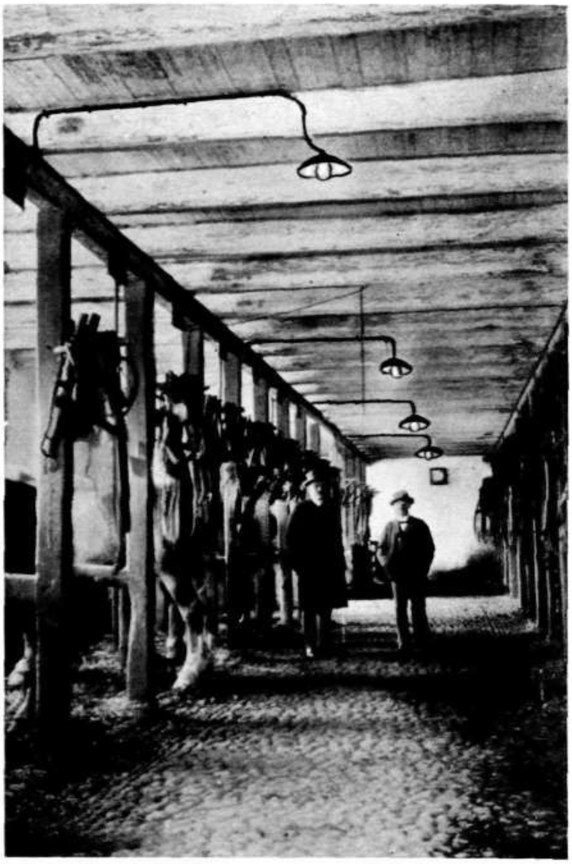
R 1889 Fig. 5.



R 1908 Fig. 6.

Even though the vulcanized conductor mounted on porcelain studs of suitable size had proved fairly satisfactory as far as the insulating stresses were concerned, this method of wiring was still too liable to suffer mechanical damage. Metal covered insulating tubes were considered to offer an equivalent to the merits of wood casings in this respect. According to the degree of mechanical strength required, the sheath would consist of leaded sheet iron or steel conduits. Wherever the leading was not considered *sufficient protection* against the corrosive effect of chemicals, the insulation was covered with brass sheeting, and as the benefit of the sheath as an insulator was sometimes doubtful, the mechanical protection alone afforded by the pipe was considered, and the vulcanized conductor was enclosed in uninsulated steel piping.

The various forms of *conduits* have been of



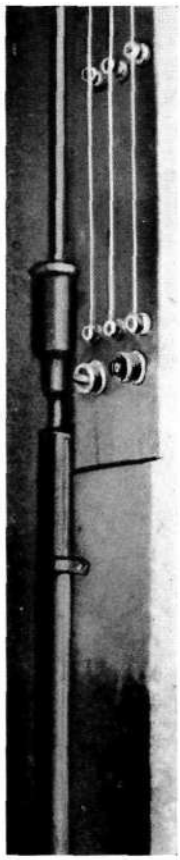
R 1893 Fig. 7.

extraordinary assistance in wiring operations, and will certainly play an important role in future also. By this method the early tendency of the wood casings—to keep the wires out of sight—could be developed to comply with any reasonable aesthetic demands. The trend of modern practice obviously is towards using vulcanized cables in steel-or steel-armoured tubing for all wiring in residential buildings, and to embed the pipes in the walls and floors. The insulated tubes covered only by leaded sheet metal are easily damabed by, for instance, nails, and are therefore gradually being superseded for use under plaster by steel pipes with or without an inner insulating tube.

Aesthetic reasons have naturally also contributed to the advent of the so called *Kuhlo conductor,* which will soon have completely ousted the twin conductor on porcelain studs. For a long time to come this type of conductor will undoubtedly be most in favour for residences and suchlike premises where considerations of cost or other reasons prevent the use of hidden conduits for the conductors.

The immediately apparent advantages over the previous methods offered by the conduit system led to its employment by some over-enthusiastic electricians even where its weak points would be particularly shown up by the working conditions. I refer to the extensive use made about 1910 of this system in stables and similar damp premises. When carefully made, and with first class materials, however, even conduits in stables may last a fairly long time before perishing of old age. Some photographs illustrate this. (Figs 4 and 5.) A pigsty is acknowledged to be a very trying locality for electric wiring, and the pipes may corrode very quickly there. The installation shown dates from a few years before the war, and the photos were taken some weeks ago. (Figs 6 and 7.) Armoured steel pipe wiring was installed in 1912 in the cow-house shown in the picture, and is still in perfect working order.

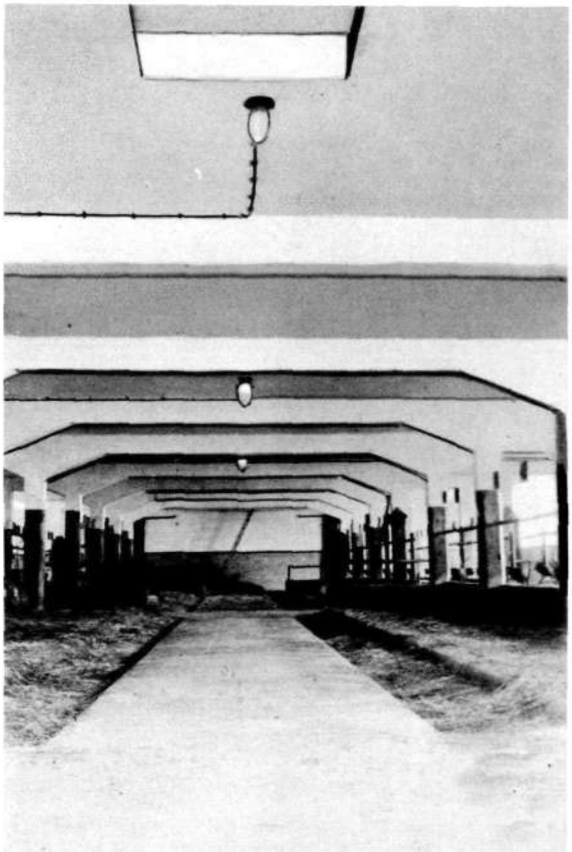
Far from all conduit wiring of damp premises, however, was done as carefully and with as good materials as those now illustrated. The weak points of the various methods therefore soon became apparent, and it became clear that the method must be varied according to local circumstances. Where mechanical damage was the principal risk, pipes were used, and where chemical corrosion was most dangerous, wires with specially prepared insulation covers were placed on ample and specially designed cup insulators. In premises of definitely one characteristic or the other, no objection could be made to this method, but working conditions involving risks of both mechanical damage and chemical corrosion will occur, and in such cases neither of these methods will be satisfactory. Another weak point was where one system was changed to another.



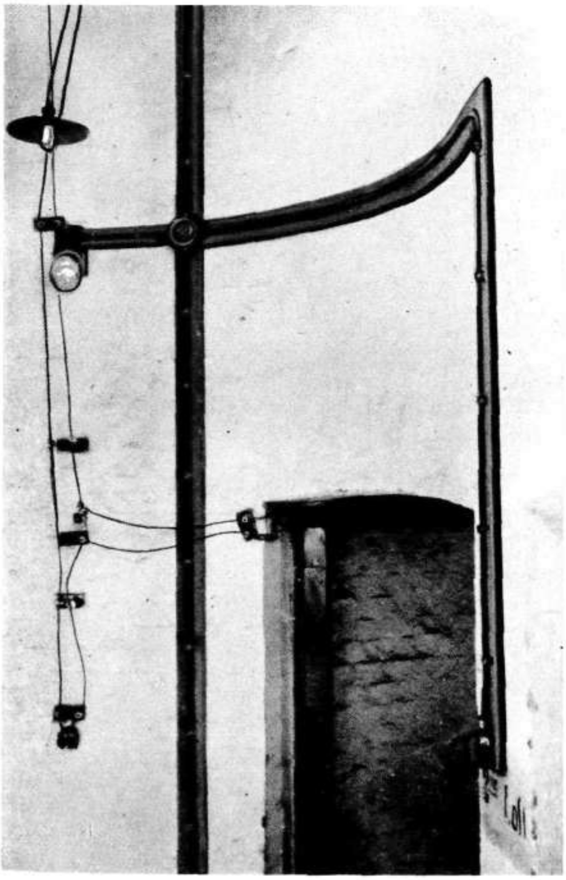
R 1894 Fig. 8.

During the war, when lack of raw materials caused a deterioration of quality in all wiring materials, the drawbacks of the different wiring methods soon showed up, and this led to the introduction of many improvements. Subsequently, when these inferior installations were overhauled, the experience gained led to a close study of the factors determining their length of life, and this was undoubtedly a consequence of the war which was of great value. The insulating properties of pipe conduits proved unsatisfactory when they were exposed to moisture or large variations of temperature. This is only natural, for, however carefully fitted, a pipe system can never be so tight that no damp can penetrate from the surrounding atmosphere. Once inside the pipes, the moisture will remain and corrode the insulating cover of the conductor and the inner surface of the pipe. Experience has also shown that conductors in pipes are unsuitable except in perfectly dry premises.

To avoid this drawback, the insulation of the



R 1897 Fig. 10.



R 1896 Fig. 9.

conductor must obviously be protected so that the surrounding air is completely excluded. The method used for earth-cables, to cover the insulated conductor with lead, did this successfully, and was therefore applied to the vulcanized conductor also. This conductor covering was proof against outside chemical influences, but a lead covering alone is not sufficient protection against mechanical stresses, and this had of course also been experienced in the case of underground cables. The mechanical stress problem was satisfactorily solved by armouring the conductor with two spiral steel bands wound in opposite directions. Just as the insulation of an ordinary vulcanized conductor was protected by an impregnated cover, the armouring of the lead-covered wire was protected by a braiding of black or red-lead impregnated hemp or cotton yarn which prevented or at least delayed the corrosion of the steel bands. Indoor conductors had thus developed into the *armoured rubber-lead conductor* or, as it used to be called, the "visi-cable" or "byre-cable".

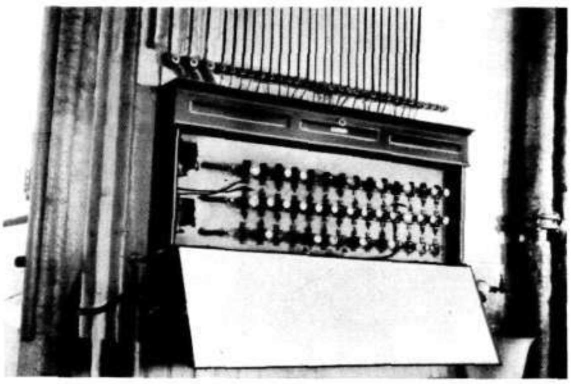
R 1899 Fig. 12.

The development of rubber-lead conductors from the earth-cables may also be traced in the development of the connexion boxes used for rubber-lead conductors. To exclude atmospheric influences, the joints must be made air-tight, and this was at first done in the same way as for earth-cables by pouring cable compound round the connexions. Most electricians will remember the compound-filled connexion boxes with glass or paper lining which was a characteristic accessory when the so-called "visi-system" was first introduced in Skåne from Denmark. To put one up was a small work of art, in which frequently even an accomplished fitter failed, and if this device had not soon been improved, the byre-cable would certainly have been very little used. We all know the radical simplification introduced by open connexions enclosed in a hermetically sealed box. The box must be exceedingly well sealed and very durable, as any deficiencies in these respects will make all the advantages of this wiring system somewhat illusory. The rapid progress of recent years in the manufacture of insulating and wiring materials leads us to hope for further perfection in connexion boxes for rubber-lead conductors. What has so far been produced can hardly be regarded as the last word on the subject. We need a less clumsy—even elegant—durable and cheap design, which will allow reliable earthing of the cover while retaining the good connecting devices of the present boxes. A rubber-lead conductor with an earth wire enclosed in it, and bakelite connexion boxes are steps towards a satisfactory solution.

The technical and economic advantages of using such wiring material as require the least possible fitting work on the spot may well be emphasized here. Standardized production will result in a more constant quality and much lower costs.

We have now followed the progress of indoor wiring methods from wood casings to vulcanized

R 1898 Fig. 11.

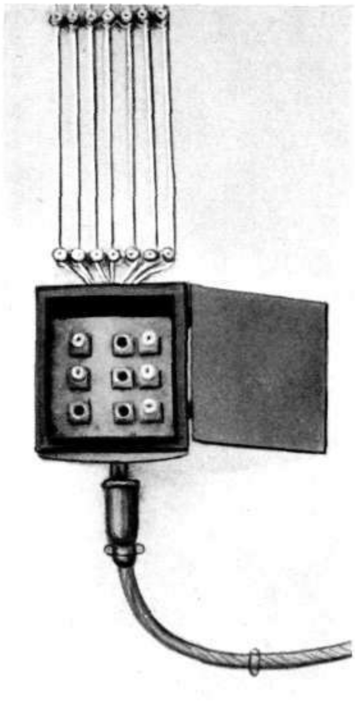


R 1901 Fig. 13.

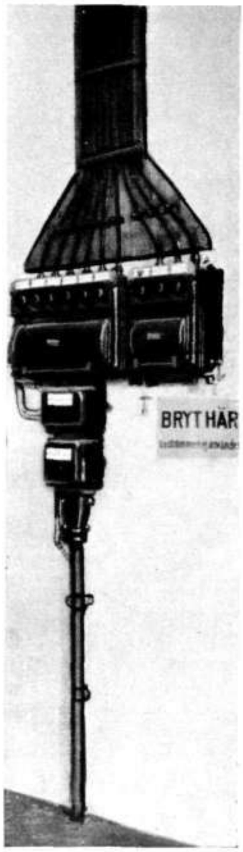
conductors in steel pipes—Kuhlo conductors—rubber-lead conductors, and will only now add that the equivalent of the rubber-lead conductor at higher voltages and powers obviously is the earth-cable, which offers the same advantages over other insulating methods as the rubber-lead conductor.

The previous illustrations have given examples of older methods of wiring, and we shall now see how these compare with the modern methods. (Figs. 8 and 9.) Steel conduits in a piggery, side by side with underground cable. The trend of progress is obvious. Rubber-lead conductors succeed the single wire conductors on studs.

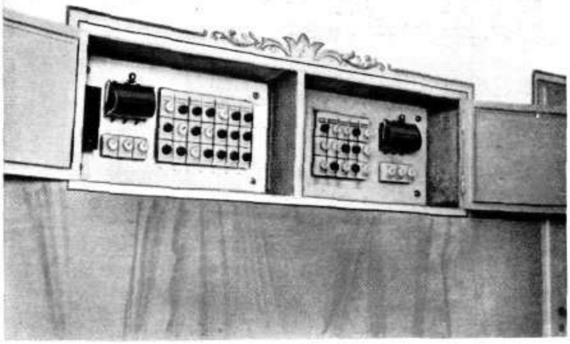
Figs. 10, 11 and 12. The latest results of progress have been utilized here. Rubberlead conductors in these large new cow-houses are perfectly in accordance with the conditions of the surroundings and the quality of the building. A correctly put up rubber-lead conductor answers the purpose well in the inflammable chaff-loft. (Fig. 12.) Obviously, the connexion through the wall between the damp and the inflammable premises will cause no difficulty.



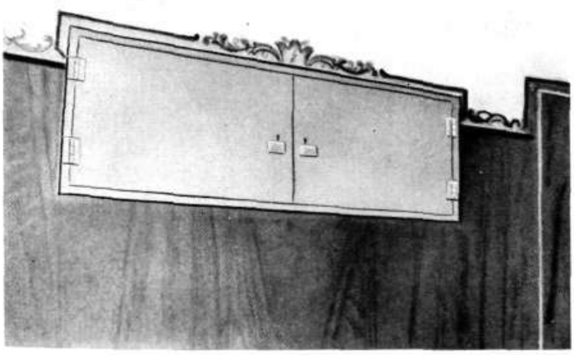
R 1905 Fig. 16.



R 1904 Fig. 17.



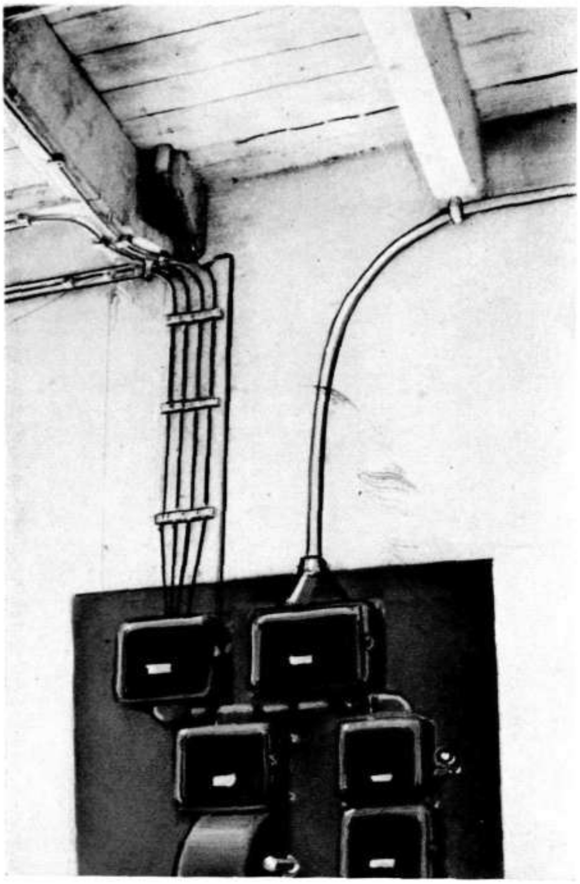
R 1903 Fig. 14.



R 1902 Fig. 15.

The wiring material being the largest and most comprehensive portion of an electrical installation, its development offers most interest. But good accessories are also required to get the full benefit from first-class wiring materials, and the improvements in these have therefore naturally kept pace with the advance in the conductors.

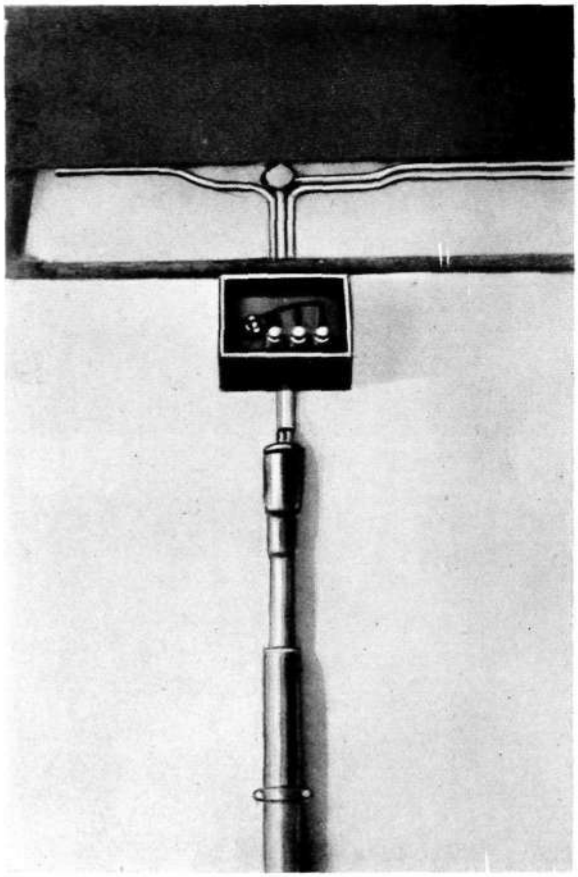
Those of us who remember the early days of high tension electricity, will probably also recollect the fuses made of a tinfoil sheet backed by a fibre slab, which might just as easily melt at double or half the rated current. The small glass tube fuses in spring holders were an improvement, but the first really great progress was the advent of the bridge-fuse device, which made a reliable rating of the protective fuse in accordance with the load capacity of the conductor possible. To reduce the cost of the part which must be replaced when the fuse melted, this was divided into a separate fuse cartridge and a screw cap. To avoid mistakes when put-



R 1906 Fig. 18.

ting in new fuses, the cartridges were at first made of different heights according to the strength of the current, but this somewhat primitive arrangement was superseded by the diazed-system, where the diameter of the live part of the fuse cartridge and the corresponding hole in the fixture increase with the strength of current, which renders any illegitimate "reinforcing" of the fuse protection practically impossible. In the two-part *fuse-box,* graduated according to the diameters of the fuse cartridges, the solution of the safety problem is perfectly satisfactory in principle. This has also made it possible to standardize dimensions and threads, and further improvement of the guiding of the fuse cartridge to make the contact between fuse and box contact absolutely reliable in fittings for high amperages also, must now be made. This demand can be fulfilled only by making the thread of the screw top fit well into the other part. For this reason a stipulation is introduced in all carefully detailed specifications to the effect that "all screw sockets in the fuse-boards must be cast solid, and the thread cut fine for currents from 100 A. upwards". Fuse boards with thin sheet brass screw sockets may now be regarded as quite out of date.

The *automatic cut-out* as protection against over-current has this advantage over the fuse, that it may immediately be taken into use again after having functioned, without any replacement of parts. The risk of making mistakes in the rating of the overload protection is also considerably reduced by the use of automatic cut-outs. For motors and other apparatuses liable to occasional overloads, the automatic cut-out is rapidly supplanting the fuse. As protection for a three-phase motor, for example, the correctly designed automatic cut-out has the great advantage over the fuse that an overload in *one* phase will cause the current to be cut off in *all three* phases, and the motor will thus be protected against overheating by "running on

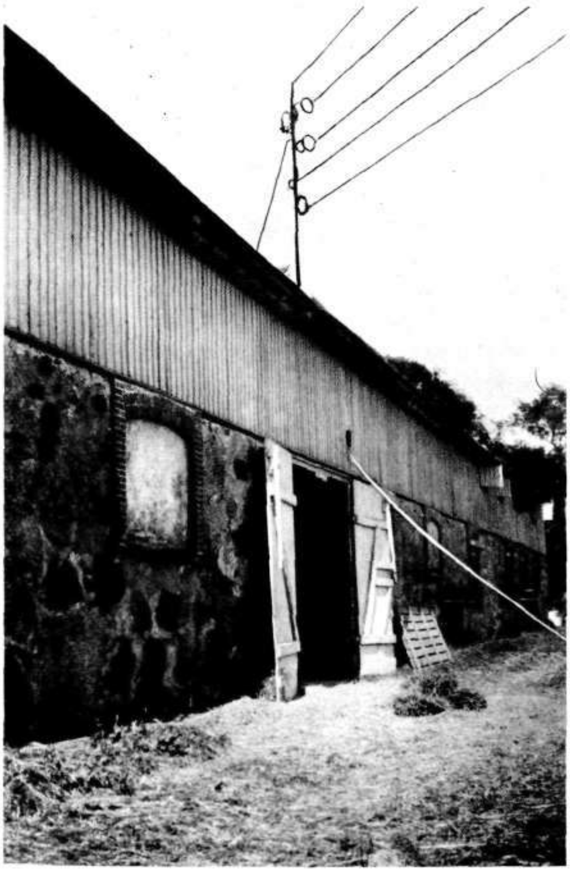


R 1907 Fig. 19.

two phases". The trend of progress in over-current protection for motors thus distinctly points to a more general use of cut-outs, designed on electrothermal or electromagnetic principles, instead of fuses.

For reasons similar to those which led to the design of cut-outs for motors and the like, attempts have also been made to replace the fuses in ordinary domestic installations by small automatic devices. In a comparatively small compass, however, these German so-called "Klein-automaten" must necessarily contain a fairly complex mechanism. But this apparatus needs, in its present form, fairly favourable working conditions to work well. The premises, anyway, must be dry.

The old bridge design is still doing duty in many places although more than thirty years old, as we see from this picture (fig. 13), and the modern diazed-apparatus, even in large and modern plants, still affords satisfactory protection against over-current (fig. 14 and 15).

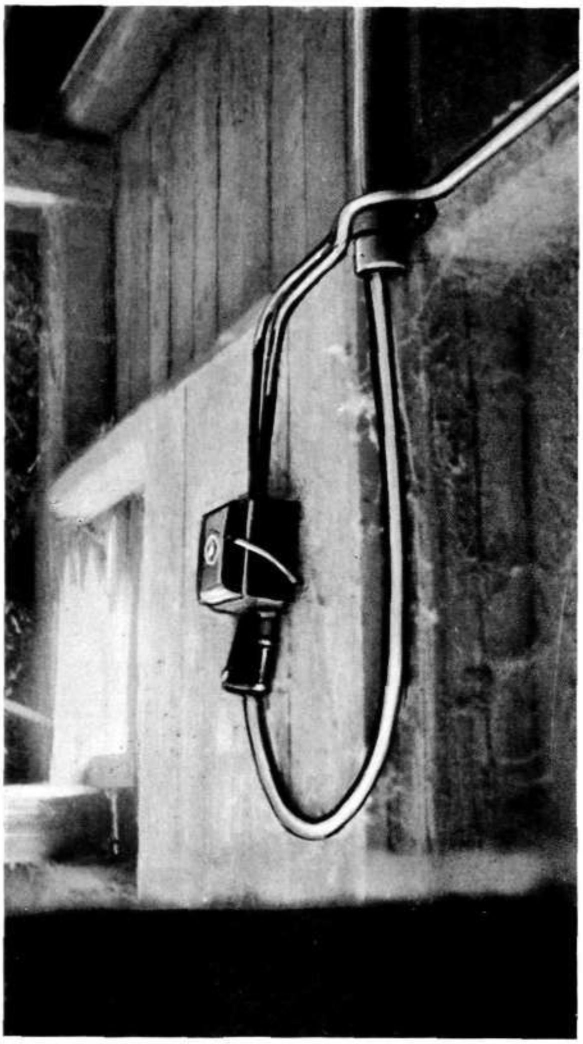


R 1900 Fig. 20.

R 1892 Fig. 21.

We have pointed out above that conductors are gradually being improved, and the same is the case with all accessories used in ordinary domestic installations. Consequently, a blown fuse will obviously become an increasingly rare occurrence, and the exchange of burnt fuses will not represent any appreciable cost or trouble. The small automatic devices (Klein-automaten) will have to be considerably simplified and their designs standardized—which might also cheapen them—before they can replace the diazed-apparatus to any great extent.

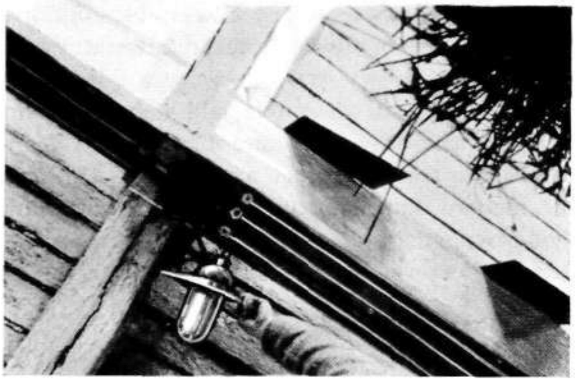
The general effort to improve the quality has also left traces in the development of *switches*. For weak currents, the rotary switch has maintained its position as a reliable fitting, and the recent efforts at uniformity of types for this mass-article must be greeted with satisfaction. The very weak designs of box-switches for 2 A and 4 A, which for reasons of cost used to be extensively employed, are now condemned by experience.



R 1882 Fig. 22.

The general principle that a switch should always be either full on or full off has been retained, while the manner of operating them has been slightly modified. Under certain circumstances it is undoubtedly more convenient to operate a switch by moving a lever up or down (the analogy of the knife-switch is obvious) or to push a white or black button, than to twist a handle. It is therefore easy to understand that the tumbler switch and the push-button switch *are gaining ground.*

The knife-switch still maintains its supremacy for breaking strong currents. It is a simple and reliable appliance, and there is no particular reason why it should be displaced by any other kind of switch, except for special purposes.



R 1881 Fig. 23.

The most remarkable feature of the development of installation accessories, however, is the incessant efforts to enclose them so that they may be used in any premises, irrespective of *prevailing conditions.* The *cast iron casings* for different voltages, manufactured on a large scale, can therefore be designated one of the most important improvements (fig. 16 and 17). About 20 years (1910—1929) intervene between these two designs. The improvement is obvious. In 1910 the apparatuses had to be fixed on the marble panel on the spot, or anyway specially designed for this particular plant, and were protected by a rather primitive sheet iron cover. 20 years later the central control station is made up of robust standard switching units.

(Figs. 18 and 19.) Fittings in cast iron *casings are not damaged by being distempered,* but open appliances in a wooden box must of course be carefully protected from any such treatment.

Suitable wiring materials, and accessories which are reliable even under difficult conditions, have enabled the electrician satisfactorily to solve certain problems which used to be very troublesome. Formerly, outside supply lines were sometimes connected to the inside wiring in a way which made the actual point of connexion a fruitful source of faults. The one-time popularity of gallows on the roofs constituted such an irregularity in installation methods, presumably unavoidable when no previous experience was available. (Figs. 20 and 21.) Earth-cables and iron-cased apparatus, however, may render even previously risky intakes from roof gallows innocuous, (fig. 22).

By connecting an underground-cable judiciously to the wiring in an old building, an iron-cased switch will, at no great extra expense, satisfy even the demands of the fire insurance companies for cut-out facilities. (Fig. 23.) The illustration next to it indicates how a still serviceable installation of TVM ki may suitably be extended by rubber-lead conductors.

The superior quality of modern installation materials naturally demands a higher price than corresponding older materials of inferior quality, and the former might therefore be expected to prove more expensive to use, but the additional cost is very small—sometimes none at all—if the wiring is carefully planned and every opportunity offered by the new materials is taken to shorten the lengths of conductors required, and to arrange the accessories rationally.

The tendency is characterized by: underground distribution lines—apparatuses enclosed in cast iron cases—great resisting power to all occurring stresses—and indoor wiring materials unaffected by local conditions.

The Railway Telephone Cable on the Electrified Malmö Lines.

Communication from the Swedish State Railways.

By *Ivar Billing.*

The electrification of the Swedish State Railways is now proceeding on the Malmö route, comprising the lines:

Järna—Nyköping—Malmö,
Katrineholm—Åby,
Örebro—Mjölby,
Nässjö—Falköping,
Malmö—Trälleborg, and
Arlöv—Lomma,

in all 534 miles of track. The system adopted is the same as on the Ore Line from Luleå to Kiruna and the Norwegian frontier and on the Stockholm—Gothenburg line, i. e. single phase A. C. of 16 $^2/_3$ cycles and 16 kV in the contact wires.

The current will be supplied not only from the previous three transformer stations at Södertälje, Sköldinge, and Hallsberg on the Gothenburg line, but from six new transformer stations as well, at Eksund near Norrköping, Mjölby, Nässjö, Alvesta, Hässleholm and Malmö (see map, fig. 1).

Regarding disturbances in neighbouring telephone and telegraph circuits and the measures so far taken to overcome them we will here only recall the following. On the Ore Line (271 miles), the electrification of which was completed in 1923, overhead telephone wires have been retained, but the poles have been moved to about 60 yds. from the track, which has been fitted

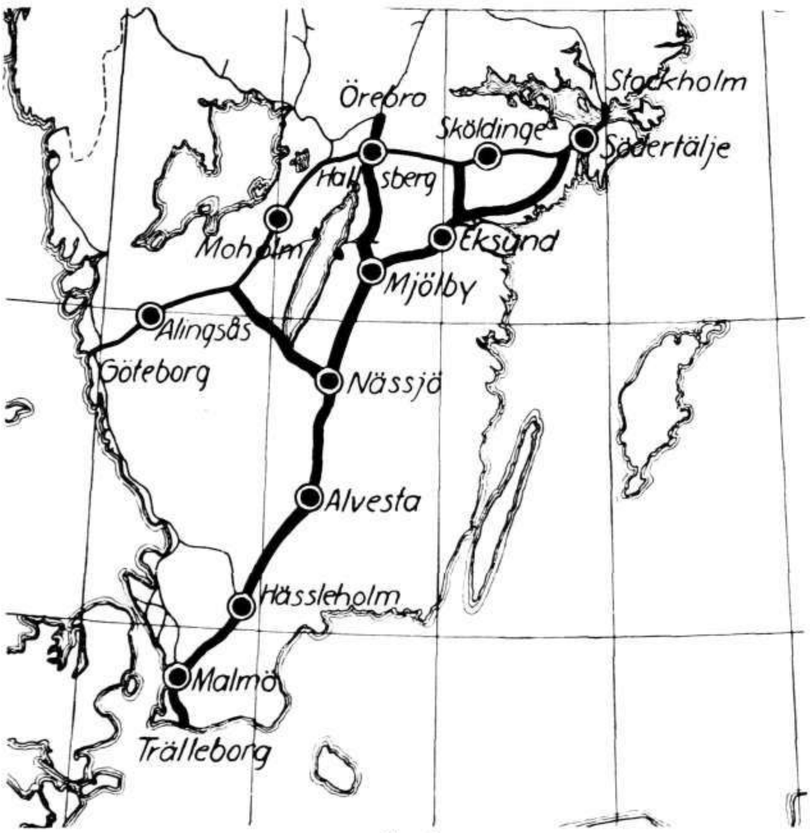


Fig. 1.

with compensating transformers connected to the contact wire and the rails. The *Gothenburg line* (284 miles), on the other hand, which was electrified 1924—26, has an underground telephone cable for the needs of the railway buried in the track, which has been fitted with compensating transformers connected to the contact wire and a special return circuit connected to the rails approximately mid-way between every two adjoining compensating transformers. Full details of the protective devices on these railways and their results as regards counteracting the disturbances have already been given in previous articles.[1]

The electrification of the Malmö lines was decided on by the 1931 Riksdag, which also granted the necessary money for the purpose, and the task was begun in the spring of the same year. This work includes the fitting of a cable for the railway telephone lines, for which a conditional agreement had been signed as early as December 1930. The greater part of this work was done in 1931 and is expected to be completed about September 1932.

Below we give some details of this cable, the method of laying it, and results obtained.

The total length of cable is greater than the length of the above railway lines, as it was found expedient to lay a new cable on the section from Stockholm to Järna also, so as to have a uniform main line cable the whole way from Stockholm to Malmö. It was also found to be an advantage to lay a separate cable for each of certain short sections where two or more of the above railways run parallel. Such, for instance, is the case on the sections Norrköping—Åby, Malmö—Arlöv, and in the yards of some of the larger stations of the line.

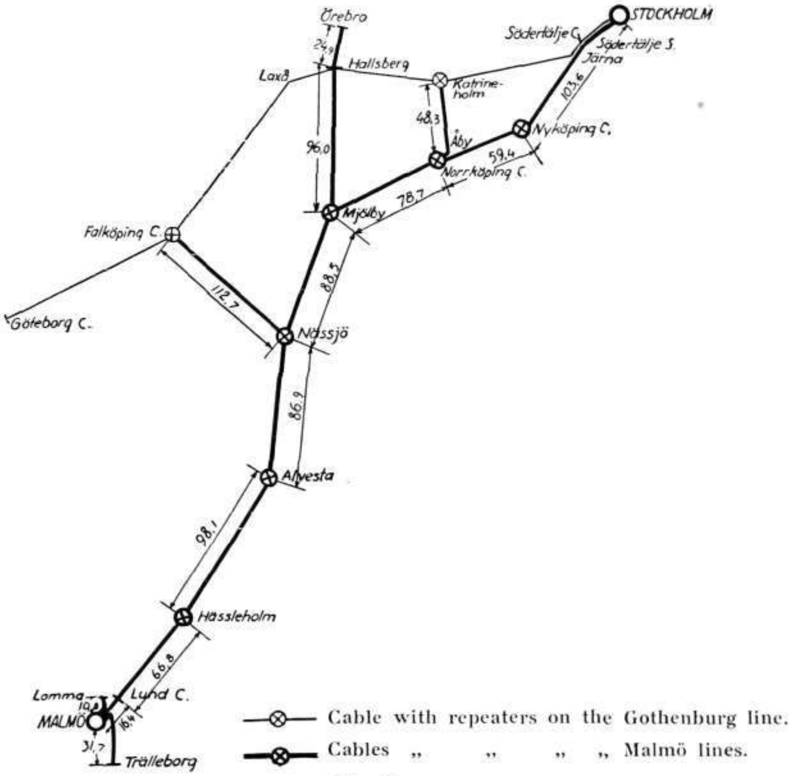
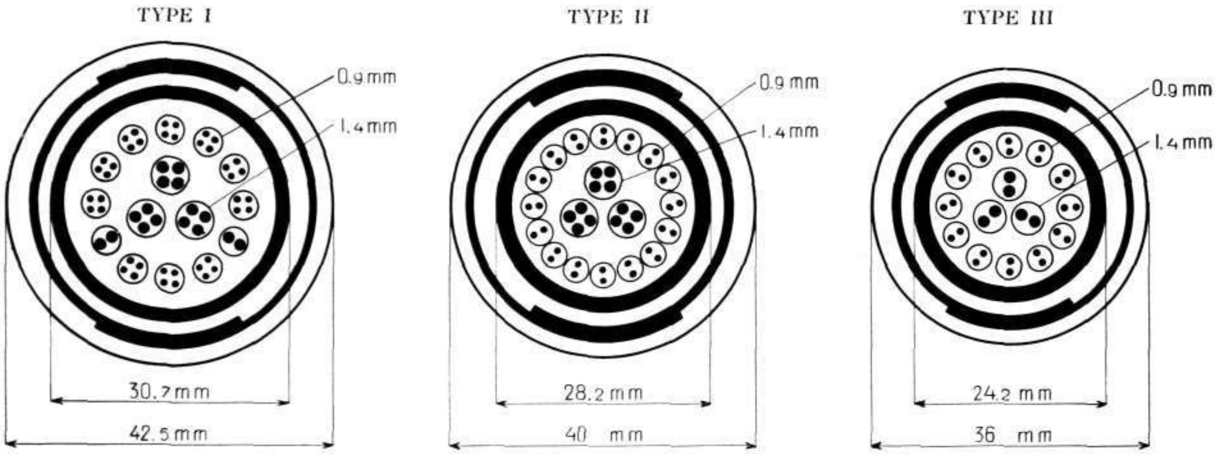


Fig. 2.

The whole plant thus comprises the following lengths of cable (see fig. 2):

Stockholm—Järna—Åby—Norrköping—Malmö,
Norrköping—Åby—Katrineholm,
Örebro—Mjölby,
Nässjö—Falköping,
Malmö—Trälleborg, and
Malmö—Lomma,

[1] See: Electrical Communication 1926, p. 199. Nord. Järnbanetidskrift 1926, p. 41, Siemens Zeitschrift 1926, p. 261 et seq., and 1927 p. 498. Elektr. Nachrichtentechnik 1927, p. 1, and Elektrische Bahnen 1928, Ergängzungsheft.



R 4090

Fig. 3.

corresponding to an aggregate track length of 573.4 miles.

Cable design and types of cables.

The experience gained in working the railway cable along the already electrified line from Stockholm to Gothenburg (see foot-note 1, page 81), was of course drawn upon when the design of the new cable was determined. It was, for instance, found possible to stipulate a lower disruptive voltage between the conductors of the cable in the Malmö lines 1 000 V. instead of 2 000 V., retaining 2 000 V between the conductors and the cable sheath.

The import of this should here be recalled. The reduction of the dielectric strength, combined with the fall in metal-prices, made it possible, without exceeding the original estimate for materials, not only to increase the length by the 30 miles of the Stockholm—Järna section, but also to increase the number of circuits in the cable considerably, thus providing a larger reserve against future needs. The main cable from Stockholm to Malmö thus contains 28 metallic circuits, as against 21 in the Stockholm—Gothenburg cable. The use of phantom circuits in the Malmö cable has also increased the circuits available there to 35, as against only 21 in the Gothenburg cable.

The Malmö cable is planned to have a greater number of direct telephone lines than the Gothenburg cable and it has therefore been possible to introduce phantom circuits, of which the Gothenburg cable has none.

Owing to the change from telegraph to telephones, a relatively large number of loaded circuits have also been found necessary in the Malmö cable. For the reasons given above, the greater expense involved by this has, however, come well within the original estimate.

With due allowance for reserves for future use, the present demand for circuits in the main cable from Stockholm to Malmö and the branches mentioned above resulted in the adoption of 3 different types of cable, (see fig. 3) namely:

Type I, the Stockholm—Malmö line, consisting of $3 \times 4 \times 1.4 - 2 \times 2 \times 1.4 - 10 \times 4 \times 0.9$ with 7 phantom circuits, in all 35 circuits.

Type II, the Örebro—Mjölby, Nässjö—Falköping and Malmö—Lomma lines, consisting of $3 \times 4 \times 1.4 - 16 \times 2 \times 0.9$ with 3 phantom circuits, in all 25, and

Type III, the Norrköping—Katrineholm and Malmö—Trälleborg lines, consisting of $3 \times 2 \times 1.4 - 12 \times 2 \times 0.9$ with no phantom circuits, in all 15 circuits.

The most important details of the technical specification of the cables are given in Table I below.

The contract for the plant was given to SIEVERTS KABELVERK, Sundbyberg, and ELEKTRISKA A.-B. SIEMENS, Stockholm, representing SIEMENS & HALSKE A.-G., Berlin—Siemensstadt, jointly, the former firm to supply all the cables, the latter to supply all the loading coils and installation materials and to carry out

Table 1.

Electrical Properties of the Cables, measured at the Cable Works.

	Specified values	Mean values measured		
		Type I	Type II	Type III
Resistance per km. at 15°C.				
in 1.4 mm. wire, ohms	max. 11.4	10.86	10.92	10.78
„ 0.9 „ „ „	„ 27.5	26.3	26.4	26.27
Difference of resistance per manuf. length between the conductors of a pair, as a percentage of the mean of their resistances	„ 3	0.5	0.6	0.4
Insulation resistance per km. with 100 v. D.C., megohms	min. 10 000	25 600	26 200	29 000
Capacity per km. with 800 cycle A. C.				
per pair of 1.4 mm., max., μF	max. 0.042	0.0330	0.0329	0.0328
„ „ „ 1.4 „ mean, μF	„ 0.038	0.0325	0.0324	0.0322
„ „ „ 0.9 „ max., μF	„ 0.039	0.0296	0.0303	0.0305
„ „ „ 0.9 „ mean, μF	„ 0.036	0.0290	0.0296	0.0296
for phantom in relation to the corresponding pair	„ 1.05×1.62	1.02×1.62	1.03×1.62	—
Capacity unbalance for cable length of 550 m. in twin conductors:				
Between adjacent pairs, mean, $\mu\mu$F	„ 100	—	18	14
„ any two pairs, $\mu\mu$F	„ 200	—	53	47
in quads:				
physical to physical in the same quad, $\mu\mu$F	„ 350	51	43	—
„ „ phantom „ „ „ „ $\mu\mu$F	„ 900	175	116	—
„ „ physical, physical to phantom, and phantom to phantom in adjacent quads $\mu\mu$F	„ 600	52	72	—
from physical to earth $\mu\mu$F	„ 1000	200	271	188
Leakage coefficient per km. at 800 cycles corresponding to $\frac{G}{\omega \cdot C}$	„ 0.005	0.0035	0.0039	0.0039

R 4076 Fig. 4.

all the installation work. The two firms were made jointly responsible for the plant as a whole.

The State Railways did the actual laying of the cable and all trenching work, etc., required.

A scheme was prepared for carrying out the contracted work during 1931 and 1932. Thus, the cables for the sections Norrköping—Katrineholm, Stockholm—Nyköping—Norrköping—Mjölby, Örebro—Mjölby, Nässjö—Falköping, Malmö—Lund, and Malmö—Lomma, a total of about 342 miles, were to be completed in 1931. Not only was all this done during 1931 according to plan but by a later agreement the cable for the Mjölby—Nässjö section of about 55 miles was also delivered, laid, and partly installed. The coil-loading and the rest of the installation work on this line will be finished in the course of the winter, before work on the remaining cables, on the Malmö to Trälleborg and Lund to Nässjö lines, is started in the spring of 1932, to be completed according to the plan by about September 1st 1932.

The Cables.

The conductors, of 1.4 and 0.9 mm. diameter, are insulated with cellulose paper, spun in pairs and partly in D.-M.-quads. The lead-sheath is 2 mm. thick and alloyed with 2 per cent. of tin. It is pressure-tested in the Works at 2 atm. extra internal pressure for 2 hours. The armouring consists of 2 iron bands, each 1 mm. thick. The marine cables used for water-crossings are provided with an extra layer of 5 mm. round steel outside the band.

The sections of the three types of cable are shown in fig. 3, from which it can be seen that external diameters of the three types I—III are 42.5, 40, and 36 mm. respectively. The weight of the cables per m. is 4.64, 4.2, and 3.48 kg. respectively.

The electrical properties required in the cables and the corresponding mean values obtained in the inspection tests at the cable works are given in Table I.

The dielectric strength of every cable was also tested with 50-cycle A. C. at 2 000 V between the conductors and the lead-sheath for 30 minutes, and between the conductors at 1 000 V for one minute.

The cables were delivered in lengths of about 600 yds., varying between 591 yds. and 602 yds. according to the fixed length of each loading-section (see below).

The coil loading.

The length of the loading-coil spacing was fixed at 2 406 yds., with a maximum tolerance of 1 per cent., and the inductivity of the loading-coils determined, with due regard to the sites already picked for the repeater stations (see fig. 2). Certain circuits used for recording the power consumption of the transformer stations, or for blocks and signals etc., have not been loaded. Each type of cable thus contains 8 pairs of 0.9 mm., conductors the are unloaded, while other pairs or physical circuits and quads are loaded as shown below:

Quads 1.4 mm., physical 160 mH, phantom 63 mH,
„ 0.9 „ „ 177 mH, „ 63 mH,
Pairs 1.4 „ 177 mH in type I, 160 mH in type III,
„ 0.9 „ 177 mH,

all values measured with 800 cycles A. C. and a tolerance of ± 2 per cent.

Every quad with phantom circuit has one coil for each physical circuit and one, common to them both, for the phantom circuit.

Table 2.

Properties of the loading coils.

	Specified values	Mean values measured
Insulation at 150 V. D. C., megohms	≧ 10 000	80 000
Self induction at 1 mA 800 cycles		
in physical coils mH	177	175.6
„ „ „ „	160	159.4
„ phantom „ „	63	63.0
variation, per cent.	≦ ± 2	≦ + 0.8 / − 1.6
Stability of self induction on magnetization of one winding with 0—2A D. C.:		
before magnetization mH	177	180.1
5 min. after magnetization mH	—	180.3
26.5 hours after magnetization mH	—	180.05
Change of induction after 5 min., per cent.	≦ 2.5	+ 0.113
Resistance to D. C.:		
coil-group 177/63 ohms	≦ 10.5	8.82
„ 160/63 „	≦ 10.2	8.32
coil 177 ohms	≦ 7.5	4.67
„ 160 „	≦ 7.2	4.23
Resistance to A. C.:		
at 800 cycles:		
coil-group 177/63 physical ohms	≦ 12.5	10.74
„ 177/63 phantom „	≦ 6.0	5.12
„ 160/63 physical „	≦ 12.2	10.01
„ 160/63 phantom „	≦ 6.0	4.79
coil 177 ohms	≦ 9.0	6.78
„ 160 „	≦ 8.7	6.55
at 2 000 cycles:		
coil-group 177/63 physical ohms	≦ 18.5	15.45
„ 177/63 phantom „	≦ 9.5	6.82
„ 160/63 physical „	≦ 18.2	14.13
„ 160/63 phantom „	≦ 9.2	6.44
coil 177 ohms	≦ 15.0	12.87
„ 160 „	≦ 14.5	13.46
Difference in resistance between the two branches of the coils		
in a physical circuit, ohms	≦ 0.1	0.07
in a phantom circuit, ohms	≦ 0.15	0.11
Difference in inductance between the two branches		
in a physical circuit, per cent.	≦ 0.1	0.073
in a phantom circuit, per cent.	≦ 0.15	0.095
Cross talk attenuation between two speech circuits in the same coil box, at 10 mA and 800 cycles, nepers	≧ 10	≧ 11

R 4073 Fig. 5.

Further, all the loading coils were also tested for disruption according to the stipulations with 50-cycle A. C. at 1 000 V. between the windings, and at 2 000 V. between windings and coil box.

The Laying of the Cable.

The cable is laid in the permanent way 6′ 3″ from the centre of the track and at a depth of at least $27^1/_2$ in. below the lower surface of the rails. The trenching-machine purchased for the Gothenburg line was used for digging the cable trench (see note on page 81).

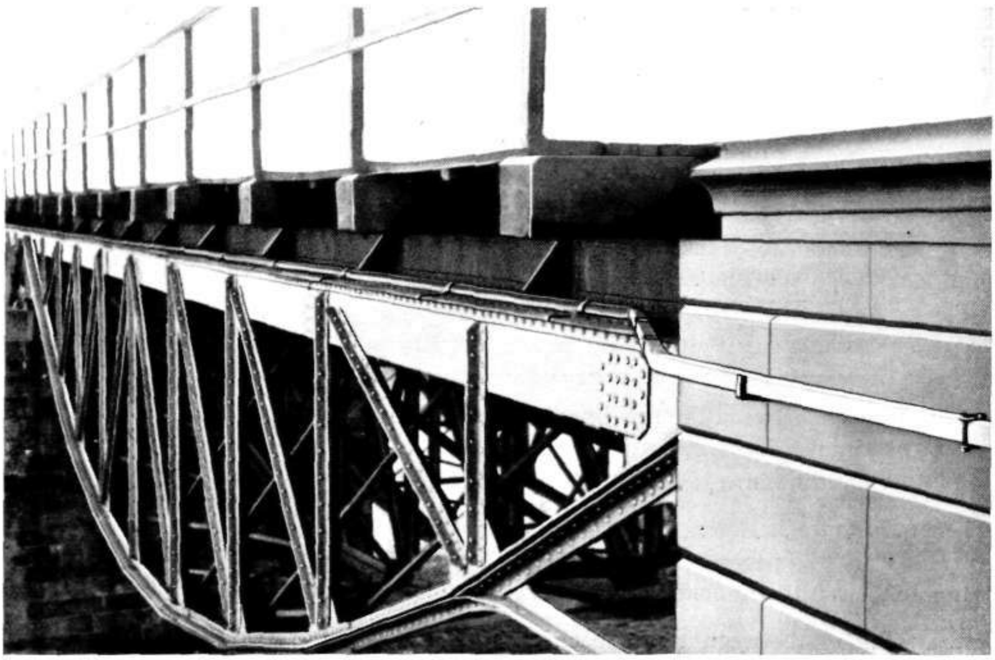
All the loading-coils at a loading point are fitted into a common box. Three sizes of loading-boxes are thus used, namely:

Type I, holding 27 loading-coils,
Type II, „ 17 „ , and
Type III, „ 7 „ .

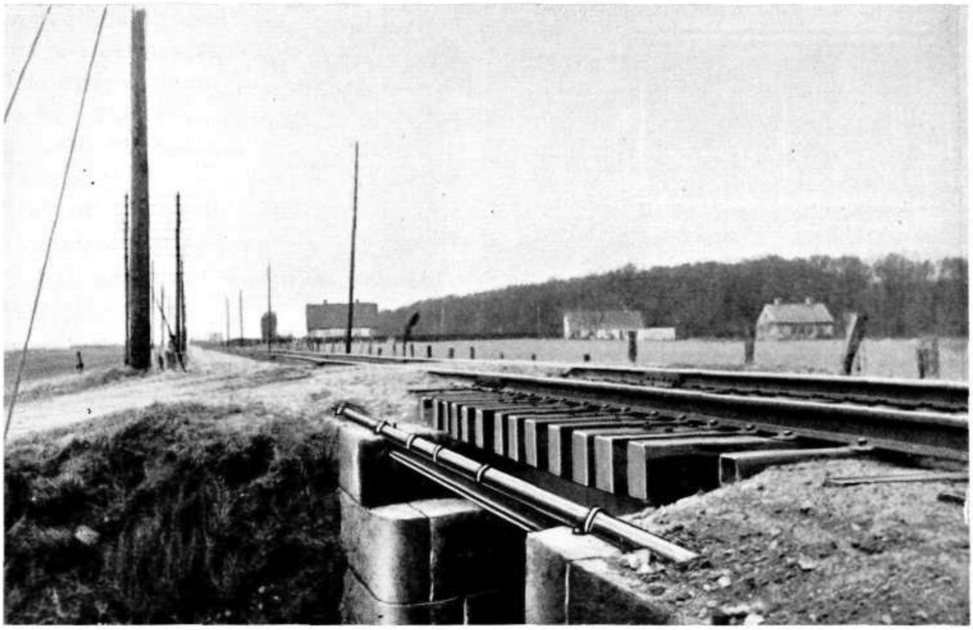
A loading-box of type I is illustrated in fig. 4.

The electric requirements for loading-coils, and the mean values obtained in the tests are shown in Table 2.

A new feature of the cable-laying may be mentioned here. Instead of the stones and gravel that fall into the trench being picked out by hand before the cable can be laid, a digging and cleaning-out plough has been used on part of the Malmö lines, so designed that the cable drops straight into the trench over a pully wheel inside the plough before the gravel can fall in again. This plough, which is illustrated in fig. 5, has also been used in suitable ground without previous use of the heavier trenching-machine.



R 4072 Fig. 6.



R 4075 Fig. 7.

As a rule the trench has been filled up by hand, but machines have been used for this purpose too wherever possible. A "permanent-way cleaning-machine", which automatically covers the cable with the gravel previously dug out, has then been coupled to the tail end of the cable train.

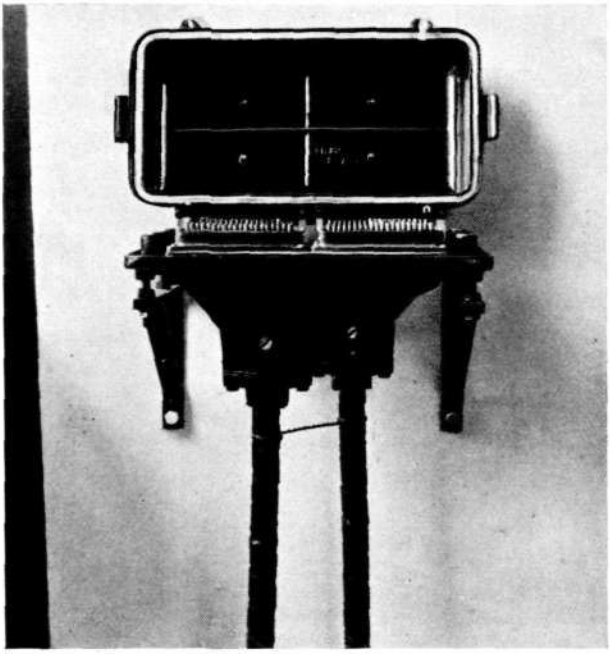
In station-yards and rock-cuttings, where the permanent way is of macadam or some similar hard material, trenching has of course been done by hand.

The cable has been laid from a cable train.

Where it has not been possible to lay the cable 27 1/2" deep, as well as where it crosses roads, railways, bridges, or culverts, and at all intakes into buildings, etc., the cable has been protected by suitable profile irons or pipes (see figs. 6 and 7).

R 4071 Fig. 8.

In laying the cable, due allowance has been made for necessary bights to be laid out at all joints and loading-points over and above the overlap necessary for the splice, so that all installation work could be carried on far enough from the track, which was in normal use, to avoid any risk to the fitters. The actual length of cable installed is therefore greater than the length previously given, or 578.9 instead of 573.4 miles, an increase of 0.95 per cent.



R 4078 Fig. 9.

The time taken for the laying has of course depended on the nature of the country, the training of the men, and so on. Thus, in 1931 progress on the line from Åby to Katrineholm, with many rock-cuttings and tunnels, was at first rather slow, but gradually became much quicker. Last year, when a total of no less than 400 miles of cable were laid between May 4th and November 13th, the average was 42 cable drums—or 14.3 miles of cable—working per week. The most over laid in a week was 78 drums, or about 26.7 miles of cable.

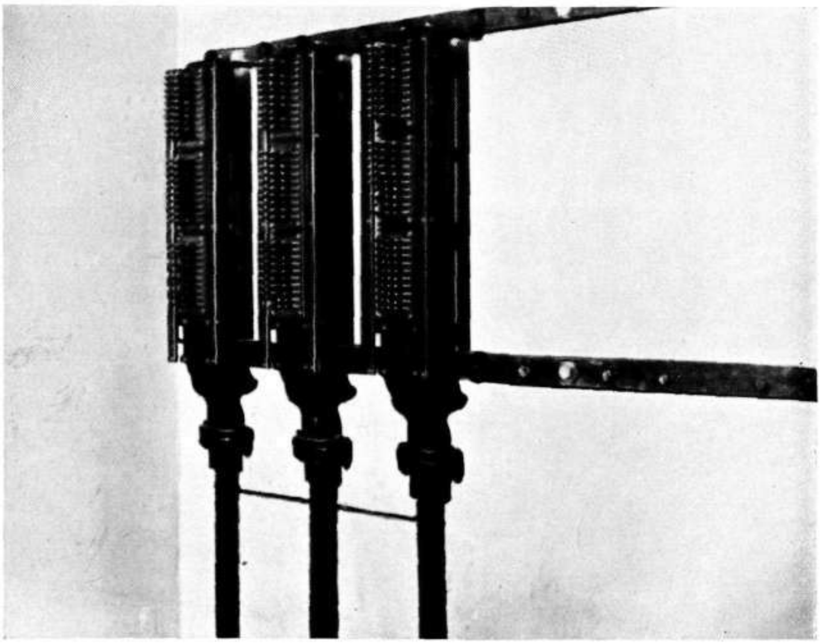
The Installation Work.

The contractors checked the lengths as the cable was laid, marking out the places for joints, condensers and loading-coil boxes. In each whole loading-section there are four lengths of cable. These have been connected in twos by ordinary simple joints. In the middle of each section a condenser box (see fig. 8) has afterwards been fitted, so that jointing and balancing of capacity has been done there in one common box when the necessary branch cables had been connected. Any further balancing of capacity required when the loading-coil boxes were afterwards put in, was effected by means of condensers inside these, according to the Siemens & Halske patent method for condensers.

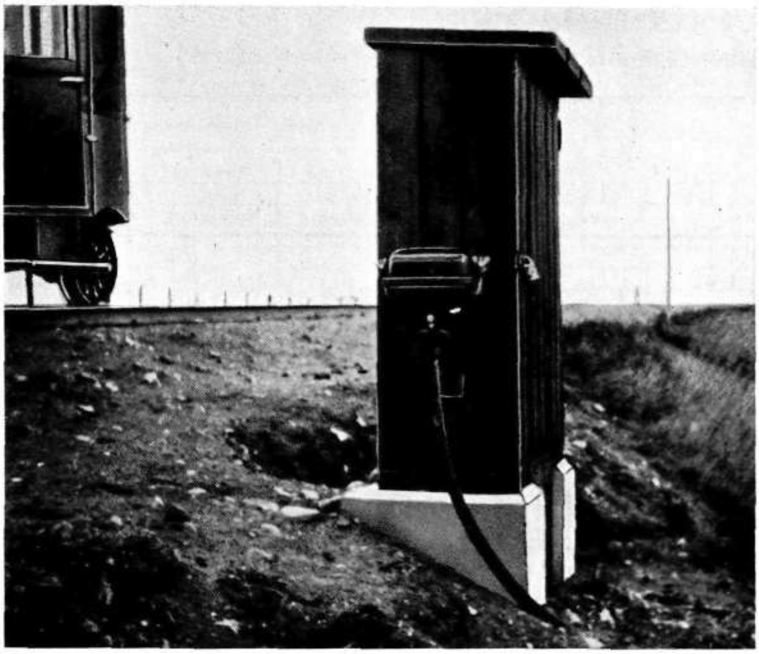
All the necessary insulation and air-pressure tests were made before the fitting of the loading-coil boxes. The loading process completed, the terminal and test boxes in the stations were put in (see figs. 9 and 10).

Boxes for joints, condensers, and loading-coils are generally buried deep enough for the cable at the ends of the boxes to be 27 $^1/_2$" below the bottom of the rails. Supports, in the shape of sawn-off sleepers or the like, were fitted as required under the larger loading-coil boxes, which weigh about 550 lbs. The boxes are protected above by impregnated boards or pieces of sleeper.

To get the full benefit of the protection afforded by the cable sheath against disturbances from the power lines, the lead sheath and armouring are well soldered together on each side of every joint. Similarly, the sheaths and armouring of all cables in the intake boxes in the stations are connected by earthed and soldered copper.



R 4074 Fig. 10.



R 4077 Fig. 11.

Intakes and branches have been made in the following way. At every station the main cable has been cut, and the two ends connected to an intake or test box with two terminals, where instruments can be connected as required for tracing faults. Branches of the following kinds have been made: type B to trackmen's cabins, type Bm to trackmasters' offices, type Block to block posts, type Signal to certain signals, and type Omf to transformer stations. Of these. 4-pair cables are used for types B and Bm, 7-pair for signal and Omf in cables of type III, and 10-pair cables for Block and Omf in cables of types I and II.

To facilitate the tracing of faults, one track-telephone circuit can easily be broken in all the stations, track men's cabins and block posts. All branch cables end in waterproof terminal boxes (see fig. 11). To make it easy to test the submarine cables used at canal crossings, of which there are seven, test boxes have been fitted at the junctions of the marin and underground cables (see fig. 12).

The length of the loading-sections, as we have already mentioned, has been fixed at 2 406 yds., but this may vary a little as, when the measurements are checked, it has been found convenient to adjust the length slightly in order to obtain equal half-sections on either side of future repeater stations (telephone repeaters). The actual installed lengths of the loading-coil spacing in the various sections are thus:

Stockholm–Nyköping and Nyköping–Norrköping	2 428 yds.
Örebro–Mjölby	2 419 „
Norrköping–Katrineholm, Norrköping–Mjölby and Malmö–Trälleborg	2 414.5 „
Alvesta–Hässleholm	2 410 „
Hässleholm–Malmö and Malmö–Lomma	2 406 „
Nässjö–Alvesta	2 401.5 „
Mjölby–Nässjö	2 393 „
Nässjö–Falköping	2 180 „

Properties of the cable when laid.

On completion of the various cable sections, each repeater section, i. e., the part of the line between two adjoining repeaters or between a repeater and the cable terminal, is thoroughly tested. The results are collected in Table 3 below, and show that the values obtained for the transmission properties of the cable are well up to the standard demanded.

As regards the *disruptive voltage* to earth, each loading-section of the cable was tested with 1 200 V before the coils were connected.

R 4079 Fig. 12.

Table 3.

Properties of the cables when laid.

	Specified values	Mean values measured		
		Cable type I Norrköping–Nyköping	Type II Nässjö–Falköping	Type III Norrköping–Katrineholm
Insulation, megohm/km.	≧ 10 000	38 500	70 000	40 000
Difference of resistance between conductors in the same pair, ohms				
0.9 mm. unloaded	—	0.60	1.20	0.40
0.9 „ loaded	—	0.80	0.60	0.50
1.4 „ „	—	0.35	0.25	0.20
Characteristic impedance at 800 cycles				
1.4 mm. physical circuit 160 mH	1480	1530	1560	1530
1.4 „ „ „ 177 „	1560	1610	—	—
0.9 „ „ „ 177 „	1640	1700	1710	1690
1.4 „ phantom „ 63 „	715	750	750	—
0.9 „ „ „ 63 „	750	780	—	—
Attenuation exponent in nepers				
1.4 mm. physical circuit 160 mH	≦ 0.0110	0.0094	0.0092	0.0086
1.4 „ „ „ 177 „	≦ 0.0097	0.0085	—	—
0.9 „ „ „ 177 „	≦ 0.0200	0.0177	0.0170	0.0172
1.4 „ phantom „ 63 „	≦ 0.0110	0.00934	0.0091	—
0.9 „ „ „ 63 „	≦ 0.0215	0.0188	—	—
Cut-off Frequency				
1.4 mm. physical circuit 160 mH	≧ 2950	> 2950	> 2950	> 2950
1.4 „ „ „ 177 „	≧ 2800	> 2800	—	—
0.9 „ „ „ 177 „	≧ 2950	> 2950	> 2950	> 2950
1.4 „ phantom „ 63 „	≧ 3530	> 3530	> 3530	—
0.9 „ „ „ 63 „	≧ 3690	> 2690	—	—
Cross-talk attenuation between any two speech circuits, nepers	≧ 8.0	≧ 9.4	≧ 9.3	≧ 9.7
Echo attenuation in a repeater section for frequencies between 300 and 2 000 cycles, nepers	≧ 2.7	≧ 4.0	≧ 4.0	—[1]

[1] Values of about 3.3 nepers were obtained in the measurements, but these comparatively low amounts must have been due to rather unsuitable terminals being used for the lines. There was no further opportunity to take new measurements, as the circuits were immediately taken into use.

CHARACTERISTICS

Cable Type I { *centre: 3×4×1.4* / *layers: 10×4×0.9+2×2×1.4* }

Norrköping C.—Nyköping C.: 59.94 km.
Pair 21+22; 1.4 mm.

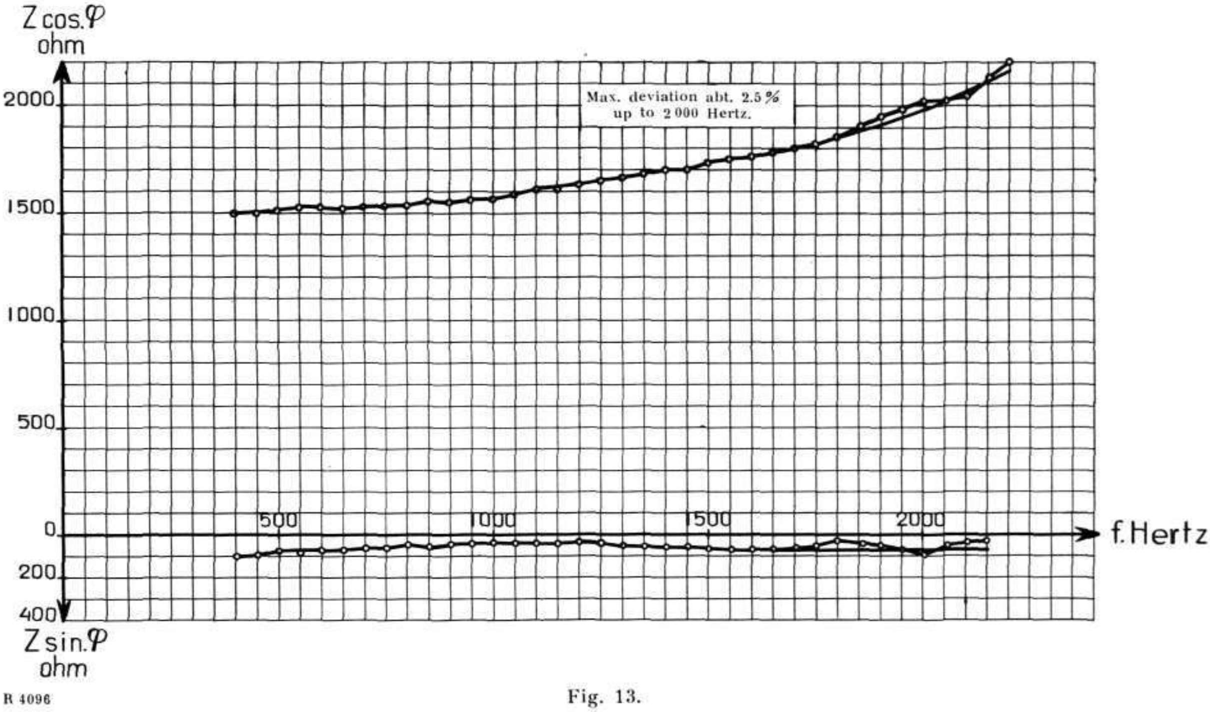


Fig. 13.

CHARACTERISTICS

Cable Type I { *centre: 3×4×1.4* / *layers: 10×4×0.9+2×2×1.4* }

Norrköping C.—Nyköping C.: 59.94 km.
Phantom 21, 22 and 25, 26; 1.4 mm.

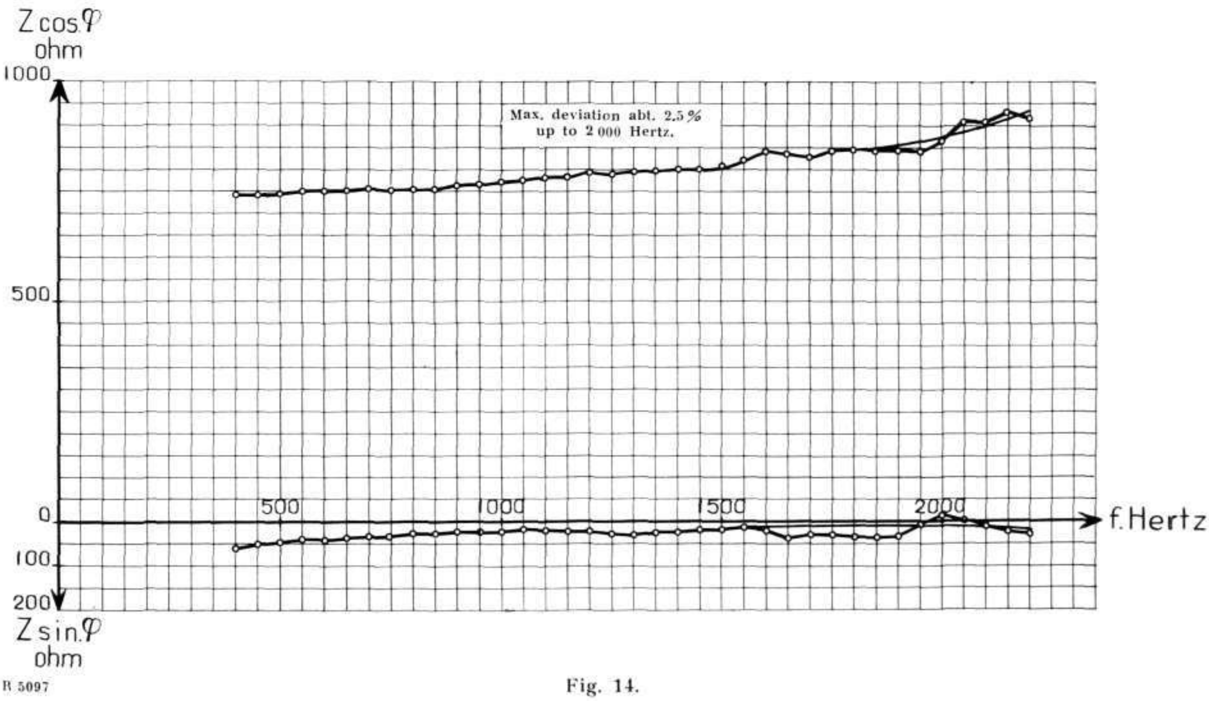


Fig. 14.

CHARACTERISTICS

Cable Type I { *centre: 3×4×1.4* / *layers: 10×4×0.9+2×2×1.4*

Norrköping C.—Nyköping C.: 59.94 km.
Pair 13+14; 0.9 mm.

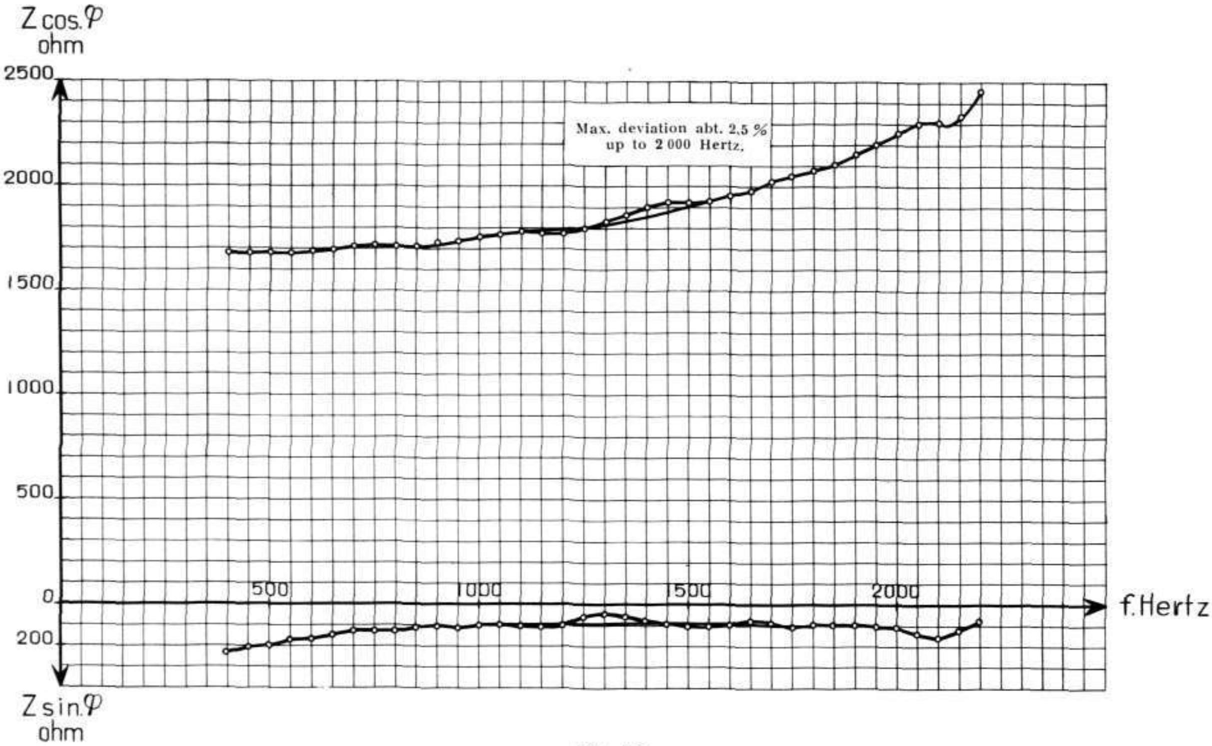


Fig. 15.

CHARACTERISTICS

Cable Type I { *centre: 3×4×1.4* / *layers: 10×4×0.9+2×2×1.4*

Norrköping C.—Nyköping C.: 59.94 km.
Phantom 15, 16 and 19, 20; 0.9 mm.

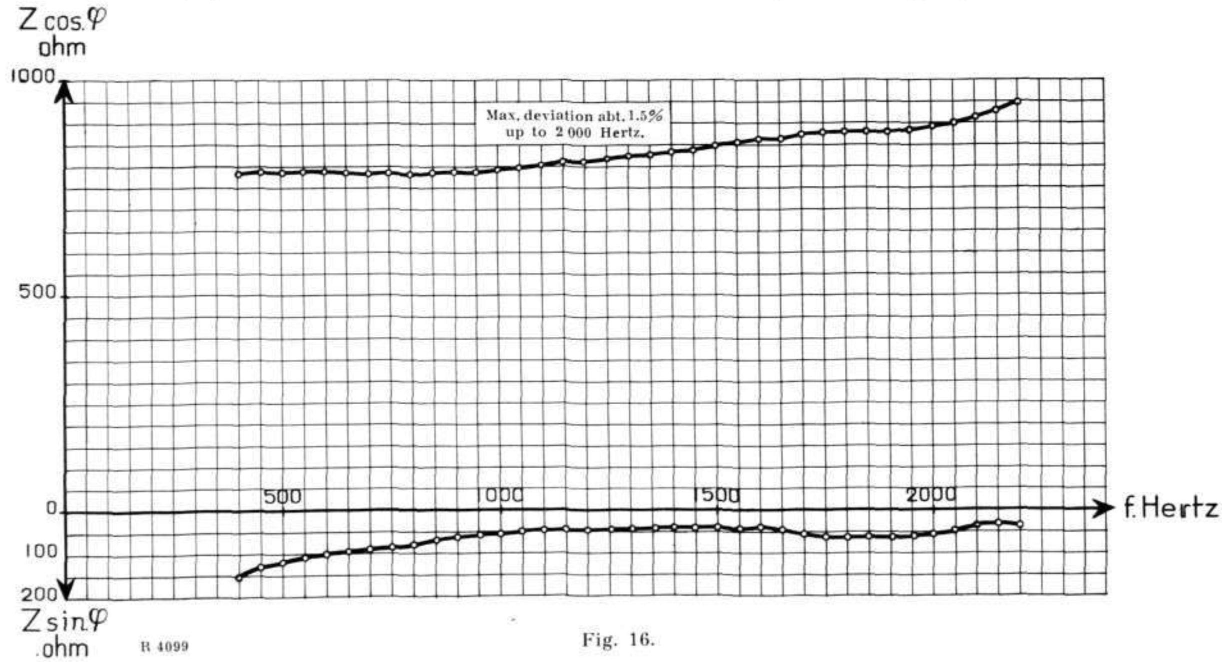
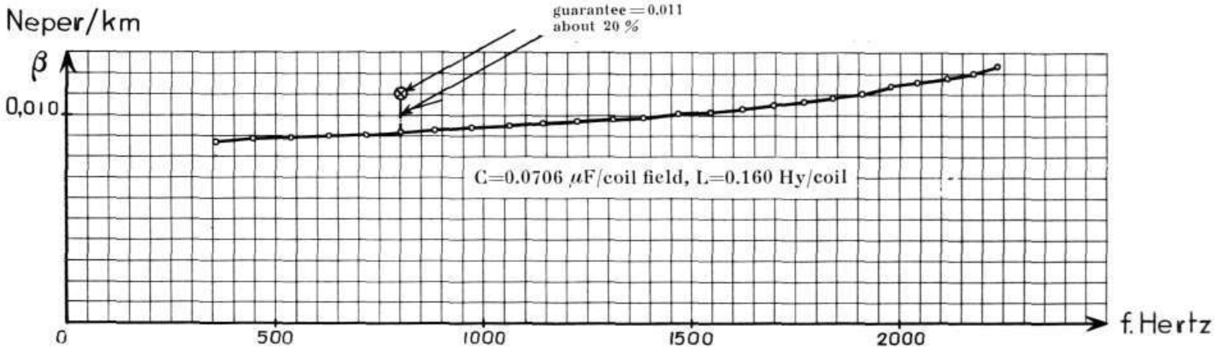


Fig. 16.

ATTENUATION CURVE

Cable Type II { *centre: 3×4×1.4* / *layers: 16×2×0.9* — *Measured at Nässjö* — *Nässjö—Falköping C.: 112.7 km.* *Pair 17+18; 1.4 mm.*

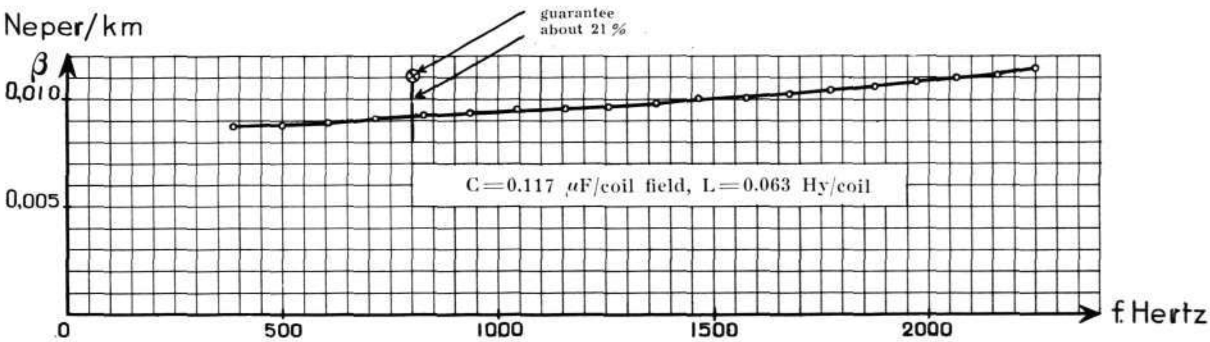


R 5000

Fig. 17.

ATTENUATION CURVE

Cable Type II { *centre: 3×4×1.4* / *layers: 16×2×0.9* — *Measured at Nässjö* — *Nässjö—Falköping C.: 112.7 km.* *Phantoms 19, 20 and 21, 22; 1.4 mm.*

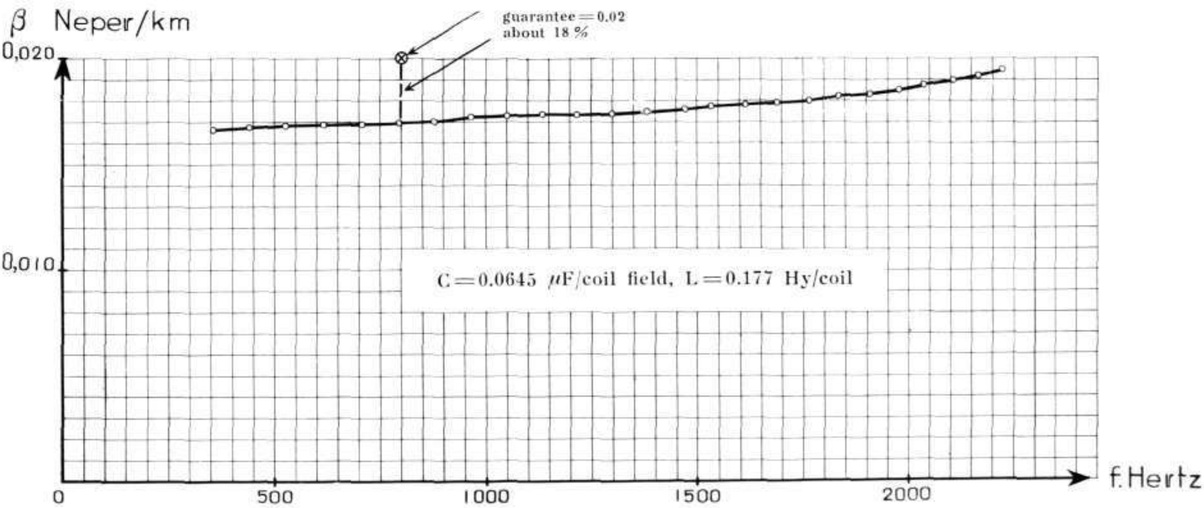


R 5001

Fig. 18.

ATTENUATION CURVE

Cable Type II { *centre: 3×4×1.4* / *layers: 16×2×0.9* — *Measured at Nässjö* — *Nässjö—Falköping C.: 112.7 km.* *Pair 16+15; 0.9 mm.*



R 5003

Fig. 19.

The following were the maxima specified for the *mean capacity* of a repeater section: for pair or physical circuits of 1.4 mm. wire 0.033 μF/km, and for the corresponding phantom circuit 0.056 μF/km, and for pair or physical circuits of 0.9 mm. wire 0.030 and 0.051 μF/km respectively. That these demands are more than complied with is proved by the figures given in the Table.

It is worth noting that the capacity of the cable when laid was about 1 per cent. less than when measured wound on drums in the cable works.

When tested, the *capacity unbalance* to earth between the two wires of a circuit in a loading-section of spliced cable turned out to be considerably less than the maximum values prescribed, which were for loaded circuits 150 μμ F and for unloaded circuits 500 μμ F.

The *insulation resistance* was measured with D. C. of about 120 V. The lowest measured values came to 38 500 megohms/km., but in other sections they rose to 40 000, 70 000, and even 100 000 megohms/km.

The *characteristic impedance* (wave resistance) was measured with a current of 1 mA at different frequencies from 400 to 2 200 cycles per sec. The values stipulated for 800 cycles were exceeded, as we see from Table 3. Some typical impedance curves are shown in the diagrams, for physical circuits of 1.4 mm. wire in fig. 13, for their phantom circuits in fig. 14, for physical circuits of 0.9 mm. wire in fig. 15, and for their phantom circuits in fig. 16.

These show that the maximum deviation up to 2 000 cycles is not more than about 2.5 per cent.

The *attenuation* was measured at frequencies between 400 and 2 200 cycles per sec. also. The measured values at 800 cycles were considerably —about 20 per cent.—below the maxima stipulated.

Some curves giving the attenuation at different frequencies are reproduced: for a physical circuit of 1.4 mm. wire fig. 17, for phantom of 1.4 mm. fig. 18, and for a 0.9 mm. metallic circuit fig. 19.

The *cut-off frequency* was calculated from the values obtained from these curves, and was found to be well over the guaranteed values given in Table 3.

Cross-talk.

It was stipulated that the cross-talk attenuation between any two speech circuits in a completely installed repeater section should be at least 8.0 nepers.

Very comprehensive measurements were made for this purpose in the prescribed manner, firstly with A. C. corresponding to a speech current and containing mixed frequencies sent out by a buzzer, and secondly with current from a valve transmitter at a series of definite frequencies corresponding to each thousand of ω from 4 000 to 12 000 (640—1 900 cycles). These measurements have also comprised near-end cross-talk (Nebensprechen) and far-end cross-talk (Gegennebensprechen) between pair, physical, and phantom circuits in all occurring combinations. Some of the values obtained are given below in Table 4.

Table 4.

Cross-talk attenuation at mixed frequencies, in nepers.

Combinations	Cable type I		Cable type II	
	Minimum	Average	Minimum	Average
Pair to pair in the same quad	10.0	10.3	9.9	10.45
Pair in a quad to the phantom of the same quad	9.5	9.8	9.4	9.7
Phantom to another phantom	9.5	9.7	9.4	9.6
Pair in a quad to phantom in another quad	9.4	9.8	9.3	9.6
Pair in a quad to pair in another quad	9.6	9.9	9.3	9.6
Between pairs and quads in separate layers of the cable	10.1	10.4	9.9	10.55

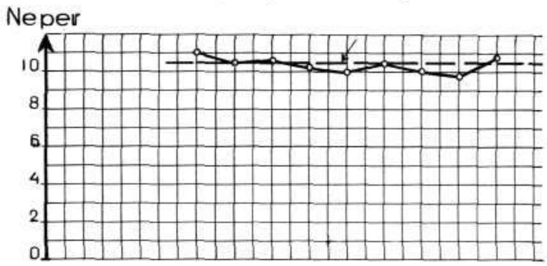
For cable type III a minimum of 9.7 was measured from pair to pair. In more than 50 per cent. of these measurements a value ≥ 11 nepers was obtained.

CROSS TALK

Cable Type I { *centre: 3×4×1.4* / *layers: 10×4×0.9+2×2×1.4* }

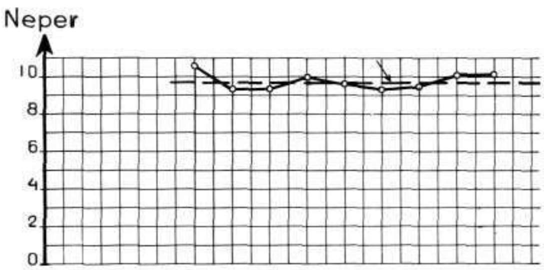
Norrköping C.—Nyköping C.: 59.94 km.

From physical to physical in the same quad ($b_1 = {}^1/_2$).
Mixed frequency $b = 10.4$ Nepers.



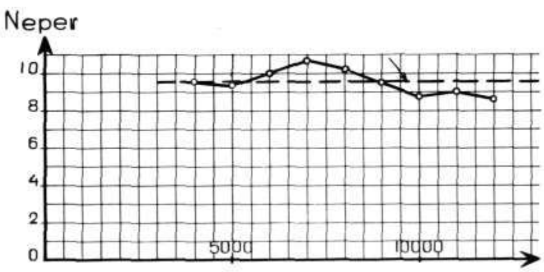
R 5002 a

From physical to 1 phantom in the same quad ($b_2 = 1/v$).
Mixed frequency $b = 9.7$ Nepers.



R 5002 b

From physical 2 to phantom in the same quad ($b_3 = 2/v$).
Mixed frequency $b = 9.6$ Nepers.



R 5002 c

Fig. 20.

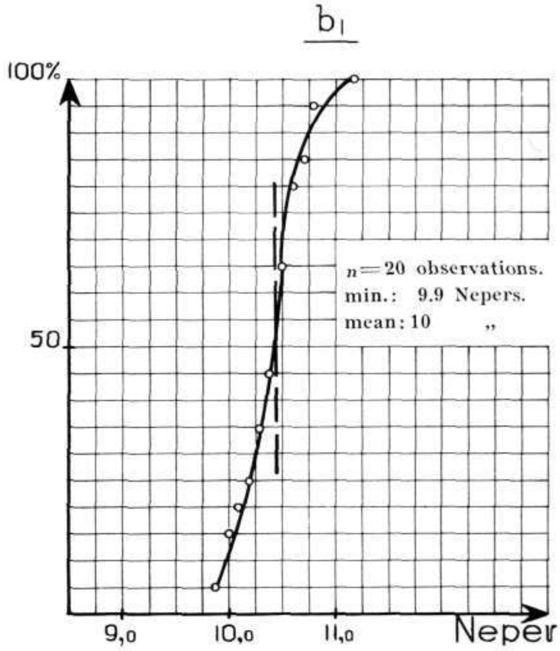
CROSS TALK

Cable Type II. { *in Quads and between adjacent pairs.* / *Coil loading.* }

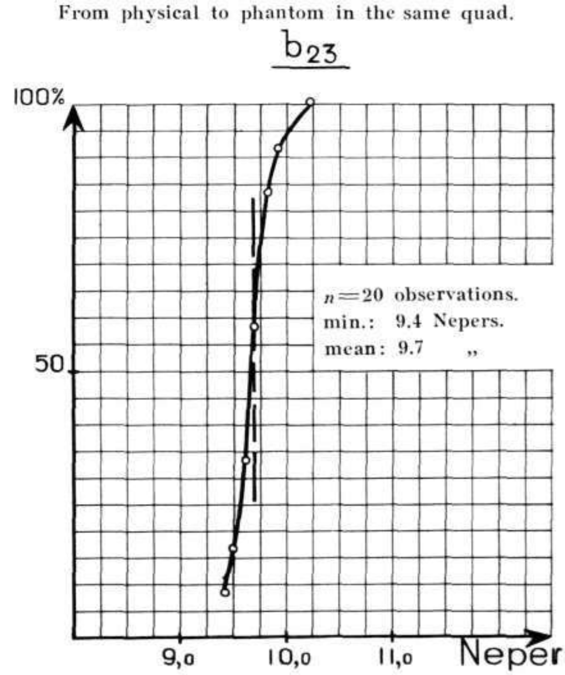
Nässjö—Falköping C.: 112.7 km.

Measured at Nässjö and Falköping C.

From physical to physical in the same quad.



From physical to phantom in the same quad.



R 5004

Fig. 21.

The dependence of cross-talk on frequency is illustrated in fig. 20, which gives the result of measurements with given frequencies in a cable of type I.

To show the results of the measurements with mixed frequencies, a couple of curves are given in fig. 21 which illustrate, for a cable of type II, how many times a certain attenuation has been observed, as a percentage of the total number of observations (Häufigkeitskurven).

Echo Attenuation.

As regards echo attenuation, which is also a measure of the variations in wave resistance and from which can be judged how easily the networks required for the use of telephone repeaters can be made, it was stipulated that in a repeater section this should be at least 2.7 nepers over the range of frequencies from 300 to 2 000 cycles.

These measurements were made over the range of frequencies from 320 to 2 240 cycles per sec. The values obtained were mostly over 4 nepers, only in certain 0.9 mm. physical circuits that had branches in the repeater section were values of less than 4.0 nepers obtained, but in any case these were considerably above the stipulated value.

To illustrate the results of the measurements in direct lines without branches Table 5 has been made out, referring to a cable of type I in the Norrköping—Nyköping line.

Table 5.
Echo attenuation in nepers.

Circuits	Minimum	Average
Physical circuits 0.9 mm....	4.15	4.3
„ „ 1.4 „	4.0	4.1
Phantom „ 0.9 „	4.8	5.0
„ „ 1.4 „	4.4	4.5

Note. In certain 0.9 mm. physical circuits with several branches the values measured were about 0.3 nepers below those given above.

Finally, it should be pointed out that before electrification the Malmö lines had a total of 5 490 miles of wire, on 3 788 miles of single-wire telegraph and double-wire telephone circuits. In comparison with this, the cables on these lines will contain 14 643 miles of double-wire circuits, or 29 286 miles of wire—considerably more than once round the equator. These cable lines contain, including phantoms, no less than 17 734 miles of metallic and phantom circuits, that is, the circuits available for distant communication have been increased by no less than 13 946 miles on 3 788 miles, or 368 per cent.

The cables are provided with station terminals in 124 stations (end-boxes were stipulated in about 131 stations). The number of branches, fixed at a maximum of one for every 1 750 yds. of cable, total about 540, corresponding to one branch per 1 914 yds. of cable. The branchings have been made thus: 444 of type B, 28 type Bm, 42 type Block, 12 type Signal, and 14 type Omf. There will be 423 loading-coil boxes.

Disturbances from the railway operating current have not yet been measured, as the contact wires and transformer stations are not ready. There is, however, no reason to suppose that higher induced voltages will occur in the Malmö lines than have previously been formed in the Gothenburg line (see reference in the foot-note on page 81).

Summary.

It is plain that in every respect the plant complies amply with the stipulations made, and that in every way it probably reaches in technical perfection as high a standard as has yet been attained. The good values for echo attenuation and cross-talk, and the excellent insulation resistance, deserve special mention. Compared with previous telegraph and telephone circuits on these routes, the cables have a considerably larger number of circuits available and will therefore be enough to meet all requirements for some considerable time to come.

The Elektromekano Copper Rolling Mill.

Svenska Elektromekaniska Industriaktiebolaget, Hälsingborg.

Number 1—3 of this journal contained a short notice to the effect that Telefonaktiebolaget L. M. Ericsson had acquired the shares of Svenska Elektromekaniska Industriaktiebolaget or "Elektromekano", Hälsingborg, which was thereby merged in the Ericsson Concern.

A factor which influenced the purchase of this business was the wish of the Concern to supply from within its own circle the rolled copper wire consumed in large quantities by the L. M. Ericsson Cable Works at Älvsjö and the Sievert Cable Works at Sundbyberg.

As early as 1918 Elektromekano installed a small wire-drawing plant to provide the copper (dynamo) wire required in their own manufacture of electrical machinery, and so become independent of outside suppliers, who at that time maintained very high prices and also had difficulty in making quick deliveries.

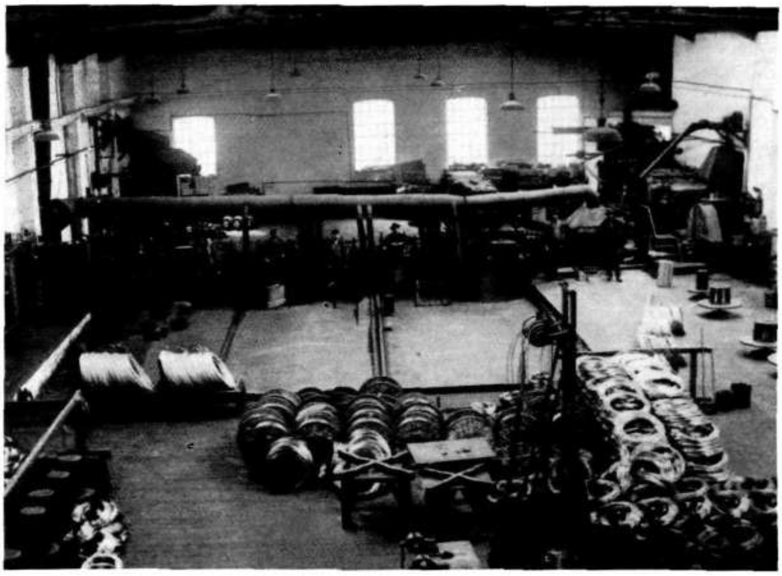
Four years later this wire-drawing plant was enlarged to allow bare copper wire to be sold also, and was further supplemented by a modern wire-rolling mill to enable them to import raw copper in the form of "wire bars", on which so far no important duty is charged in this country.

In 1925 Elektromekano, and particularly its copper works, was ravaged by an extensive fire. In rebuilding it, several new roll stands were added to the rolling mill to allow of the production of smaller gauge wire than before, viz. 6 mm., which is the "raw material" of most large cable works. From this time on, the firm succeeded in securing most of the orders for this commodity from the Danish market also.

Elektromekano is still the only copper rolling mill in Scandinavia able to supply such fine gauge rolled wire.

When at the beginning of this year the Ericsson Concern acquired Elektromekano, the latter firm took over the supply of all the rolled copper wire for the Concern's two cable works, the L. M. Ericsson Cable Works at Älvsjö and the Sievert Cable Works at Sundbyberg.

This work will fully occupy the Elektromekano rolling mill, in spite of its large capacity, and the output may in future be estimated at about 7 500 tons per annum. Of this, about 1 200 tons will be made into bare drawn copper wire, in the firm's wire-drawing plant, partly for sale and partly for its own requirements. This latter copper wire is spun over in their own spinning mill to make so-called dynamo wire, which is used for winding electrical machines, partly in the Elektromekano workshops and partly in a large number of repair shops for electrical machinery. It may be of interest to note how year



R 3092 General view of rolling mill.

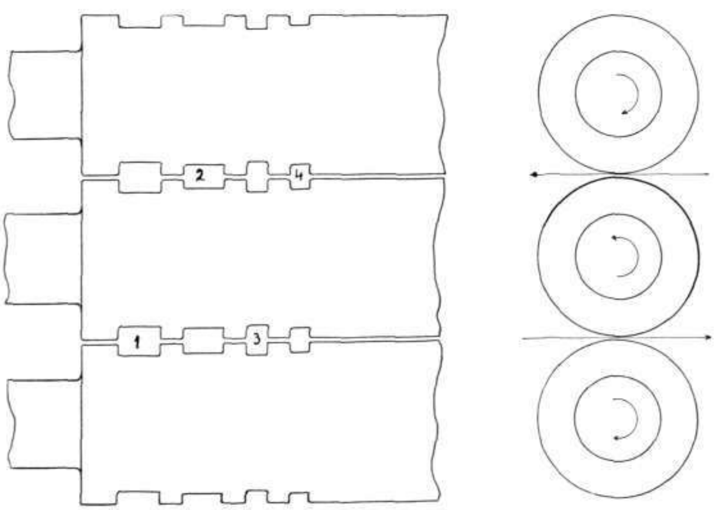
by year the production of rolled wire has increased, as is shown in the table below.

"Output of rolled wire."

In 1922	480 050 kg.
„ 1923	732 963 „
„ 1924	1 240 454 „
„ 1925	751 704 „
„ 1926	1 607 642 „
„ 1927	1 378 818 „
„ 1928	2 628 008 „
„ 1929	2 973 773 „
„ 1930	4 193 420 „
„ 1931	4 246 450 „
Total:	20 233 282 kg.

As we see, the production has increased nearly tenfold between 1922 and 1931, and an output of about 7 500 tons will henceforth, as we said above, be reached.

In its simplest form the rolling process consists of introducing the material between two rolls revolving in opposite directions. This movement grips the bar or billet, and carries it forward, while subjecting it to a pressure which alters the shape or dimensions of its cross section. The rolling may thus be considered a modified forging process adapted to mass production. The first rolling mill was built in Nuremberg in the 16th century and was used for the production of high-quality iron. The first iron rolling mill in Sweden was designed by Kristoffer Polhem, and even today they are largely made on the same principles.



R 4043 Diagram of a three-high stand.

A rolling mill usually consists of a number of specially designed pedestal bearings or standards, in which the rolls are placed. Two standards form between them a roll stand, in which there are usually two or three rolls placed one above the other. One speaks of two-high or three-high stands, according to the number of rolls in each.

In this particular rolling mill, the first roll stand, called the roughing mill, is three-high, with grooves cut in the rolls, as shown in the sketch below.

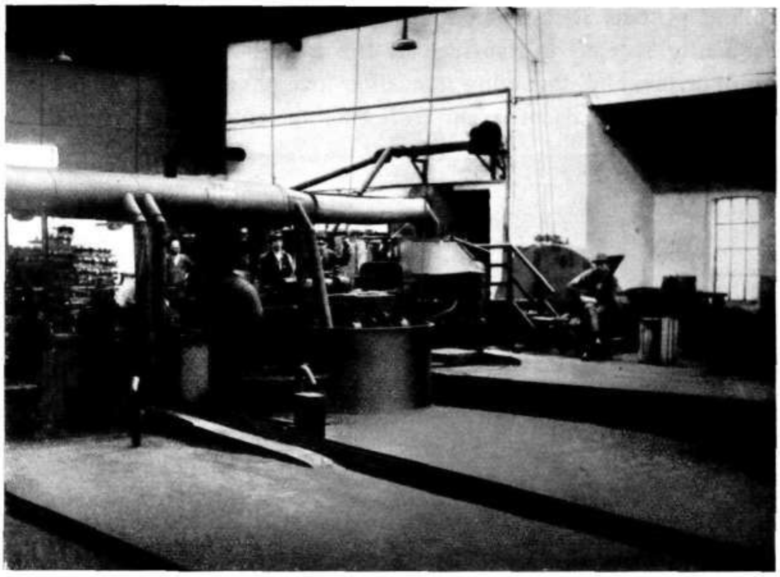
The other stands (five in number) are called the finishing train, and the first of these is three-high and the other four are two-high.

The rolling mill is driven by an electric motor of 35 H. P. and 365 r. p. m. By an elastic coupling this is direct-coupled to a through shaft, at the other end of which there is a kind of gear box, or bushing for the gear pinion, of the finishing train. On the same shaft a large flywheel and a spur gear are also fixed. The latter transmits the driving power to the middle roller of the roughing mill, from which in its turn it is transmitted to a gear pinion bushing for the other roughing mill rolls. It also reduces the speed of the roughing mill to about 130 r. p. m. The speed of the finishing train is the same as that of the motor, or about 365 r. p. m.

The raw material for the rolled wire is supplied in long narrow copper billets called wire bars, weighing from 132 to 200 lbs, which are heated in a furnace to about 850° C. The wire bar is then passed through the first groove of the roughing mill, between the lower and the middle roll. This groove is smaller than the thickness of the bar, and the bar is flattened out by the rotating rolls, its cross section being reduced, and its length instead increased. Wen the billet has passed through to the back of the roll stand, it is lifted up and inserted in the second groove, smaller than the first, between the upper and the middle roll. Its direction of movement is now reversed and, while it is still further compressed and lengthened, the billet is therefore forced back to the front of the stand.

This process is repeated again and

again in smaller and smaller grooves, and finally the bar, now some 10 meters long, is carried *on to the finishing train* and there passed through the grooves of the first roll stand. The remaining stands of the finishing train have, as mentioned above, only two rolls each so that the wire can only be passed in one direction through each stand; alternate stands are therefore revolving in opposite directions. The first "pass" in the second roll stand of the finishing train is thus followed by a "pass" in the third stand, the next pass is again in the second, and so on. The sketch below shows how the cross section is altered groove by groove. At the same time the bar becomes longer and longer, until the finished wire has a length of up to about 200 m.

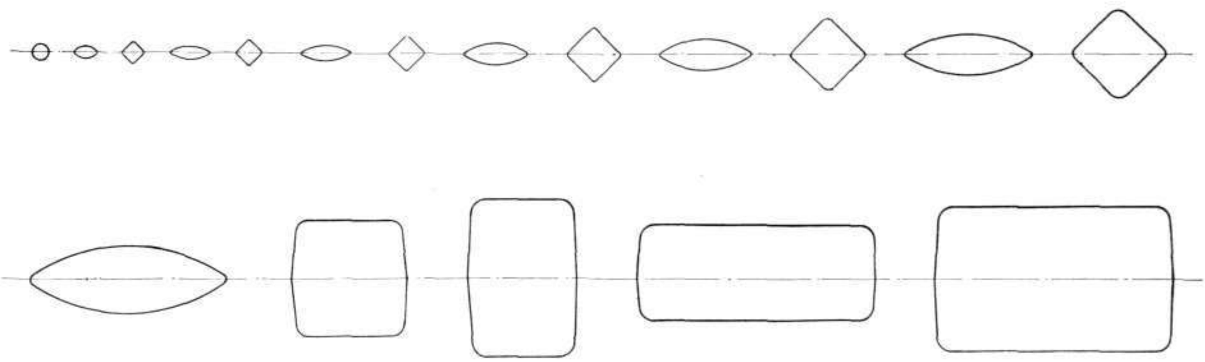


R 3087 The first stand of the finishing train, and the roughing mill.

As we see, the grooves are not of uniform shape, but are sometimes square, sometimes rectangular, and sometimes oval. During these forcible changes of shape, the material is as well worked as if it were forged. By using alternately square and oval grooves, the cross section in particular is reduced very rapidly, which is necessary both in order to loose as little heat as possible in the bar during the rolling process and to attain the largest possible output.

When the bar has passed the last round groove in the finishing train, the wire is ready and is wound on a reel which automatically coils the wire. It is then cooled quickly in cold water. When tied, the coil is ready for sale. The wire is rolled to a number of different gauges, ranging from 6 to 19 mm. round.

Both the rolls and their journals get very hot during the rolling process, partly by contact with the hot billets and partly by the heat generated by the friction. Both rolls and bearings are therefore cooled by cold water constantly running over them. To prevent the scale formed in the



R 4044 Series of grooves in the rolling mill.

rolling process sticking in the grooves and subsequently scoring the surface of the next billet more or less and damaging it, each groove passed by the billet is also flushed with cooling water.

R 3088 Unloading copper billets at the mill from a railway truck.

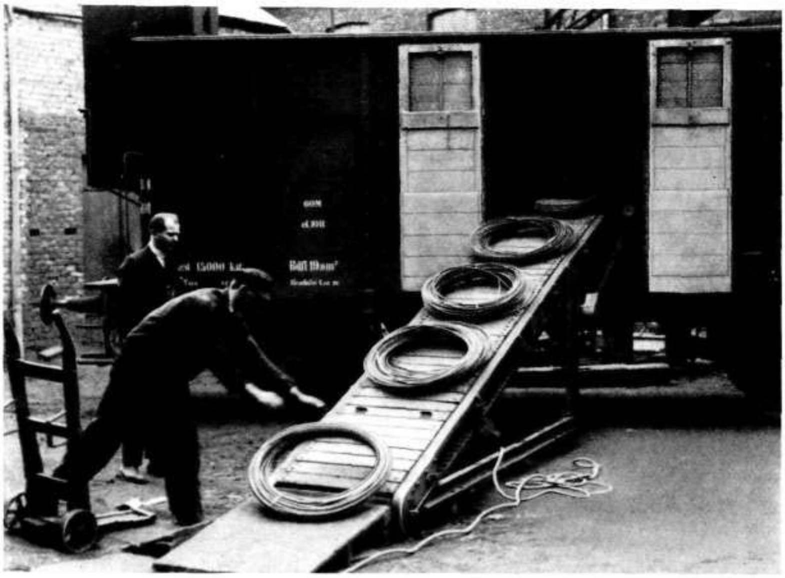
One of the largest orders for bare copper wire ever received by Elektromekano was to supply the State Railway with wire for the electrification of the main line from Stockholm to Malmö. The order comprised about 615 tons of contact wire and copper cables for this electrical installation, the delivery being spread over the next few years as work proceeds.

The great increase in the quantity of copper handled in the rolling mill has made it a necessary economy to reduce manual labour by various mechanical contrivances. The copper is thus now unloaded by electric cranes direct from the ships into railway trucks, which are then shunted on to the factory line. Electrical overhead travelling cranes take the billets thence straight to the furnace in the rolling mill.

The water consumption of the rolling mill is as a matter of fact so large that in order to reduce costs Elektromekano has built its own water conduit from a river near by to supply all the water required.

The effect of the air and the cooling water on the hot billets during the rolling process is to make the wire black, and this layer of oxide has to be removed, before the drawing process, by pickling in dilute sulphuric acid.

In cold-drawing, the end of the wire is pointed and put through a hole in a drawplate of case-hardened iron or steel. The hole is slightly smaller than the wire, which is drawn through the hole by fixing the end to a rotating winding block. This naturally reduces the diameter while increasing the length. The process is repeated several times until the required gauge is obtained. For lack of space we will not enter into any further technical details of wire-drawing.



R 3099 Loading wire coils by the "moving gangway".

Mechanical devices have also been introduced to deal with the finished wire coils. A "moving gangway", for instance, loads the coils into the railway trucks which carry them to their various destinations.

THE L.M. ERICSSON REVIEW

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EXCERPT
SR 50 E

SOME DETAILS OF THE NEW ELECTRIC SIGNALLING PLANT AT GOTHENBURG CENTRAL STATION: ITS DESIGN, INSTALLATION, AND USE

Communication
from the
Signalbolaget
33 KUNGSGATAN, STOCKHOLM

SR 50 E (VII. 32)
200

Some Details of the new Electric Signalling Plant at Gothenburg Central Station: its Design, Installation, and Use.

By *T. Hård.*

The program for reconstructing the Gothenburg Railway Yards, which was started after long investigations and negotiations in 1927, included the building of a new main passenger station, for the common use of all the standard gauge railways entering Gothenburg except the purely local one to Särö. The new station was to replace the two former passenger stations, Gothenburg B. J. and Gothenburg S. J., of which the former belonged to Bergslagen Railway Co, and the latter to the State Railway. Gothenburg B. J. was the terminus of the Gothenburg—Borås Railway and the State-owned West Coast Line as well as the Bergslagen Line. Two State Railway lines ran into Gothenburg S. J., viz. the Bohus and the Western Main Line. The new passenger station was to be on the site of Gothenburg S. J. and to be built and managed by the State Railway as a union station called Gothenburg Central. The new station was finished in May 1930.

The amalgamation was made principally for the purpose of providing greater facilities for the public, two separate passenger stations being inconvenient for the travelling public and hindering through traffic between the various railway lines. It was also thought that the amalgamation would reduce operating expenses, so that an immediate profit on the capital outlay might be expected.

A modern electric signalling plant was an important part of the great project. The installation, from the very first, of such a plant in the passenger station made it possible, in estimating the track facilities, to allow shorter intervals between necessary shunting and train movements than could have been done if a less modern signalling system were to be used, thereby making it possible to limit the number of platform tracks required from 12 to only 10, with a reserve track for departing trains. Right from the beginning this meant a considerable saving, which might be credited to the account of the signalling system.

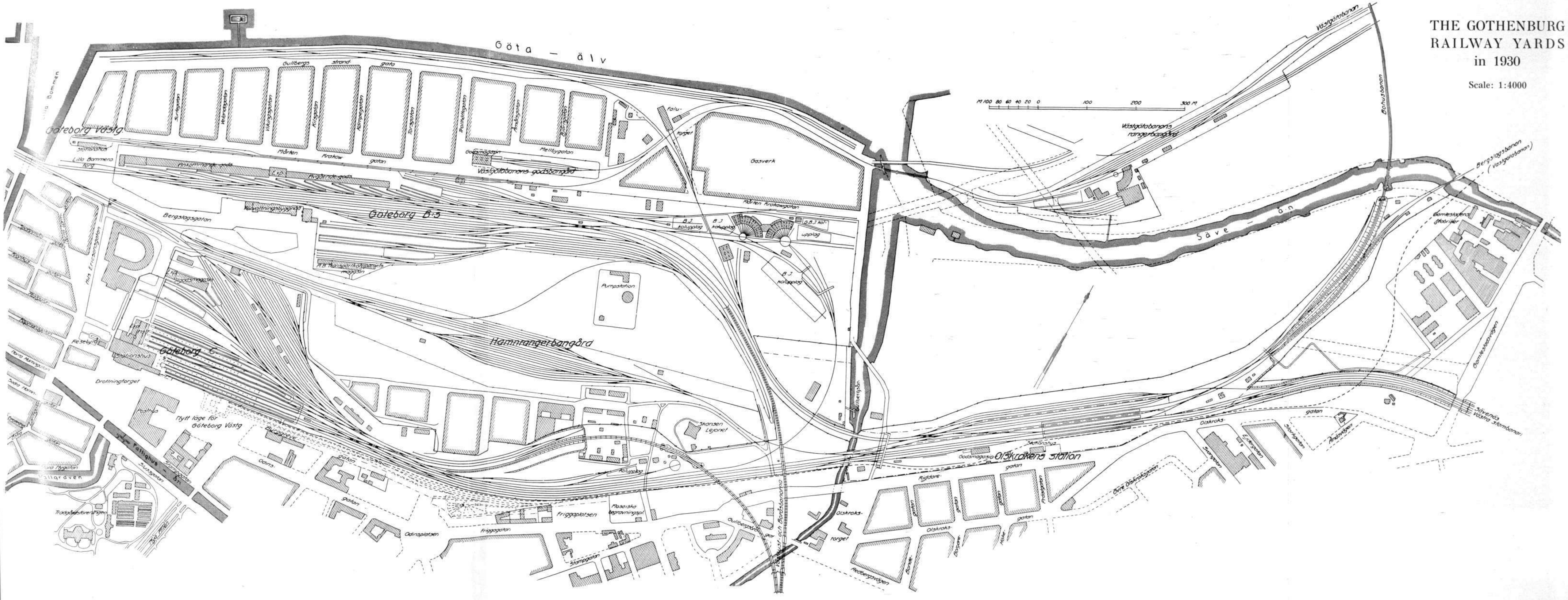
Incoming and outgoing lines.

The railways entering the new passenger station are shown in fig. 1. The trains arrive either by Almedal or Olskroken. The West Coast Line from Trälleborg and Hälsingborg via Varberg, and the Borås Line from Alvesta via Borås, come in by Almedal. The Western Main Line from Stockholm via Falköping, the Bergslagen Line from Oslo and Falun via Mellerud, and the Bohus Line from Strömstad via Uddevalla come in by Olskroken.

The connexions between Gothenburg C on the one hand, and Almedal and Olskroken on the other, can be seen from the plan of the Gothenburg Railway Yards in fig. 2, and also from the track diagram of the passenger station shown in fig. 5, in which the names or numbers of tracks, points, and signals are indicated.

Between Gothenburg and Almedal the trains from the West Coast and Borås Lines run on a joint single track, the *Almedal line,* 3.4 km. in length; at 1.2 km. from the outermost points of Gothenburg there is in this line a branch for the goods trains. These do not enter the Central Station, but are taken directly to and from the joint goods yard, which occupies the site of the former Gothenburg B. J. passenger station. The goods trains accordingly do not touch the new passenger station except in so far as the diverging points, and the signals for train movements through these points, are controlled from the passenger station signal-cabin.

Between Olskroken and Gothenburg C, the trains of the Western Main Line and the Bohus Line run on a joint double track, the *Up Line* and the *Down Line*, the length of which between the outermost points is 800 m. Between the same



THE GOTHENBURG
RAILWAY YARDS
in 1930

Scale: 1:4000

Fig. 2.

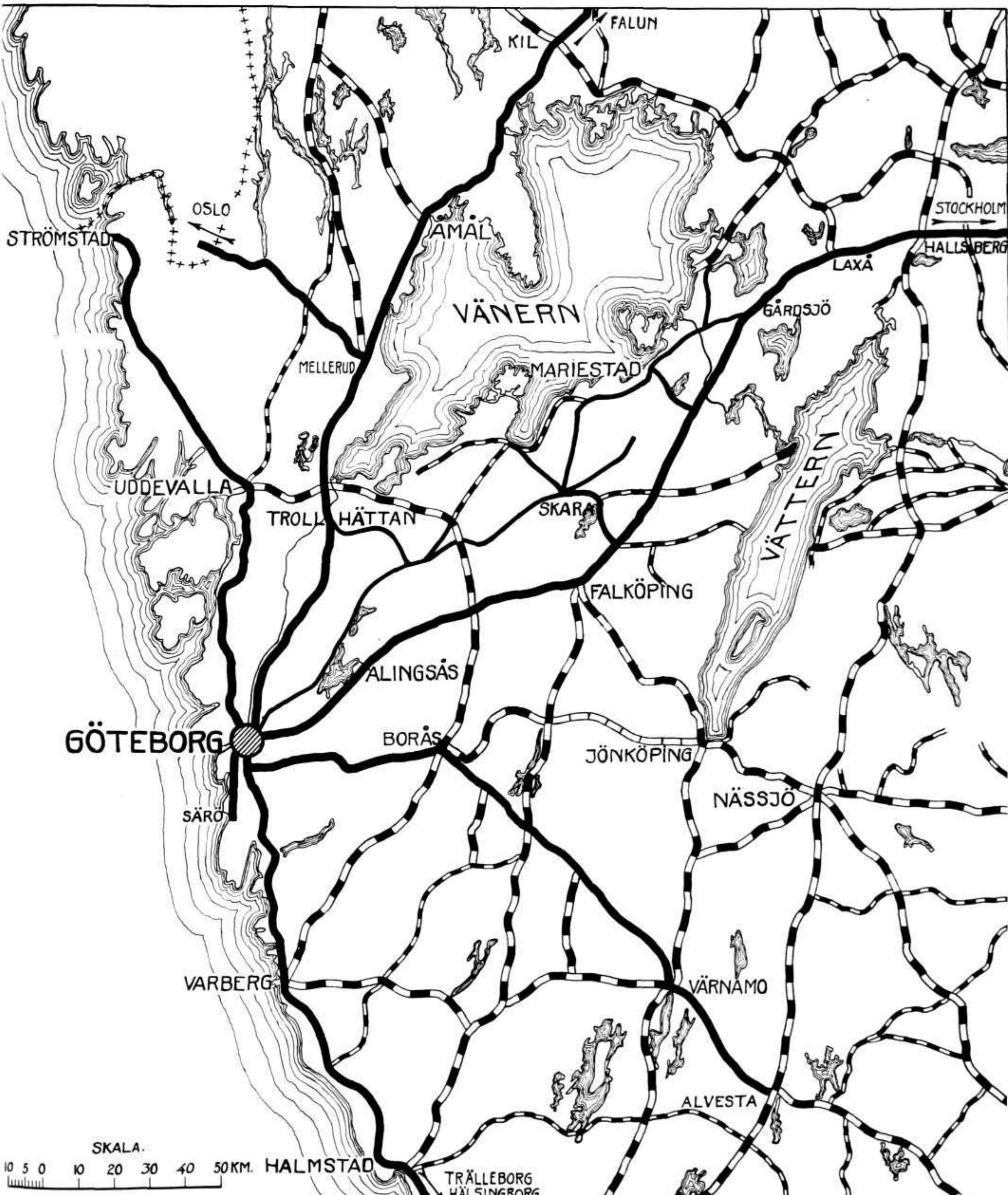


FIG. 1. MAP OF THE RAILWAY LINES RUNNING INTO GOTHENBURG.

stations there is also a separate single track, 800 m. in length, the *B. J. Line* for the trains of the Bergslagen Railway.

Besides these, there are two other tracks, the *North Loop* and the *South Loop*, between Gothenburg and Olskroken, used principally for locomotives on their way to and from the sheds, which are situated beyond Olskroken station. The locomotives of the Bergslagen and the Borås Railways use the North Loop and the State Railway the South Loop. A number of single locomotives are also run on the double track.

The number of scheduled movements on the lines running into the passenger station is shown in the table below.

	Express trains	Ord. trains	Local trains	Engines	Goods trains
Down Line	6	18	26	10	—
Up Line	6	20	27	7	—
South Loop	—	—	—	83	—
B. J. Line	10	15	7	—	—
North Loop	—	—	—	61	—
Almedal Line	15	24	29	—	19
	37	77	89	161	19

The total number of scheduled movements to or from the lines using the terminus is therefore 383 in twenty-four hours.

Line Blocking.

All the lines connecting Gothenburg C with the adjacent stations were provided with signals for line blocking. The Olskroken lines were provided at each end with a fixed signal automatically controlled by a track circuit over the whole line. In fig. 5, for example, the signals 5h and 5v apply to the North Loop, 7h and 7v to the B. J. Line, etc. The signals are controlled by signal levers (5,7 etc.), one for each line. If a lever is reversed to the right (h) or left (v), the signal is cleared at the Olskroken or Gothenburg end of the block section respectively, but will not show "proceed" unless the track circuit is free from vehicles. The line blocking works entirely automatically for trains in the same direction, as long as the signal lever is kept in the reversed position.

The signal lever can be restored to the normal position when a train occupies the block section and the signals at both ends are at "stop". If the signal lever is to be restored when there is no vehicle on the track circuit, a special time-switch has to be used, so arranged that the signals for both directions must show "stop" for a certain length of time before another "proceed" can be given. This permits a train which has already received the proceed indication but fails to see or sees too late a subsequent stop indication to enter the block section and shunt the track before "proceed" can be given to a train in the opposite direction. The time switches are operated by springs and can be adjusted for time intervals of up to 2 minutes. There are 6 of these time switches, one for each track.

Both the Up and the Down Lines were provided with signals for both directions, although under normal conditions double track working is arranged, that is, the Down Line is used for trains to Gothenburg and the Up Line for trains from Gothenburg. If one of the lines must be given up on account of work on the permanent way or on the trolley wires, or if there is an exceptional density of traffic in one direction, single track service can be arranged on the other track with complete protection, by means of signals.

While only one block section was required for each of the tracks between Olskroken and Gothenburg C, the Almedal Line, to make shorter intervals between trains possible, had to be divided into two block sections, 1 800 m. and 1 400 m. in length. The difference in length was due to the fact that in the section next to Gothenburg C, the northern section, speed has to be reduced considerably when passing a viaduct running in a sharp curve over the Olskroken lines.

The northern block section also includes the track between the outlying points 2 and the entrance signal to the goods yard. When a signal is cleared to enter the northern block section, the tracks must be free from vehicles both to the goods yard and to the passenger station.

The signals for the northern block section are arranged on the same principles as those for the lines between Gothenburg C and Olskroken. There are entry signals to the block section both from the direction of Almedal (1h) and from the goods yard (1va) and Gothenburg C (1vb). The signals are also intended to safeguard movements of trains through the outlying points 2, which are operated from the Gothenburg C signal cabin.

The signals B1 and A2 control the entrance to the block section nearest to Almedal, i. e. the *southern* block section. These signals are all-automatic and no levers for them are provided in the frame in the Gothenburg C signal cabin. In normal position "proceed" is shown. When the lever 1v at Gothenburg C is reversed to prepare for a train movement from Gothenburg C or the goods station towards Almedal, A2 will show "stop", where it is held by the track circuits until the train has arrived at Almedal. When a train leaves Almedal for Gothenburg C the conditions are different. The signal A2 is then independent of the track circuit of the northern block section and may show "proceed" for a following train from Almedal as soon as the first train has passed out of the southern section.

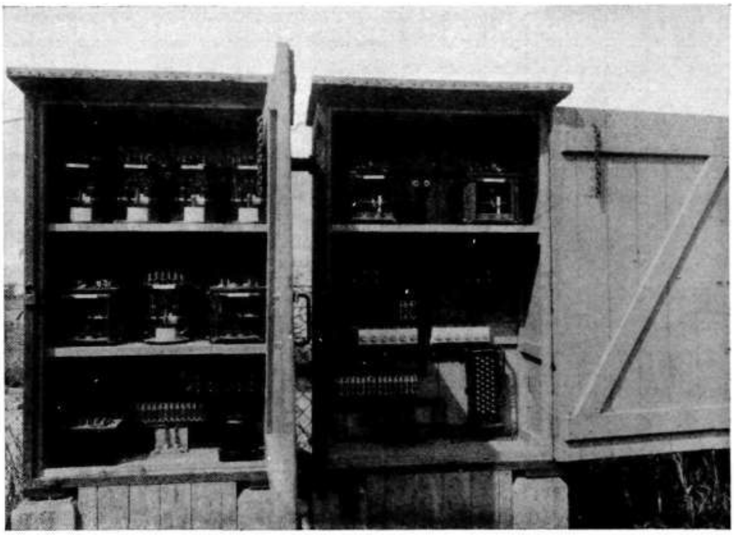


Fig. 3. Automatic block post on the Almedal Line.

When a train from Almedal has entered the southern section, signals 1va or 1vb cannot show "proceed" until the train has arrived at Gothenburg C or the goods station. For trains from Gothenburg towards Almedal, on the other hand, 1va or 1vb can indicate "proceed" for a following train as soon as the first train has cleared the northern block section.

The signal arrangements for the Almedal Line thus allow shorter intervals between following trains than between opposing trains. The arrangements used are made on the same principles of wiring as the well known A. P. B. block system for single lines.

The Passenger Station.

The station is of terminus type with making-up tracks for the trains to one side of the arrival and departure platforms. Between the making-up yard B and the platforms there is a special smaller yard F, for mail and express parcel coaches, which are often run with passenger trains. As the number of platform tracks is small compared to the number of trains and incoming and outgoing lines, the trains must not be kept longer at the platforms than is necessary for the passengers to get on or off. Incoming trains that are not due to leave immediately after arrival should therefore be moved over as soon as possible to the making-up yard, and outgoing trains be brought to the platform as shortly before departure as possible.

Figs 4a and b show graphically how the platform tracks have been used for the time-table in force in 1930, when the station was put in use, and for the present time-table. The time is marked by a vertical line for each hour of the day, and each platform track is represented by two parallel horizontal lines. These latter are filled in at times when a train is standing on a platform track. The part filled in is terminated by a leaning cross line on the left for incoming and on the right for outgoing trains. When the train begins or finishes at Gothenburg C and has therefore to be brought to or from the platform track by shunting within the station limits, the part filled in is terminated by a vertical cross line on the left or right respectively.

Usually empty trains are moved between the making-up yard and the platform tracks via the draw-out tracks K, L, and M. When a train is shunted away from the platform, its own locomotive backs it into one of these tracks, generally K, whence it is backed by a shunt engine to the making-up yard B.

The locomotive is then moved as soon as possible to the shed unless it has to be taken to the local engine depot of the passenger station for taking fuel or water, or to be turned; or, as is often the case with electric locomotives, it goes to the waiting track A to wait there until the next train to use it is due to depart.

When traffic is particularly heavy it may hap-

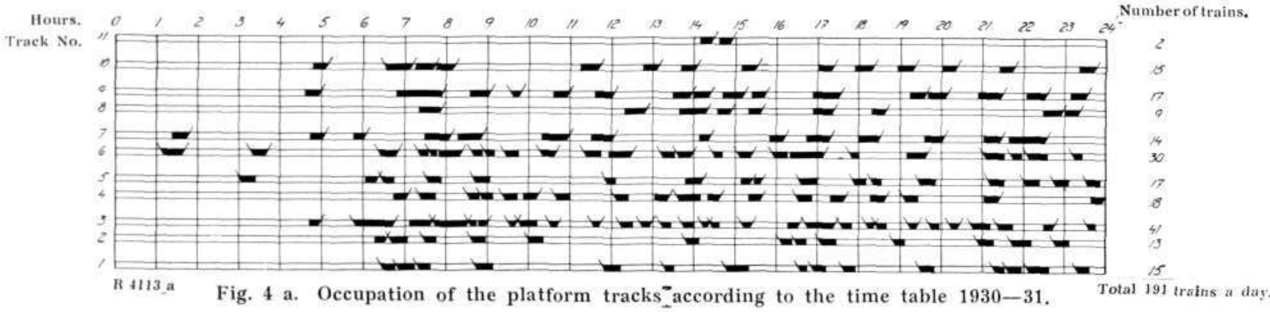


Fig. 4 a. Occupation of the platform tracks according to the time table 1930—31.

Fig. 4 b. Occupation of the platform tracks according to the time table 1931—32.

pen that none of the pull-out tracks is available. In this case the tracks O are used for storing empty trains until the traffic allows them to be shunted to the making-up yard.

Empty trains are usually moved from the making-up yard to the platforms via the pull-out tracks by shunt engines. The train locomotive is attached after the train has been brought to the platform. During busy hours new trains already made up can be stored in tracks O in order to release the draw-out tracks, which are occupied to a great extent also for rearranging trains in the making-up yard.

As far as possible locomotives arriving from the sheds go direct from the incoming line to their train if the latter is standing at the platform. If the train is not ready when the locomotive arrives, the latter is held on the track inside the incoming signal or moved to the waiting track A or to any other available track where it can wait for its train.

Local trains made up of electric rail motor cars generally leave after only a short stay in the station, during which the train is standing at the platform. Other local trains using a locomotive as tractive power are backed out from the platform track during their stay in the station, whereupon the locomotive is shifted to the other end of the train over suitable points, and finally the train backed to its departure platform.

A special category is formed by the large express and passenger trains which carry the traffic between the Continent and Oslo via the West Coast and Bergslagen Lines. These trains also have through-carriages from or to Gothenburg C, which have to be coupled to and fro during a short stay in the station.

There are also through coaches between the Western Main Line on the one hand and the Bohus and West Coast Lines on the other. These are moved from one train to the other by shunting at the ends of the platform tracks, while the trains are standing at the platforms.

Finally mail vans and express goods cars are shunted to or from the long distance trains, as well as cars carrying fish and other fresh goods, which are transferred from the harbour station to the passenger station via tracks B to be connected to passenger trains just before their departure.

From the above it will be seen that in designing the signalling arrangements, allowance must be made not only for train movements to and from the outgoing or incoming lines but also just as much for shunting operations within the station limits. Some mechanization of the signalling also for shunting movements was essential, to make it possible to deal with the expected traffic rapidly and without undue risk. It was therefore clear from the beginning that at Gothenburg C a system would have to be employed that would allow the use of fixed signals and interlocked points for the shunting movement as well, i. e. an installation similar to the one taken into use a few years before at Malmö, where traffic conditions are similar.

The division of the tracks into signal sections.

A shunting movement generally affects only a small part of the track system, while an incoming or outgoing train makes a continuous movement along the whole length of the yard between a platform track and one of the lines. In order not to impede other movements unnecessarily, the locking of points in shunting should not extend over a longer stretch at a time than is necessary for safeguarding the movement. The track lay-out was therefore, for signalling purposes, divided into sections, each of which was provided with signals for both entering and leaving.

The outgoing signal of one section is at the same time the entry signal for the next. Only sections ending with a dead-end track or leading to a siding, have no outgoing signals. The outgoing signals for the sections at the extreme end of the station are at the same time the entry signals for the block sections ahead.

Where a track branches off, one signal may serve as the entry signal for several sections. We find, for instance, in fig. 5 that signal 43h is for 5 different sections, ending at signals 53h, 51h, 49h, 47h and 45h. Similarly one and the same signal may be the outgoing signal for several sections meeting at that signal. 49h, for instance, is the outgoing signal for 3 sections, the entry signals of which are 43h, 39h, and 35h.

The determining of the positions and lengths of the signal sections is of course of great importance if the installation is to give good results, and should be done with careful attention to the demands of the traffic. The signal sections for inward and outward movements do not as a rule coincide either in position or length. The cost of construction increases with the number of signals, and the number of sections should therefore not be greater than necessary. Some of the principles employed in determining the signal sections at Gothenburg C are given below.

The station boundary, i. e. the outer ends of the sections next to the incoming and outgoing lines, has been pushed so far out that shunting may be done as a rule without affecting the use of the block sections. The departure of trains or single locomotives from Olskroken or Almedal is thus not interfered with by shunting operations in the station.

Each platform track is arranged as a separate signal section, the outgoing signal of which is placed at the fouling point behind the points at the outer end of the platform track.

The sections lying between the platform tracks and the station boundary have been determined by the position of the sets of points, which are drawn across the track system in two main directions. Movements in the cross-overs leading to the pull-out track K, which is generally used for moving empty trains to and from the platforms, ought to be possible with a minimum of disturbance from other movements. The signals were therefore grouped on either side of these cross-overs.

Draw-out movements from the platform tracks to move carriages from one train to another should be made as short as possible, in order to cause the least possible obstruction to other movements. The signal sections nearest the platforms are therefore made shorter than the others. When, for instance, coaches are being moved from track VI to track VII, they can be pulled out to signal 45va without interfering with movements further out in the track system.

Another advantage of having short sections at the ends of the platform tracks is, that a train for which there is not enough room in the signal section formed by the platform track will nevertheless be standing just inside a signal for outward movement. The platform track may be said to be extended by the short signal section adjoining it.

In all groups of points where a pull-out movement is often followed by backing the same set of cars, the signals are so arranged that there is always one available for showing "proceed" for the backward movement at the point where the pulling out movement usually ends. When a carriage is moved from track VII to track VI, signal 65va may for example be used for the pulling out and signal 65h for the backing in.

The division into signal sections is of course mainly a matter of judgement for which no exact rules of general validity can be given, and which must therefore be settled in each case according to local conditions.

The total number of signal sections within the station boundaries is 73 for inward movements and 84 for outward movements. Besides these there are the six block sections outside the station boundaries, which may be regarded as signal sections lying in the lines.

When a train arrives or departs it will pass several signal sections one after the other, the entry signals of which must each be at "proceed". It would of course be possible to let the movements of incoming and leaving trains be controlled entirely by shunt signals. This is also done in certain cases, namely, in the case of trains running on the double track against the normal direction of traffic, and trains on the Loop Lines, besides all cases where for some reason the whole route between the line and the platform cannot be cleared at once, e. g. for a locomotive going up to the platform track occupied by its train. Again, when a departing train is so long that its locomotive is standing outside the outgoing signal from the platform section, it must be moved in stages on the same signals as are used in shunting.

Movements in stages under control of shunt signals alone would, however, mean much loss of time to long and heavy trains, as they would then always have to be moved at comparatively low speed. Special signals have therefore been arranged for movements between the platform tracks and the incoming and outgoing lines generally used by the passenger trains enabling a "proceed" indication to be shown for the whole train route through the yard. All the signal sections on the route must then be clear and the signals for them showing "proceed". Such special signals are arranged for trains coming in from the Down, B. J., and Almedal Lines and departing from any of the platform tracks towards the Up, B. J. and Almedal Lines.

The track arrangements frequently allow of several routes for a train between a line and a platform track. In such a case the installation also allows for the use of any of these routes, that one to be chosen which can be used at the time of the train movement with the least possible interference with other simultaneous movements. Between the B. J. Line and track VI, for instance, there are three routes, viz. over points 66a, 36 or 54a. The total number of routes between the incoming and outgoing lines and the platform tracks is therefore greater than that indicated directly by the number of lines and platform tracks, being 52 for incoming and 57 for outgoing trains, instead of 30 and 33, as it would be if for each line there were only one arrival and one departure route per platform track.

The division of the tracks into track circuits.

Besides signal sections, the tracks have also to be divided up into insulated track sections or track circuits, the function of which is to effect that part of the signalling whose object is to show if the route is free of obstruction, i. e. clear of vehicles, to a shunting or train movement. The track circuits must further prevent the points being operated too early, either just under a vehicle standing over a pair of points when a derailment certainly would occur—or just in front of a movement, which otherwise might be turned on to a different track from the one intended. Track circuits also serve to prevent a second movement taking place before the first one started has been safeguarded.

The division into track circuits is shown in fig. 5, where the limits between them are indicated by a break in the track. Track circuits are denoted by an S, followed by the number of the points in the track circuit or, where there are no points, the name of the track or the number of the signal at the inner end of the track circuit. This symbol is printed in the plan only when referring to a signal number or a track.

A division between two track circuits is always located opposite or quite close to every signal. A signal section may, however, include several track circuits. The divisions between these are often placed at rail joints in front of points or opposite fouling points between converging tracks. In the parts of the lay-out where shunting is frequent and the lengths of these movements therefore ought to be limited as far as possible in order not to hinder simultaneous movements, the track circuits have been made shorter than elsewhere. This is, for example, the case at the ends of the platform tracks.

Considering the cost of installation and maintenance, an increase in the number of track circuits is a disadvantage, and the division must therefore not be carried further than the traffic conditions make necessary. The dividing into track circuits is mainly a matter of experience and judgement, and greatly affects both the economy and the usefulness of an installation.

Fig. 6 shows part of the track lay-out on a larger scale, so that the position of insulating rail joints and connectors between the rails can be seen. The two rails opposite one another in a track circuit must not be in metallic contact. All the tie-plates and connecting rods between the rails at the points must therefore be provided with insulated joints, which are made by having the plates and rods made in two parts separated from one another by packings of vulcanized fibre. One of the rails of a track circuit is called the plus, the other the minus rail. The same polarity must not prevail in rails opposite one another for a length of track greater than the minimum distance between the axles of the vehicles. At Gothenburg this length in no case exceeds 1.5 or 2 m.

The length of a track circuit ought to be greater than the max. distance between axles on the vehicles, as, when a train is passing, the plus and minus rails must be in constant connexion through the wheels. The shortest track circuit at Gothenburg is 17 m.

Only the plus rails of two adjoining track circuits have been separated by insulating rail joints, the minus rails being metallically connected with one another to serve as return conductor for the traction current.

The total number of track circuits within the station boundary is 65, that in the outside lines 10. Frequency-selective vane relays, Westinghouse Type L, are used as track relays except in seven circuits which are not affected by the currents of the electric trains. The frequency-selective relays are immune to the frequency of the traction current, $16\,^2/_3$, and are fed with a 50-cycle signal current via condensers and inductive track transformers, so that the supply of current is practically constant. The principles of this device are described in an article in the L. M. E. Review 7—9, 1931. Relay transformers of ratio 1 : 4 are connected between the track and the relay to reduce the voltage drop in the long cable conductors to the relays, all of which are located in the signal cabin.

Interlocking of the levers in the locking frame.

Tables I and II give extracts from the locking tables prepared for this installation. To make the tables clear it may be mentioned that points and signals are denoted by the numbers of the levers with which they are operated. Even numbered levers are used for the points, odd numbered for the signals. When two pairs of points are operated by the same lever only one number is given in the table, but in the plan these points are distinguished by the letters a and b after the number. A letter h or v after the number of a signal means that the signal is operated by moving the signal lever to the right *(höger)* or left *(vänster)* respectively. The former is used for inward, the latter for outward movements. Thus in clearing the signals the levers are moved in the direction of the train movement, which makes the arrangement easy to survey and facilitates operation.

Before a signal can be cleared, the point levers must first be brought into proper position for one of the signal sections beginning at the signal. If the position of a point does not correspond to the position of the lever in the cabin, it must be impossible to show "proceed". When the signal lever is reversed, the point levers in the signal section are locked in their proper position and interlocking is then said to be arranged between the signal and point levers. Further, it must be impossible for the signal to be cleared if a "proceed" indication has already been shown for any conflicting movement, by which is meant a movement in another signal section having some tracks common with the first one. This is prevented either by direct interlocking between the signal levers or by locking some suitable points, safety points, in positions preventing entry to the signal section to be protected. The locking of safety points is not always possible, but has been used in this installation as far as admitted by the track lay-out and by practical considerations in general.

Movements across the track just outside the signal section run the risk of a collision should the movement for which the signal is to be cleared not be stopped at the outgoing signal of the section but slip over into the next section, and are therefore regarded as conflicting. Such movements crossing the track just outside the signal section have been prevented by locking

TABLE I

Lever	With	Requires	Unless	Lever	With	Requires	Unless
25h		13v, *43h*		41v		26, 38/*38*, *40*	
»	26, 38	*53h*		»	38	23h, 28, 39h	
»	26, *38*, 40	48, *51h*		»	*38*	25v, 43h	
»	26, *38*, *40*	46, *49h*					
»	26, *38*, *40*, *42*	*66*, *67h*		43h		26/*26*	
»	26, *38*, *40*, *42*	*72*	68/70	»	26	38/*38*, 53v	
»	*26*	44		»	26, 38	48	
»	26, 32	*47h*, *66*, *67h*		»	26, *38*	40/*40*, 41v, 51v	
»	*26*, 32	*72*	68/70	»	26, *38*, 40	46	
»	*26*, *32*	*45h*		»	26, *38*, *40*	37v, 42/*42*, 44	
»	*26*, *32*, 36	*67h*		»	26, *38*, *40*, 42	49v	
»	*26*, *32*, 36	*72*	68/70	»	26, *38*, *40*, *42*	47v	
»	*26*, *32*, *36*, 56	*65h*, *72*		»	26, 38, *40*, *42*	67v	66
»	*26*, *32*, *36*, *56*	*59h*, 60		»	*26*	*28*, *30*, 32/*32*, 33v, 34	
				»	*26*, 32	42, 44/*44*, 47v	
				»	*26*, 32, *44*	49v	
				»	*26*, 32, *44*	51v	46
				»	*26*, 32, *44*	53v	46/48
				»	*26*, 32, 44	67v	66
				»	*26*, *32*	36/*36*, 45v	
				43v		25v, 26, 38	

TABLE II

Lever	Requires	Unless	Lever	Requires	Unless
14	*16*		16	29h, 55h/v	
16	14, 18, 22, 34		»	45v	*32*
18	16, *20*, 22				
20	18		18	55h/v, push-button 55t	
22	16, *18*, 32				
26	*28*		20		
28	26, *30*				
30	28, 34		22	29h, 45v, 55h/v	
32	22, 34, 44		»	33v	34
34	16, 30, 32, *44*		»	57v	*50*
36	66		»	65v	*50/54*
40	44				
42	44, 46		26	23h, 33v, 37v, 39h, 41v, 43h/v	
44	32, 34, 40, 42, 65		»	45v	32
46	42				
50	52, 56		28	33v, 39h	
52	50, 54		»	35h	*12*
54	52, 56		»	37v, 41v	*38*
56	50, 54		»	45v	32
58	60		24	33v	22/34
60	*58*		»	45v	22/*32*
64	70		»	55v	*16*
66	*36*, 44		»	55h	
68	70				
70	64, *68*				
4	12				
10	24				
12	4				
24	*10*				

trailing points also beyond the signal section. The trailing points have been locked over the whole of the next signal section whenever demanded from the safety point of view. For movements from the platform tracks, the risk of pulling out past the outgoing signal of the section was not important, and, as the locking of all the trailing points in the next section would restrict other movements too much, it was for those movements considered sufficient to lock trailing points within a distance of 50 m. from the end of the signal section. This would, for instance, allow movements in the cross-overs leading up to track K to be made independent of movements in, for example, the signal section 65va—45va, as the distance between the outgoing signal 45va and the fouling point behind points 34a is about 50 m.

In a few cases a ground signal of the type used in connection with scotch blocks has been mounted at the fouling point between converging tracks and so connected with the points that the signal shows "stop" for the main track when the points are reversed. Behind points 26a there is such a signal, which allows the points to be free for movements on, for instance, signal section 53v—43v. For the section 43v—13v, which contains points 26a, these are of course locked in spite of the ground signal.

Signal levers for routes direct between the lines and the platform tracks are made to depend on the signal levers for all the signal sections of that route, so that the latter levers must be reversed before the first ones can be operated. This locks the levers of the sections in the reverse position, and they cannot be restored to normal unless the lever of the direct route has first been put back.

Looking at table I, we find on the right examples of interlockings for a signal lever (43h) connected with a signal for a signal section, and to the left for another signal lever (25h), connected to an incoming main signal from a line to a platform track.

The table for 43h reads as follows, line by line:

The reversal of lever 43h to the right requires point lever 26 to be at + (normal position) or — (reverse position);

with point lever 26 at +, 38 to be at + or —, and signal lever 53v not to be reversed to the left;

with 26 at + and 38 at +, 48 to be at +;

with 26 at + and 38 at —, 40 to be at + or —, and 41v and 51v not to be reversed to the left, etc.

The table for lever 25h reads line by line as follows:

When lever 25h is reversed to the right, 13v must not be reversed to the left, but 43h must be reversed to the right;

with 26 and 38 at +, 53h to be reserved to the right;

with 26 at +, 38 at —, and 40 at +, 48 to be at +, and 51h to be reversed to the right etc.

Table II shows on the right how the point levers are locked by the signal levers. For point lever 28 the table gives the following lockings:

of 33 and 39 always;

of 35h unless point lever 12 is at —;

of 37v and 41v unless 38 is at —;

of 45v unless 32 is at +.

A special kind of locking is sometimes arranged direct between the point levers, so that these must be operated in a definite order. This kind of interlocking will to some extent simplify the arrangements, but may on the other hand necessitate points being operated simply to release point levers, thereby increasing the number of point operations. In a busy station this is a disadvantage and may, when in a snowstorm snow and ice render the operation of points difficult, add to the difficulties of keeping the traffic going. In view of this, interlocking between point levers has at the Gothenburg installation, as well as at other similar installations on the State Railways, only been employed where obvious advantages from the safety point of view are thereby obtainable without complicating the operation of the points.

The principles followed in using this kind of interlocking at the Gothenburg installation may be made clear by some example. For this purpose we will again refer to the diagram in fig. 5.

Points 26b and 28a form together a single slip switch. For all movements with points 26 at minus it is necessary for points 28 also to be at minus. The levers of these points have therefore been interlocked so that 28 always has to be reversed before 26 can be reversed. The removal of the points to normal must then be effected in the reverse order.

Before reversing point lever 4 12 must be at normal, because points 12 must always be at plus when a movement takes place with points 4 at minus.

At a double cross-over, as between points 50a and b, and 52a and b, the point levers are interlocked so that only two pairs of points at a time can be brought in position for movements through the crossing.

When points 18 are set for movements from track K into the running lines, 20 must first be set to track L, so as to serve as safety points for these movements.

On the left in table II, examples of direct interlocking between point levers are shown.

Locking by track circuits.

Track circuits are used in the installation to lock signal levers in reverse positions, *route locking*, and for direct locking of point levers, *point locking*.

The levers of signals governing movements direct between the incoming and outgoing lines and the platform tracks are equipped with route locking, which works in the following way. On reversing the signal lever it becomes locked in the reverse position and cannot be restored to normal until the train has passed over the route and entered the last track circuit of the route, i. e. the track circuit of the platform track for incoming trains, or the track circuit nearest to the block section for outgoing trains. The signal lever has to be restored after each train, as it is arranged that otherwise "proceed" cannot be shown for a following train.

Should it be necessary to restore the lever and change the route without a train having passed, the lever can be released by means of a time switch. The signal must then show "stop" for a suitable length of time before the signal lever can be brought into such a position that the point levers of the route are free to move. This time-switch is also used, should the devices for automatic release of the route fail to function. A sealed switch for emergency release could therefore be dispensed with. The time switches are of the same type as those used for the line block sections and have been installed for routes to and from the Up, Down, B. J. and Almedal Lines.

Route locking can only be used where the clearing of a signal is followed by a continuous movement from one end of the route or signal section to the other. This is not the case in shunting, as every movement cannot be expected to pass the whole signal section, but might stop within the section and continue in the opposite direction. The levers of the signal sections are therefore not provided with route locking, but are left free to restore to normal at any time. Point locking is therefore arranged to prevent points in a signal section being operated before this can be done with safety.

Two different methods of point locking, one *direct* and one *indirect*, are used in this installation side by side, supplementing one another. By the first method the point levers are locked as long as certain track circuits are shunted by vehicles. The locking is effected by contacts of the track relays directly breaking the current to the lock magnets of the point levers. The following track circuits are considered for such locking of point levers.

Track circuits in which points connected to the lever are situated, e. g. S26 and S26/28 for point lever 26;

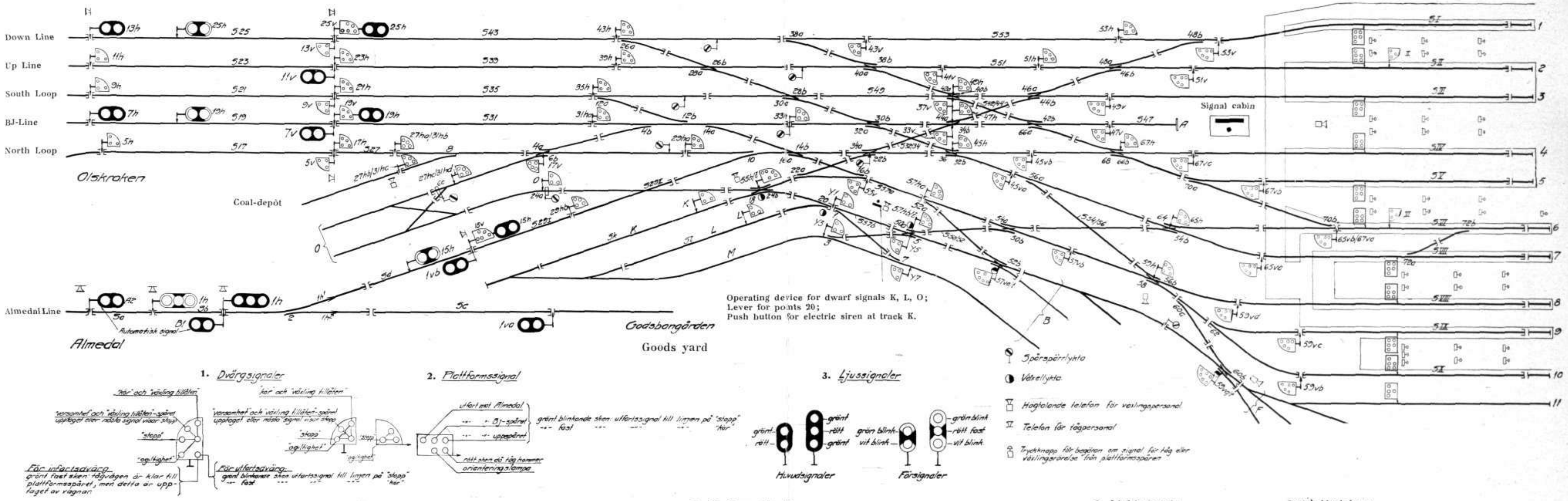
the track circuit between two pairs of points connected to the lever, e. g. S54/56 for lever 56;

track circuit behind a pair of points connected to the lever, when wanted to prevent the throwing of the points until a previous movement has passed clear of the fouling point, either behind a shunt signal showing "stop" for movements towards the points or behind a pair of points which can be laid in a position to prevent such movements, e. g. S49, S28/30 and S12a for lever 40; S32/36 and S30/32 for lever 34;

track circuit situated in front of a pair of points connected to the lever when wanted to prevent the points being operated before a signal controlling movements in the opposite direction has been passed, e. g. S12a for lever 30, or when a vehicle is immediately in front of the points, e. g. S57b for lever 20.

The direct method of point locking acts independently of the direction of movement, but may be changed by varying the positions of other points. The locking between S12 and lever 40 for instance does not function if point lever 12 is reversed.

By the second, indirect method, the locking of the points only takes place if a vehicle enters the signal section with the lever for the incoming signal of that section in reverse position. The locking is effected by means of special lock



1. Dwarf signals.

"Proceed" and "Shunting allowed".
"Caution" and "Shunting allowed"-track engaged, or next signal at "stop"
"stop"
"neutral"

For incoming dwarf:
Green steady light: route clear to the platform track, which is occupied.

"Proceed" and "Shunting allowed".
"Caution" and "Shunting allowed"-track engaged, or next signal at "stop"
"stop"
"neutral"

For outgoing dwarf:
Green flash light: entry signal to the line at "stop".
" steady " " " " " " "proceed".

2. Platform signals.

Departure towards Almedal
" " BJ-Line
" " Up-Line

Green flash light: entry signal to the line at "stop".
" steady " " " " " " "proceed".

Red light when train arrives.
Marker light.

3. Light signals.

	green		green flash light
green	red	green flash light	red steady "
red	green	white "	white flash "
Main signals		*Distant signals*	

Scotch block lamp.
Point lamp.
Loudspeaking telephone for shunting staff.
Telephone for train staff.
Push button for requesting the signal for a train or shunting movement from a platform track.

Fig. 5.

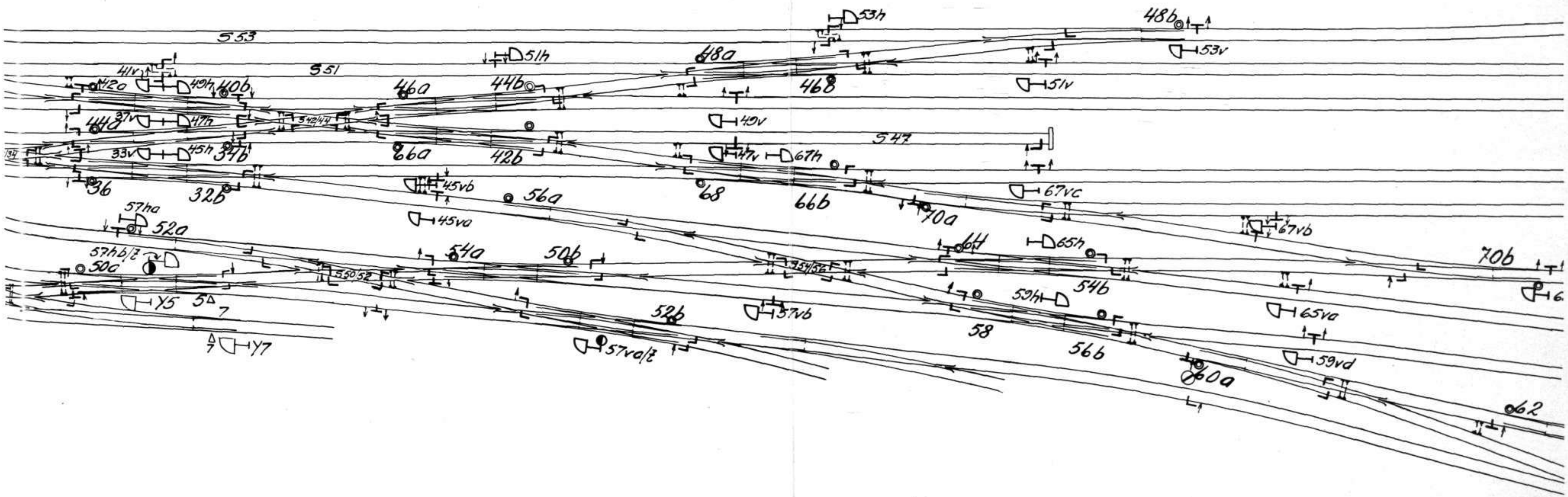


Fig. 6.

relays and functions differently according to the direction of movement. The use of this locking method is best explained by an example.

Supposing signal 45va has been cleared for a movement into the signal section between 45va and 9v over points 30. Point lever 30 is made dependent on a relay which starts locking the lever at the moment the signal is passed by a vehicle. Even if the signal lever is restored to normal immediately the signal has been passed, the relay will continue locking the point lever until the track between signal 45va and points 30 b is free of vehicles. The purpose of these lock relays is thus to prevent points being operated while movements are going on between the incoming signal of the signal section and the points, i. e. until vehicles have entered one of the track circuits directly affecting the point lever by the direct method of point locking.

Fig. 7. Dwarf signals.

Point locking requires the use of quick releasing track relays in which the contacts will open at practically the same moment as the track circuit is shunted by a vehicle. But if a signal is put back from "proceed" to "stop", and a pair of points just behind the signal are thrown over immediately afterwards, there may still be the risk of derailment at the point, if a locomotive just in front of the signal has observed the proceed indication and started moving towards the points, and has afterwards failed to notice the stop indication, or, after noticing it, has not been able to stop at the signal. Point locking by means of the lock relays would also be out of action should the signal lever be restored before any vehicle has passed the signal.

To prevent this, some signal levers have been fitted with a delaying device which, if there are vehicles on the track circuit in front of the signal, will act so that the lever cannot be restored directly from reverse to normal but has to be kept in an intermediate position for a certain time after the signal has occupied the "stop" position. During this time the points remain locked. A delay does not occur if the track circuit in front of the signal is free. Loss of time on account of the delaying device will thus only result if a proceed indication after being given has to be cancelled while there are still vehicles on the track circuit in front of the signal. This device has only been considered necessary for some ten signal sections situated rather far away from the cabin, where movements are hard to survey from the cabin. The delay is obtained by special relays with a clock-work arrangement of the usual type. The signal lever is locked by a back contact of the corresponding time relay, this contact being opened when the relay, on reversal of the signal lever, is energized. The contact is shunted by a front contact on the track relay of the track circuit in front of the signal, the delaying device thereby being out of action when that track circuit is free.

For route locking and indirect point locking, D. C. relays of the L. M. E. standard type have been used, with 4 front and 4 back contacts, and a 2 000 ohm coil for 12 V. There are 37 of these relays, of which 16 are used for route locking for arriving and leaving trains and 21 are lock relays for point locking.

Signal aspects.

As incoming and outgoing signals for the signal sections within the station dwarf signal according to the safety regulations of the State Railway were to be used. Such a signal is illustrated in the regulations with the figure shown in fig. 8 and described as follows:—

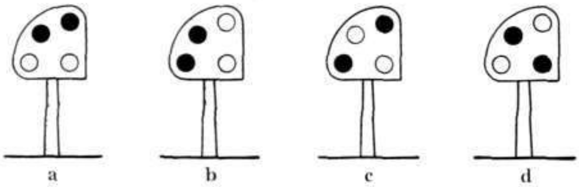


Fig. 8.

"Dwarf signal: a lamp device from which are shown 2 steady white lights either in a horizontal line (a), or in a vertical line (b), or in a line sloping at an angle of 45° to the left (c) or to the right (d), the lights to be visible in daylight also."

The signal aspects "b" and "c" in fig. 8 should be understood as an authorisation from the signal men in the cabin to carry out some intended movement. The shunters and drivers have to make use of this permission as soon as possible, thereby enabling the signal men to determine the order in which the shunting operations are to be performed. The aspect "c" means a restriction of the proceed indication so far as the signal may be shown even when the signal section ahead is occupied. This aspect therefore means "caution" and only makes sure that the points are in proper position and that signals for conflicting movements indicate "stop". The aspect "c" will always appear on reversal of the signal lever unless the following conditions for showing aspect "b" are already satisfied.

For aspect "b" to appear there is the additional condition that the whole signal section must be clear of vehicles. If there is an outgoing signal from the signal section that one must also show "proceed" or "caution (aspect "b" or "c"). If these conditions are fulfilled aspect "b" will appear automatically instead of "c" without any special action of the signal men. Aspect "b" is not used for signal sections leading on to make-up tracks where there are no track circuits, for example, when a movement is to be made on signal 57ha to tracks B or F. Nor will indication "b" appear for movements towards the short waiting track A.

Position light signals as in fig. 8 have also been used for controlling movements on the Loops, and for movements against the normal direction of traffic on the Up and Down Lines. These signals only show the aspects "a" and "b", i. e. "stop" and "proceed", the latter requiring the whole block section to be clear of vehicles.

Aspect "d" (neutral) has been used in this installation for controlling movements which are not supervised from the signal cabin. This is the case with all movements between tracks O, K, L, and M on the one hand and B and F on the other, i. e. shunting movements taking place outside that part of the lay-out which is used by incoming and outgoing trains. Therefore, signals 55h/t, 57hb/t, 57va/t, and 59va/t show the neutral aspect for movements not leading on to the running lines. When this aspect is to be shown, the lever normally used for the signal section must not be reversed, but a special switch, mounted on the frame above it, used instead.

The neutral aspect is automatically controlled by the position of the points which must then be diverging from the running lines.

The neutral indication is also used on dwarf signals O, K, and L, which are intended to prevent simultaneous movements towards points 24b and 20 from tracks O, K, and L, and on signals Y 1, Y 3, Y 5, and Y 7, the object of which is to prevent simultaneous movements through the cross overs 1—7 and 3—5 when the points are so set that a collision may occur between crossing movements.

Signals O, K, and L are selected automatically by the position of points 20, 24, and 18, but may be set to "stop" either by a lever in the signal cabin or by a lever at a place near points 1, 3, 5 and 7. These latter are always operated on the ground. Normally, points 20 are connected to the signal cabin, but they can also be operated on the ground by means of a ground lever near the points just mentioned (dual electric control). Points 18 and 24 are always operated from the signal cabin.

The aspects of the signals Y depend on the position of the points 1, 3, 5, and 7. "Stop" appears automatically when movements must not take place; otherwise the neutral aspect is shown. Y 1, for instance, indicates "stop" when points 3 or 5 are reversed, or when points 1 are at plus and 7 at minus or vice versa.

While thus position light signals are used for movements on the signal sections within the station limits and on the Loop lines as well as for movement against the normal direction of traffic on the double track, colour light signals are used for movements direct from the block sections to the platform tracks. In the safety regulations of the State Railway a colour light signal is illustrated by the figures shown in fig. 9 and described as follows:

Fig. 10. Incoming and outgoing signals for the Olskroken Lines.

A light signal with two different proceed aspects is used on the entry signal (1h) to the southern block section of the Almedal Line, to show if the route is cleared to Gothenburg C or to the goods yard. One green light is used for the former, two for the latter.

The incoming signals from the Down, B. J., and Almedal Lines are provided with distant signals, which are described in the regulations as follows:

"Light signal: a lamp device by means of which a green or white flashing light (in some cases also a red light, steady or flashing) is shown, all lights to be visible in daylight also."

The distant signals were mounted about 300 m. from the incoming signals. This short distance was considered convenient because the speed of the trains had to be reduced already before entering this terminus, in which the maximum speed is limited to 40 km. p. h.

Further, it was considered that the incoming signals ought to be cleared as short a time as possible before the arrivals of the trains in order not to cause unnecessary delay to other movements in the yard. If the distant signal were placed too far from the incoming signal, the signals would generally not be cleared until after the train had passed the distant signal. That being the case, the benefit of the distant signal would be illusory.

The block signal 1h on the Almedal line was

"*Light signal*: a lamp device which can show a red light, steady or flashing, or else one, two, or three steady green lights placed vertically above one another, all lights visible also in daylight."

Note 2. Such a light signal may be arranged to show also a flashing green or white light, then assuming the function of a distant signal.

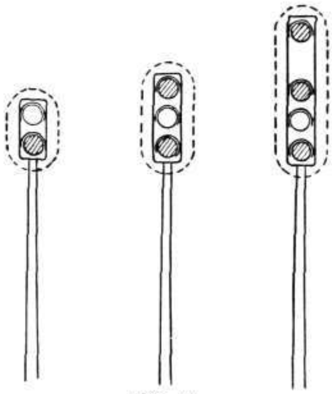


Fig. 9.

Note 3. The main signal allowing trains to enter a station is called the incoming signal, to leave a station the outgoing signal.

A steady green light is used in the installation as proceed indication on the incoming signals. The speed-limit for all movements through the station being 40 km. p. h., and all routes beeing practically equal in length, different signal aspects were not required for indicating the speed. Nor was it considered necessary to indicate at the incoming signal the platform track to which the train should proceed. While the train is passing through the yard a certain direction as to the route is, however, obtained by means of the dwarf signals of the various sections of which the route is made up.

Fig. 11. Incoming and outgoing signals for the Almedal Line.

Fig. 12. Dwarf signal at platform track.

the object of showing when a route is clear from that platform track to a block section. The signal consists of a green light placed immediately below the lower right-hand light of the dwarf signal. This green light will appear together with aspect "b" when a route is clear up to the entry signal of the Almedal, B. J., or Up Line. A flashing green light is shown when the entry signal is at "stop" and a steady green light when the signal shows "proceed". A train can leave the platform track even though a flashing green light is shown, but must then proceed with caution and be prepared to stop at the station boundary, should the entry signal to the block section still be at "stop" when the train reaches it. The route through the station is thus treated as a line block section, into which a train can be admitted before a previous train has arrived at Olskroken or passed the southern block section of the Almedal Line.

A signal of the type used for outgoing trains has in some cases been used at Gothenburg C for incoming trains also. It was necessary to be able to admit a train to platform tracks II or VI while a short train was still standing at the platform. Special dwarf signals, indicated in fig. 5 by II and VI, were therefore fitted about half-way down these platform tracks, the signals normally showing the neutral aspect. When the first train has arrived behind this signal the platform officer (the train clearer) will set the platform signal to "stop". This enables the signalman in the cabin to direct a new train to the platform track although this is already occupied. The clear signal will then not be given with the main incoming signal, but with a green light on the dwarf signal underneath the incoming signal, which shows a red light all the time to indicate that the movement is directed against a train already standing at the platform.

Signals of a special kind are those put up at the platforms, in order to announce to the plat-

provided with a distant signal at a distance of 700 m., full line speed being allowed here. This distant signal was also provided with a red light, which is shown when a train is moving in the direction of Almedal, or when there is a train on the track circuit between 1h and the distant signal.

It would of course have been possible to use light signals as in fig. 9 as outgoing signals for trains going direct from the platform tracks towards the lines. But as in this case the signals are given to a train standing at the platform, it was considered sufficient to provide the dwarf signals already existing at the outer ends of the platform tracks with a special signal aspect with

Fig. 13. Platform signal.

form officer and other platform staff that a proceed signal has been shown for a train to leave or enter the station. There is one of these platform signals at each platform track. The signals face the platform and each one has three green lights and one red, besides a white marker light.

When proceed is shown for an outgoing train to leave the station, one of the green lights is switched on. The left-hand light indicates departure towards the Almedal Line, the middle one towards the B. J. Line, the right-hand to the Up Line. The platform officer has to check the indication to see that it is for the right line, before he gives the driver the order to start.

The red light of a platform signal is switched on when an incoming signal is cleared for a train to enter the platform track in question. The red light means that a train is expected, and serves to call upon porters and other staff concerned to turn up on the platform to receive the train.

In each platform track the last 30 m. next to the buffer stops have been left without a track circuit, so that vehicles can be standing there when a train arrives. For these, and also for trains standing at the platform when another train enters the same track, the red light means "stop".

The signal cabin.

The principal connexions between the tracks of this yard being the set of points ending at the platform tracks, it was quite natural to centralize the operation of the signals and points to a single cabin. This ought to be placed at a suitable spot near the platforms used by the long distance trains, where a direct supervision of the shunting movements appeared to be most wanted.

At first it was intended to build the signal cabin in three storeys, of which the ground floor would be used for the power plant, the first floor for the relays, etc., and the top floor for the interlocking frame itself. It was, however, found necessary to prepare for a future extension of the platform beyond the place between tracks III and IV which had been reserved for the signal cabin. The ground floor therefore had to be dispensed with and replaced by columns to carry the building. The lower part of the building had to be reduced in width at the same time, to allow free passage along the future platform on both sides of the building. After this reduction a width of slightly more than 2 m. could be used for the first floor. It was not possible to make the building higher to compensate for the loss of the ground floor, the subsoil being very unsuitable for a high building. It was therefore decided to

Fig. 14. The signal cabin.

make only two floors, as it proved possible to find room for the necessary apparatus by careful disposition of the available space.

The top floor was made wider, projecting beyond the first floor as far as the 16 000 V trolley wires of the electric traction allowed. Thus a width of about 4.5 m. was obtained for the top floor.

The length of the building had to be chosen according to the floor space necessary on the first floor, and it was therefore made longer than was necessary for the interlocking frame, the length of which was only 6 m. The length of the cabin was thus made about 14 m., which allows for a considerable extension of the frame in the future.

Besides the interlocking frame and other apparatus requiring the direct attention and supervision of the signal men, all the switch-boards for the power supply, as well as the rectifiers and some of the transformers and resistances, were placed on the top floor.

On the first floor were placed a rotary converter and the larger transformers; also the end boxes and distribution panels for the underground cables coming into the cabin, and finally a relay shelf to carry the relays, etc. The disposition of the available space appears from the cross sections of the signal cabin shown in fig. 17.

The interlocking frame.

The motors of 57 points and 2 scotch blocks were to be controlled from the frame. The majority of the points belonged to cross-overs between parallel tracks, and could in most cases be combined in pairs on the same point lever, only 9 points requiring separate levers. In addition, one lever was required for locking the points in a cross-over between platform tracks VI and VII, the points of which were to be operated from the ground and only locked from the cabin on train movements on those platform tracks. The total number of levers required for points and scotch blocks was 35.

In addition, 55 dwarf signals, 10 main incoming signals and 11 outgoing signals, 76 signals in all, had to be operated from the frame.

The number of routes for shunting movements was 157, for train movements within the station limits 109, and for the lines 13, or a total of 279.

Owing to the large number of routes, an interlocking frame with the German type of locking register, in which not more than two route combinations can be connected to the same lever, was considered out of the question. This kind of interlocking frame would have needed at least 150 signal levers which would, with the 35 point levers and necessary spare levers, have necessitated a length of 16 to 18 m.

With the American type of interlocking register the number of signal levers is not fixed by the number of routes but by the number of signal units operated from the frame. The number of signal levers is thus not increased if, as is often the case, the same signal is used for a number of different routes. Each signal lever corresponds to a certain signal or group of signals. In this case the number of signal levers required was less than a third of what a German register would have needed.

The difference between the two types of register lies in the method of interlocking the signal levers and point levers. In a German register the locking pieces are always firmly fixed to the locking bars, which are only used with signal levers. Each locking piece will always affect a certain other lever in exactly the same way. The reversal of a signal lever therefore always presumes the same relative position of the points, i. e. the same route.

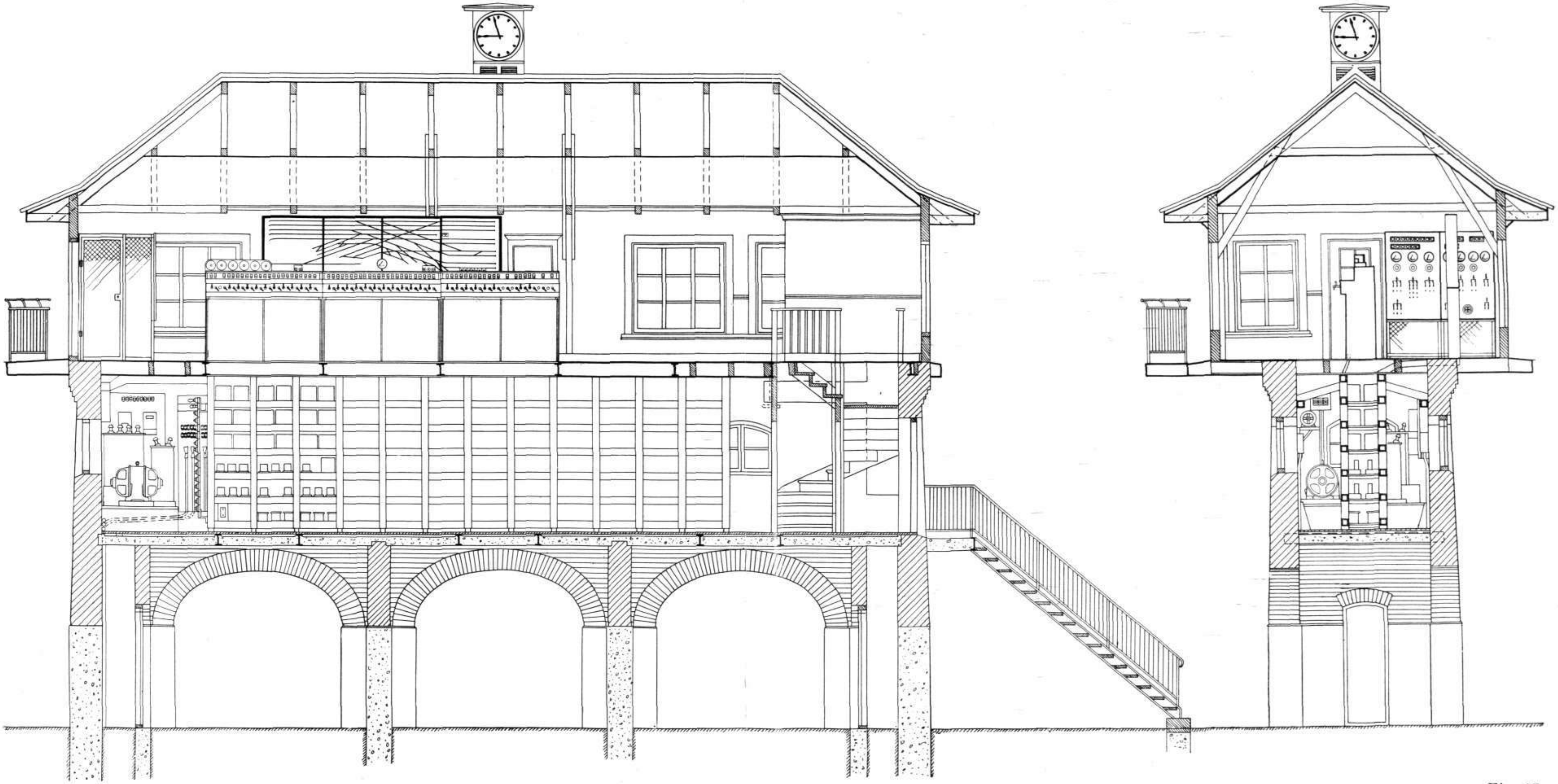


Fig. 17

Fig. 15. The top floor of the signal cabin.

In an American register also the point levers are provided with locking bars. The levers are interlocked by means of locking pieces which are movable relative to the locking bars. The same locking piece may be affected by several different levers, and also lock several different levers. When a signal lever is reversed, the point levers are locked in position for one of the routes to which the corresponding signal applies. Such locking of several routes by the same lever is rendered possible by what is called the conditional locking typical of this kind of register, which enables levers to be locked only if one or more other levers are in a certain position. According to the interlocking table, conditional locking is required for every combination showing lever numbers under the heading "with" or "unless".

American registers of two different designs had so far been used on the State Railway, viz. one with a plain mechanical register, as at Malmö (L. M. E. Review No. 1—2/26) and one with an electro-mechanical register, as at Hässleholm and Lund (L. M. E. Review No. 1—3/27, 10—12/30). The latter type of register was selected for Gothenburg C also. The final reasons for this were the considerably smaller first cost and the lower maintenance costs due to the elimination of the complicated mechanical register. Another advantage of importance is the greater ease with which alterations and extensions can be made with an electric register. Finally, such a register required considerably less space, as will be seen in comparing the cross sections shown in fig. 18 of the Gothenburg interlocking frame and various frames with mechanical registers of the size needed for this station.

The interlocking frame was provided with 72 levers, of which 30 were signal levers, 35 point levers, and 7 spare levers. The design of the frame is that described in the L. M. E. Review, No. 1—3/31. The point machines are of the type described in the L. M. E. Review, No. 10—12/1931, with motors for 120 V. D. C., controlled by contacts on the horizontal axles of the point levers.

The positions of the points are checked by Westinghouse Type G2 three position relays with local and indicating windings for 110 V, 50 cycles, all the relays being located in the cabin. Each relay has 6 normal and 6 reverse contacts, and there are 35 of them, i. e. one relay for each point lever.

The signals are operated by means of relays of L. M. E. standard type with 4 front and 4 back contacts, and a 2 000 ohm coil for 12 V. D. C.

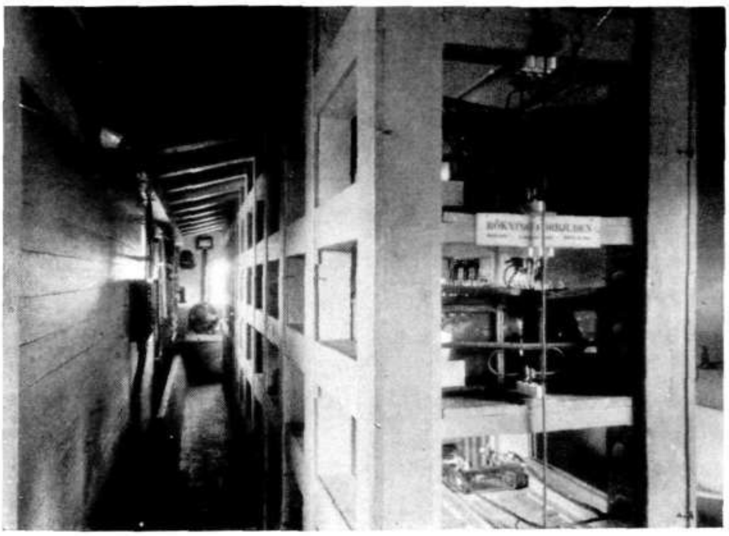


Fig. 16. The first floor of the signal cabin.

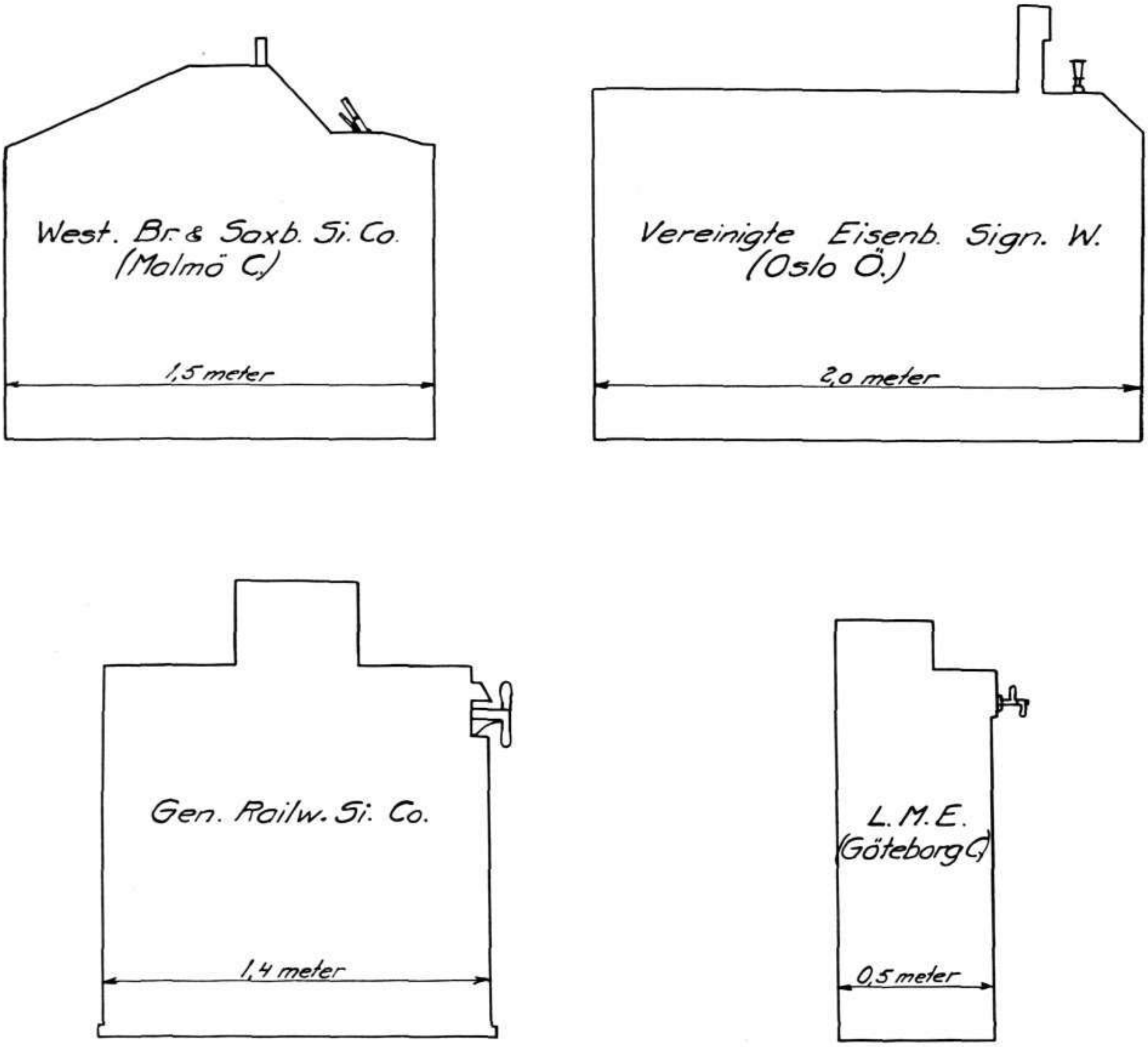


Fig. 18.

A total of 102 relays are used for controlling the dwarf signals, and 22 relays for colour light signals and platform signals. All these relays are fitted on the relay shelf on the first floor.

The illuminated track diagram.

The track diagram put up above the interlocking frame is of very great importance in the use of an installation of this type, by giving the signal men a concentrated picture of the lay-out with its signals and points.

For each track circuit there is a corresponding lamp in the diagram, this being "on" when there is any vehicle on the track circuit. Consequently when the track is empty, the lamp is "out", the result being that comparatively few track lights are seen at a time, thus making it easy to distinguish exactly those track sections that are occupied for the moment. The lamps are burnt a comparatively short time and in consequence interruptions by burnt out lamps are infrequent.

In the track diagram the indications of the signals are also repeated. The changing of the signal indications can therefore be seen simultaneously by all the staff in the signal cabin. "Caution" on a dwarf signal is repeated by a yellow light in the diagram, "proceed" by a white light. "Stop" on a dwarf signal is not repeated. The colour light signals are repeated in the plan by red light for "stop" and green for "proceed".

Other lamps in the track plan indicate when a train is ready to leave Almedal, Olskroken, or the goods yard. These lamps are placed in the plan at the outer ends of the block sections. For the Almedal Line there are two lamps, of which one is used as indication for trains to the passenger station and the other for trains to the goods station. The lamps are switched on from the signal cabins at these stations by special levers controlling annunciator relays in the Gothenburg signal cabin. There are eight of these relays, two of which are connected with Almedal, one with the goods yard and five with Olskroken.

By means of these annunciators warning is given to the Gothenburg signal cabin in due time when a train is coming. The routes in the passenger station need therefore not be cleared until it is necessary, thereby preventing movements in the station being delayed on account of the routes being set too early when trains are late.

The following lamps have finally been arranged in the track diagram at the inner end of each of the platform tracks:

one red lamp, which is automatically switched on when an incoming signal is cleared for a train from any of the three main lines to the platform track;

one lamp showing a white light when a push-button on the platform close to the track is pressed to inform the signalmen that a train is ready to start from the platform track, or that an empty train is ready to be moved from the platform track to the making-up yard; the push-button controls a time relay equipped with a contact which keeps the lamp alight for a certain time after the current to the relay is broken;

three lamps with green lights, one of which is switched on when "proceed" is displayed for a train to leave the platform track for the Almedal, B. J. and Up Lines respectively.

A small, illuminated track diagram, on which the various incoming and outgoing lines with their signals are shown, was also put up in the train office in the station building to indicate to the staff there the arrival of trains. For each incoming line there is a lamp, which is switched on when a train enters the block section. A green lamp also indicates when the incoming signal is displaying "proceed".

The power supply.

The following power consumption was estimated to be required for the installation.

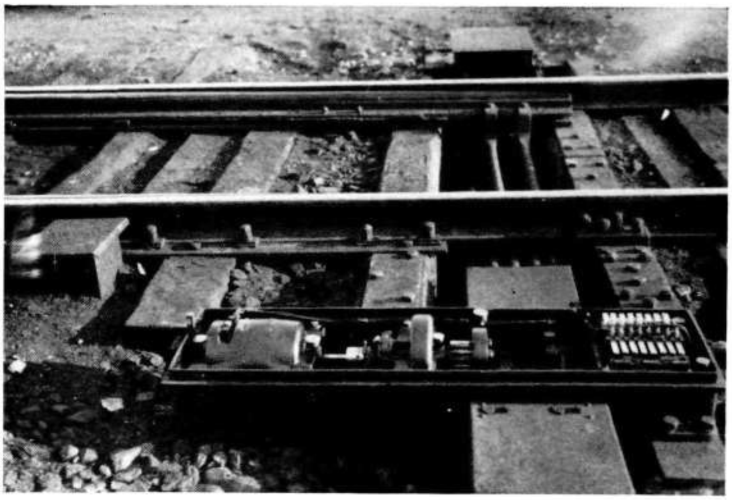


Fig. 19. Point machine.

Alternating current of 50-cycle frequency:

For lighting of the signals and the lamps in the track diagram	4.2 kw.	cos φ 1.0
For track circuits, which are provided with condensive limiting resistances..	4.8 »	» φ 0.9
For point indication relays (SS relays)	1.0 »	» φ 0.6
Total	10.0 kw.	

The above consumption is practically constant all the day round and thus corresponds to an annual consumption of about 87 000 kwh.

Direct current, 120 V:

The point motors were estimated to take a maximum of 3.6 kw. which corresponds to ten simultaneous point operations, each requiring a current of 3 amps. The total consumption depends on the actual number of point operations. With an average of a hundred operations per point motor and day, a power consumption of 360 watt per motor, and an operating time of 3 secs., about 700 kwh. per annum would be consumed.

If the A. C. supply is interrupted D. C. is used for driving a rotary converter. To obtain 10 kw A. C. of 50-cycles, about 12 kw of D. C. is required, this however being taken out only for short periods. Assuming 15 interruptions in the A. C. supply of 8 hours duration each time, a reserve D. C. supply of 1 500 kwh *per annum* would be required.

Direct current, 14 V:

About 70 watts was estimated to be required for lock magnets, control relays, telephones, etc. and had to be taken from metal rectifiers connected to the 50-cycle A. C. supply. This means an approximately constant A. C. consumption of 140 watt, i. e. about 1 200 kwh per annum.

Since the completion of the installation the power consumption for 1931 has been measured —in round figures 84 000 kwh of A. C. 50-cycles and 1 800 kwh of D. C. The power factor of the A. C. consumption has proved to be practically unity. According to the measurements the point motors are taking about 2 kwh a day, making about 800 kwh per annum, 1 000 kwh per annum thus being the power used for generating alternating current during interruptions in the normal supply.

Electric power for the installation was available from two sources, one 3-phase 50-cycle A. C. of 220 V., obtained by stepping down the high tension voltage supplied in the station from the power works of the state, and the other 2 × 120 V. D. C. from the municipal power supply. Simultaneous interruption in both sources was considered unlikely. By making provision for taking all the power required from either of these two sources, it was considered unnecessary to provide reserve power from a storage battery or a gasoline motor driven generator, a considerable saving in installation and maintenance costs thereby being made possible. In this instance the resulting reduction in floor space was also of great importance.

Considering the price of power—5 öre per kwh for A. C. and 15 öre per kwh for D. C.—the main part of the power required ought to be taken from the A. C. supply. Consequently all the power that could be used in the form of 50-cycle A. C., i. e. for signal lighting, track circuits, point indication relays, and rectifiers was taken direct from the A. C. mains.

In order to obtain A. C. from the local D. C. supply in the event of an interruption in the A. C. supply, a converter was installed in the signal cabin for converting D. C. into A. C. This converter can give 15 kwa. Should an interruption occur, both the starting of the converter and the switching over from one supply to the other have to be done by hand. An unexpected interruption must therefore cause an interruption of the work for one or two minutes, this however being of minor importance as it happens very seldom.

The point machines being designed for 120 V. D. C., the current for them could be taken direct from the local supply. To allow the power for the motors to be taken also from the A. C. supply, a metal rectifier, connected to 3 × 220 V., 50 cycles, and giving off 30 amps. D. C. at 120 V. was installed. Owing to the no-load losses in the rectifier, it proved advantageous normally to use the local D. C. supply in spite of its higher price.

The switching over from the normal supply to this rectifier is done by hand. To prevent the voltage coming back after an unexpected interruption at an unsuitable moment, for instance when a point lever is in such a position that the point motor can be affected, a "no voltage" circuit breaker is connected in the D. C. feeds, in order to break the supplies to the motors, if the voltage fails for a moment or drops to too low a value. The circuit breaker must then be restored by hand, which must not be done before taking the necessary precautions.

The 12 volts tension required for the locking magnets in the frame and for the D. C. relays is taken from a metal rectifier connected to the A. C. mains and giving a maximum of 10 amps. If this rectifier is out of order power can be taken from a rotary converter connected to 120 V. D. C. and giving 5 amps. D. C. at 14 V.

This rectifier and converter are only used for circuits inside the signal cabin, a special rectifier giving 2 amps., 12 V., being installed to feed circuits extending outside the signal cabin.

In fig. 20 the circuit diagram of the power plant is shown.

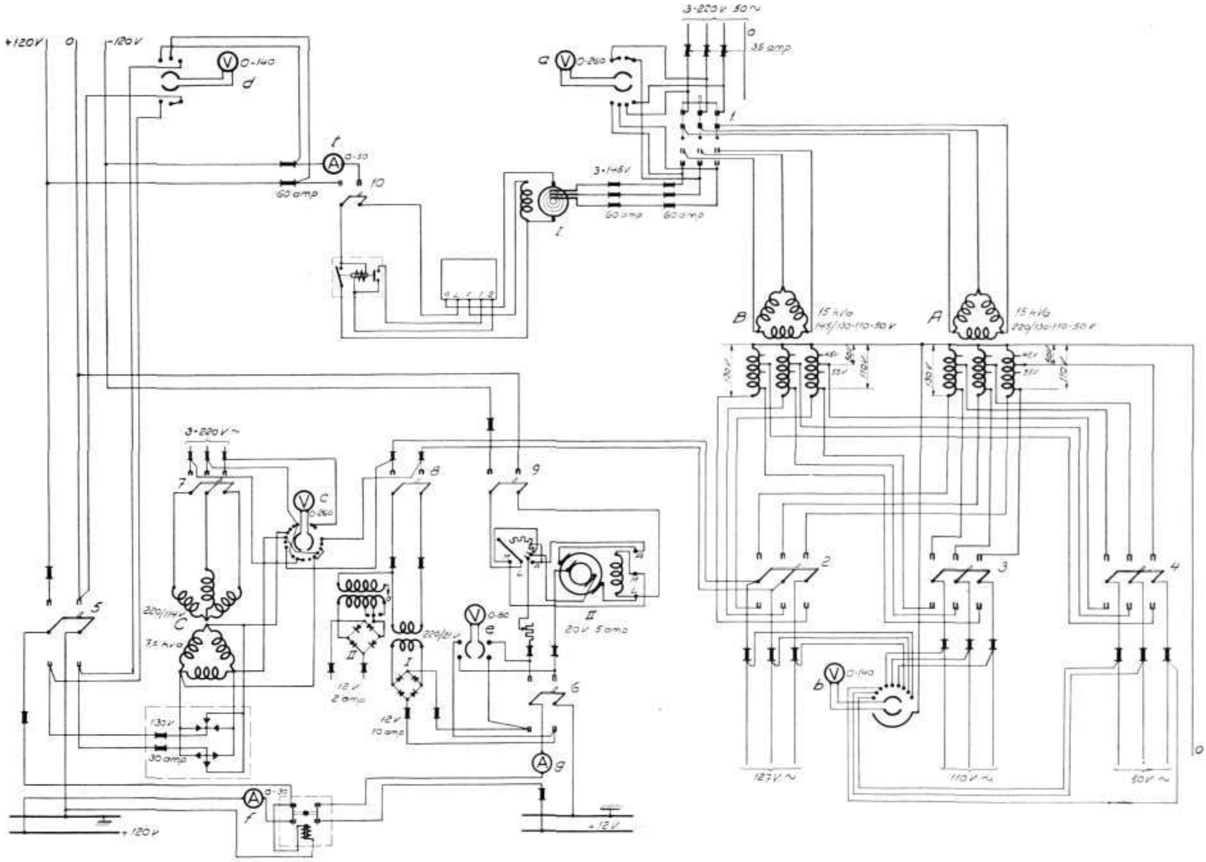


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Fig. 20.

Telephone equipment.

An important part of the installation is the telephones by which orders and information from the stationmaster and other officials are received in the signal cabin, and by which the signal men can communicate with the employees working on the ground.

A special telephone system using loud-speaking telephones with fixed microphone and telephone was installed for communications required between the signal men and the shunters when changes are to be made in the normal course of the work. A telephone is mounted in the signal cabin and connected to six telephone posts in the yard. A call can be made from any of these telephone posts to the signal cabin as well as from the cabin to any of the posts. In the latter case the call is made by means of a bell mounted at the telephone box, which summons the shunters in the yard to the telephone. The installation is energized from an accumulator charged through a metal rectifier, the same power source being used also for the bells.

A special telephone system is also arranged between the signal cabin and all the main incoming signals controlled from it. The telephones in this system are of the usual type with micro-telephone. Those fitted at the signals are mounted in water-proof cast-iron boxes (so-called mine telephones), and are all connected to a common telephone in the signal cabin. This telephone system is used for communicating with the driver of the locomotive when a train has been stopped at an incoming signal. The signal men are authorized by telephone to order the train to pass the main signal at "stop" and to proceed on dwarf signals alone, which may be necessary in case of a failure in the installation or when the whole route up to the platform track for some reason cannot be cleared at the same time.

The locations of all these telephone posts are shown in fig. 5.

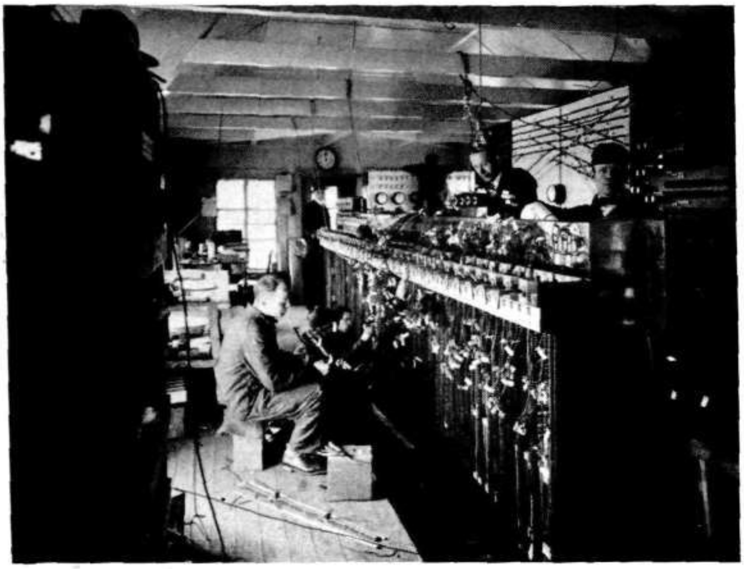


Fig. 21. Fitting of the interlocking frame, front view.

A special telephone with a line selector is also installed in the signal cabin, by which communication can be obtained with the train office in the station building, the train dispatchers office, the telephone posts on the platforms, telephones in the signal cabins at Olskroken, Almedal, and the goods yard, and finally with the local telephone system on the lines beyond Almedal and Olskroken. Incoming calls on these telephones are given by means of bells with a trembler indicator, showing the line from which the call is coming, whereupon the calling line can be connected through the selector to the cabin telephone.

Finally, an instrument connected to the automatic telephone system of the station is also installed in the cabin. This provides communication with all the offices of the Gothenburg yards, the train order telephone systems of the different railways as well as the public telephone system.

The construction and placing in service of the plant.

All designing work and purchase of material were carried out at the office of the Signal Department of the State Railway at Stockholm. The installation work was under the general control of the engineer in charge of the whole rebuilding of the railway station and was supervised by an engineer appointed from the staff of the Signal Department. The inspection of the wiring and other details and the technical arrangements in connexion with the placing of the plant in service were made by this engineer, who also assisted in instructing the traffic and locomotive staffs in the use and meaning of the signals.

The installation work began on Oct. 6th, 1929. The track lay-out was then far from completed, which made the work considerably more difficult. The signal cabin, on the other hand, was at that time practically finished. The greater part of the material for the plant was furnished on the spot, the remainder being on order for successive deliveries at the dates when, according to the working scheme, they would be required.

According to this scheme the outside work— i. e. cable work and works on points and tracks —was to be completed first, so as to be ready if possible before the beginning of winter. Work inside the signal cabin, being independent of the weather, was to be done last, which was also most convenient on account of the interlocking frame and the track diagram not being expected to be ready for delivery until the end of the year.

The installation was favoured by a comparatively mild winter, and the plant was ready for use on the appointed date, May 2nd, 1930. At that date only the traffic previously dealt with by the Gothenburg S. J. entered the station. It was not until May 15th that the traffic was taken over from the Gothenburg B. J. station, which then ceased to be a passenger station. At the same time a new time-table, to meet the altered conditions, came into force. This meant that the traffic in the new passenger station was suddenly increased by 150 per cent.

After the station was taken into use, it was found that the sidings for storing and marshalling of empty trains was not quite sufficient for the demands of the traffic, and some new tracks and points had therefore to be laid out immediately. Track M and track group O (see fig. 5), as well as the scissors cross-over at points 1-3-5-7, and also points 4a and b, and 24, were thus added when the installation was already in service. This naturally involved some alterations and extensions of the signal installation also. The frame with electric register chosen for this plant now came in very well as the necessary alterations of the frame could easily be done without interfering with the traffic.

Fig. 22. Fitting of the interlocking frame and illuminated track plan, back view.

Including these alterations and other adjustments and final touches, which were not completed until June 1930, the whole installation had required the following labour:—

1 engineer from the Signal Department, in all	200 working days
2 fitters from the L. M. E. Signal A.-B., in all	300 » »
3 signal maintainers from the State Railway, in all	780 » »
8 skilled workmen (blacksmiths etc.)	1980 » »
helpers in varying numbers per day, max. 40, in all	3240 » »
Total	6500 working days.

In the installation work electrically operated tools were largely used for drilling holes in point tongues and rails for the fitting of point machines and insulating joints. Rails and tie plates were cut with acetylene burners. Rail bonds and leads were electrically welded to the rails within the station boundaries, while on the line, where electric power was not easily available for welding, they were soldered with the aid of acetylene flames.

Fig. 23. Welding of rail bond.

The rolling out of the cables was as far as possible done from trucks moved by shunt engines. The wiring and cable work, both in the signal cabin and on the ground, was as far as possible done by piece-work, i. e. the work was divided among gangs, each doing one particular detail of the work. Piece-work rates could thus be largely applied, and the training of labourers for the various tasks was facilitated.

The cost of the plant appears from the table below, which gives the actual expenditure in round figures, and includes all costs except designing and other work carried out at the Signal Department's office at Stockholm:—

Signal cabin	Kr.	42 000
Interlocking frame, track diagram, and relays	»	117 000
Point machines, with connecting rods and plates	»	75 000
Signals, with transformers, foundations, masts and suspending devices	»	27 000
Cables, with boxes, compound, and cover	»	60 500
Power plant	»	11 500
Telephones, with accessories	»	6 500
Push-buttons and relays for controlling starting indicators from the platforms	»	2 000
	Kr.	341 500

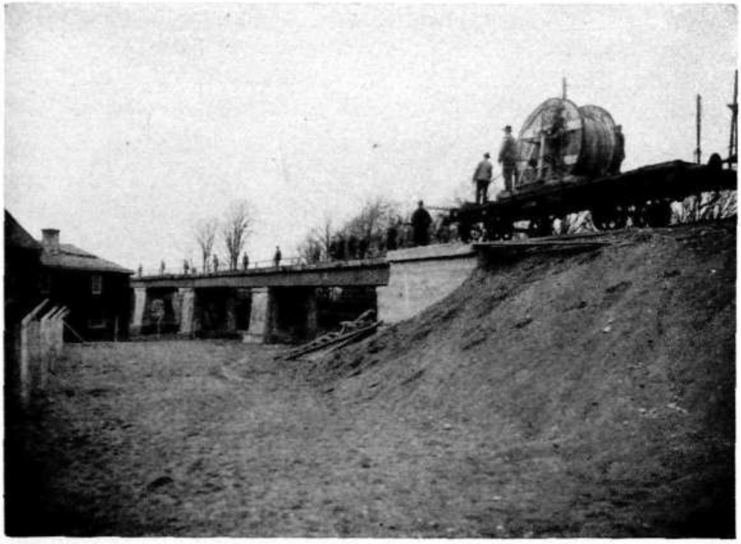


Fig. 24. Cable laying.

	Kr.	341 500
Track circuit materials, including rail bonds, leads, insulating joints, track transformers, condensers, relay transformers, connecting boxes, housings for transformers, wood conduits for leads between the rails, rail bolts, spring washers, gas, electrodes and other materials for welding and soldering bonds and leads	»	25 000
Duplicate apparatus for reserve	»	2 500
Labour	»	93 000
Total	Kr.	462 000

Operating and maintenance.

Between 1 a. m. and 6 a. m. only one man is on duty in the signal cabin for operating the interlocking frame, and at other times of the day two men. Between 7 a. m. and 10 p. m. a foreman is also stationed in the signal cabin for the special purpose of supervising and directing the work, receiving incoming orders and information regarding traffic movements from the stationmaster or from adjoining stations, and forwarding information to the shunters and train men. The total number of working hours in the signal cabin is thus 58 per day, of which 15 hours for the foreman and 43 hours for the signalmen. With a working day of 7 hours and a half for each man, this necessitates a signal cabin staff of 7 to 8 men.

For the maintenance of the installation one chief maintainer, 2 maintainers and 2 signal helpers are employed. The same men also are charged with the maintenance of other interlocking plants within a certain area round Gothenburg. The chief maintainer also does work on other plants within the second district of the State Railway, having its headquarters at Gothenburg. The supervision of the maintenance rests on the District Signal Engineer.

To prevent interruptions of the traffic should a failure occur, the service is arranged so that always at least one signal maintainer or helper is on duty near the signal cabin to be rapidly called for when needed. Attendance has so far also been arranged at night on account of the important trains then arriving or departing. Attendance is also arranged on Sundays, the number of trains being then practically the same as on weekdays. A workshop for overhauling old signal material having been located near the cabin, the men on duty can be employed in productive work even though no maintenance work would be needed on the installation itself.

The technical supervision includes on the one hand a general inspection of the plant to be done daily, on the other hand tests to be made from

Fig. 25. Testing a cable distribution box.

time to time to make sure that all important safety devices are functioning properly. According to the general rules, a more complete inspection is to be made at least 4 times a year, at regular intervals.

The following tests are usually made. The points are tested by trying to lay them over when an object 5 mm thick is applied between the stock rail and the tongue. The point motors are also checked to see that their field windings are properly short-circuited by contacts on the respective point levers, when these are in their extreme positions. Similarly, the shunting of the indicating windings of the point indication relays, when the points are in an intermediate position, is duly checked. Tests are also made to see that there is no earthing of circuits which ought to be insulated from earth. The shunt values of the track circuits are measured, and the tracks examined to see that all parts are properly connected to the track relay. In the regular inspections, special attention is finally given to the interlocking frame itself, this being gone through to inspect the proper operation of locking devices, magnets, and contacts.

In spite of careful inspection, it may be unavoidable in an installation of this size that failures sometimes occur on account of defects in one or other of the many details of which the plant is composed, this being the case particularly during the early days of its use before all the parts—many of which could not be finally tested until the plant was placed in service—are properly adjusted.

To obtain a full knowledge of failures occuring, a record has since the installation was placed in service been kept of all defects, including even such as have been of little or no importance to the work and have been immediately put right. From this record the Signal Engineer has month by month made up failure statistics, from which a survey of the number and causes of the failures can be obtained. By studying the individual causes it has been possible gradually to increase the reliability of the installation and to reduce the number of failures either by improving certain details or by directing the attention of the maintainers to devices requiring special care.

The point machines were of a new type, which had not been tested under such trying conditions. It was found necessary to exchange the contact blocks of the circuit controllers of the machines, as the contacts were not firmly enough fixed in the insulating material. Further, the design of the breaking device of the point motor was altered, and the friction clutches were gradually adjusted to increase the pulling power. In the locking frame the fixing of certain movable parts of the lock magnets was amended, as they had shown a tendency to loosen after frequent operations. A thorough adjustment was also made of the attachment of the wires to the binding posts of the contact springs of the frame, after loose contact had been found in some cases on account of shrinking of the insulating material of the contact blocks. By these measures about 50 per cent, of the failures occuring during the first period of the plant's being in service could be prevented for the future.

One category of disturbances which it is not possible completely to eliminate are those caused by purely external obstacles to the operation of the points, such as snow or ice in the points, gravel or stone from the ballast occasionally getting in between the tongues and stick rails, or displacements in the track which may prevent the points from being completely laid over. In 1931 the total number of such cases was 18, which, however, is a very small fraction of the one or two million annual point operations.

Failures have been reported in the locking devices of locally operated points in 4 cases, due to the formation of ice on the contact surfaces. The same kind of failure has not occured in the point motor, as these are provided with a higher contact pressure.

Only in 4 cases during 1931 has it happened, owing to contact failures in points and relays, that "proceed" indication has not appeared when it might.

A strikingly small number of failures—in all 8 during 1931—have been traced to the track circuits, and were caused by wear of the fibre packings in insulating rail joints (6 cases), metallic contact between the rods in a pair of points (1 case), and defective contact in a terminal screw of a cable distributing box (1 case). This small number proves the high degree of reliability obtainable by means of track circuits even in a comparatively complicated track lay-out with rather heavy traffic.

Disturbances by small failures of various kinds in the interlocking frame from other causes than those mentioned above, which could be eliminated in future by certain measures, occurred in 7 cases during the year, in addition to which there were 6 cases caused by too high resistance in the contacts of various relays in the signal cabin.

15 cases of interruption of the A. C. supply, when the installation had to be switched over to the converter connected to the D. C. supply, have also been resorted as failures.

Finally, there is a special category of failures caused by burnt out signal lamps. Periodical changes of lamps after a certain period of use have not been arranged, but the changing of lamps has been made as they have become useless, through either the filament breaking or the light being dimmed by a dark deposit in the bulbs. Records have been kept of replacements made at each light, in order to find out the durability of the lamps, and also to discover any abnormal consumption of lamps possibly caused by too high voltage or by defective lamps. The total number of lamp replacements was only two in colour light signals, where 12 V., 24 W, lamps are used, and 20 in the dwarf signals, where 55 V., 20 W, lamps are in use. The number of lamp replacements in the track diagram was 25 during the year.

Economic results.

A direct estimate of the saving in operating costs by this plant is difficult, as it was installed in connexion with an extension of the station by more than 100 per cent. as far as the number of platform tracks, points, and incoming or outgoing lines are concerned, and was from the very first organized for taking over a traffic exceeding in volume the previous one by about 150 per cent. There is therefore no material available for a direct comparison showing what savings of staff have been possible by centralizing the signalling of the station and the adjoining lines into one single cabin at Gothenburg C.

Some assistance in estimating this saving may be obtained by a comparison with Malmö, where an installation of the same type and approximately the same size and with similar operating conditions, was placed in service in the year 1925. The new installation at Malmö, which was the first of its kind on the State Railway, was built without extensions being made of the station or alterations as to the amount of traffic. A direct comparison was therefore possible between the number of staff required before and after the installation was placed in service. The former installation at Malmö was a mechanical one, divided between two signal cabins in the station and 2 manual block posts on the incoming lines, one having a junction for goods trains at about the same distance from the main station as the outlying points on the Almedal Line. Experiences from Malmö indicate that the staff could be reduced by 22 men on account of centralization of the signalling.

By courtesy of the Station Master an attempt has been made to estimate what increase of staff would have been required if a mechanical system instead of an electric one had been installed at Gothenburg C. The result indicates that, beyond the present staff, the following additional personel would have been required:

for the outlying points on the Almedal Line	3 men
for operation of route releasing instruments in the station master's office	2 »
for the main cabin	2 »
for an additional signal cabin	5 »
locomotive pilots	2 »
signalmen on the ground	2 »
Total increase of staff for Gothenburg C	16 men

In addition more men would have been required at neighbouring stations also, Olskroken in particular, where the work would increase considerably if the line-blocking of the 5 lines to Gothenburg C had to be operated manually without the aid of track circuits and automatic signals. If this increase of staff is estimated at 2 men, the estimated total saving in personel compared with a mechanical plant would be 18 men.

To use this station without interlocking and with all the points operated on the ground must be considered as an alternative beyond practical possibility. Even with a further heavy increase of the number of staff it would meet with insuperable difficulties in such a case to be able to handle the trains in the time available.

The initial cost of a mechanical plant would probably have been less than the cost of the plant now in service, but the interlocking of the 60 points and about 30 signals would certainly have cost at least 300 000 kr., even if a mechanical system had been used. In view of the technical limitations of a mechanical system, however, the use of such a system would necessarily have meant a considerable reduction of the number of available routes for the trains, as well as of the efficiency of the installation from the safety point of view.

A plant with electrically operated points and signals but otherwise made on the same principles as a mechanical plant, without track circuits and shunting signals, i. e. on the system used in several installations of an earlier date on the State Railway, might have been installed for approximately the cost estimated above for a plain mechanical installation. Owing to the greater ease of operating an electric interlocking frame, the signal cabin staff might then be reduced by about 4 men, so that the increase of staff over the present might be estimated at 14 men, instead of 18 with a plain mechanical system.

It should be observed, however, that it is an open question whether the present number of platform tracks and track facilities for shunting empty trains to and from the making-up yard would suffice, even with the additional personel estimated above, should a signalling system without track circuits and shunt signals be used, as the operation of points for shunting and train movements could not be performed as quickly as it is now.

It is mentioned in the introduction to this paper that the choice of the signalling system from the first affected the design of the station in making it possible to reduce the number of platform tracks, a reduction which would not have been advisable with a signal installation of less efficiency. An estimate of cost carried out by direction of the Railway Management indicated that an extension of the number of platform tracks from the present 10—including an eleventh track for outgoing trains—to 12 complete platform tracks would mean a capital outlay of 525 000 kr., i.e. an amount considerably exceeding the total cost of the present signal installation. In a letter from the Railway Management to the Government of Sept. 27th, 1930, on the subject of granting money for new improvements of various kinds on the railway, a report was given on the effect of the newly finished rebuilding of the Gothenburg station. The Railway Management stated in this report that the traffic taken over by the new station had proved to be considerably greater than expected when the station was designed some years ago. It might therefore become necessary to extend the station by two additional platform tracks in order to safeguard the regular handling of the traffic. The Management was however not convinced that an extension would prove absolutely essential, and wanted therefore to postpone the question of granting money for this purpose—to be able to make further observations before a final decision could be made.

Since then two years have passed, during which the number of trains, as indicated by the the diagrams given in fig. 4, has been further increased without causing any difficulties for the proper handling of the traffic. The Railway Management has consequently in subsequent propositions definitely abandoned the project of extending the platform tracks.

Without exaggeration it might thus be said that the additional sum of 160 000 kr.—being the difference between the cost of the present plant and a simplified one—spent on signalling facilities at Gothenburg C, has resulted in not only a saving in personel of no less than 14 or 18 men, but also in putting off a direct capital outlay of about 525 000 kr., as not needed in the reasonably near future.

The collection of the information given in this paper has been greatly facilitated by the courtesy of Teodor Lundberg and John Olofsson, of whom the former has been in charge of the installation work, and the latter of the designing work. The author has further taken the liberty of using the occupation diagrams made out by the Yard Construction Department of the State Railway, as well as the information supplied by the District Signal Engineer, regarding the organization of the maintenance, and is finally indebted to the Chief Station Master at Gothenburg, E. Ericsson, for the valuable information regarding the operation of the plant.

Reduction to a Constant of a Variable E.M.F. with the Assistance of the Heaviside Operator Calculus.

Communication from the Research and Development Department.

By *H. Pleijel.*

In the case of a system of conductors in which an E. M. F. $E(t)$ is working, a solution in operator form is obtained by means of the equations given by Kirchhoff's laws. We introduce $\frac{\delta}{\delta t}=p$, and regard p as an algebraic quantity.

On solving the system of equations thus obtained, we get y, the unknown quantity to be determined, in the form

$$y=\frac{g(p)}{h(p)}\cdot E(t)$$

where $g(p)$ and $h(p)$ are whole functions of p. If we are not dealing with conductors with distributed capacity and inductance, $g(p)$ and $h(p)$ will be polynomials in p.

If $E(t)$ is constant we can obtain the real solution by means of Heaviside's expansion theorem. But if $E(t)$ is of some other form, that theorem cannot be directly applied. Such cases frequently occur. In technical problems, sinusoidal E. M. F:s are nowadays actually more frequent than constant ones.

We can, however, always obtain the solution by integrating. If we represent the solution of

$$\frac{g(p)}{h(p)}\cdot 1$$

by $f(t)$, the solution, it will be remembered, will be obtained from the formula

$$y=E(t)f(o)+\int_o^t E(t-u)f'(u)\,du \quad\ldots\ldots\ldots\ldots (1)$$

$E(t)$ may here have an arbitrary number of discontinuities.

Another method would be to start directly by expanding $\frac{g(p)}{h(p)}$ into a series of partial fractions.

If p_m is a root of the denominator $h(p)=o$, $\frac{g(p)}{h(p)}$ may be expanded in partial fractions, in the form

$$\frac{g(p)}{h(p)}=A_o+A_1\frac{p}{p-p_1}+A_2\frac{p}{p-p_2}+\ldots.$$

(We assume for the time being that $p=o$ is not a root of the denominator, and that all the roots are simple.)

By making $p=o$ we get

$$A_o=\frac{g(o)}{h(o)}$$

Multiplying by $p-p_m$ and making $p=p_m$ we get

$$A_m=\frac{1}{p_m}\cdot\frac{g(p_m)}{h'(p_m)}$$

Our solution will thus be

$$y=\frac{g(o)}{h(o)}\cdot E(t)+\sum\frac{1}{p_m}\frac{g(p_m)}{h'(p_m)}\cdot\frac{p}{p-p_m}\cdot E(t)$$

But

$$\frac{p}{p-p_m}E(t)=e^{p_m t}\frac{p+p_m}{p}e^{-p_m t}E(t)=$$

$$=e^{p_m t}\left[1+\frac{p_m}{p}\right]E(t)\,e^{-p_m t}=$$

$$=E(t)+p_m\int_o^t e^{p_m(t-u)}E(u)\,du$$

For a variable E. M. F. the expansion theorem can thus be written

$$\frac{g(p)}{h(p)}E(t)=\frac{g(o)}{h(o)}E(t)+\sum\frac{1}{p_m}\cdot\frac{g(p_m)}{h'(p_m)}\left[E(t)+\right.$$

$$\left.+p_m\int_o^t e^{p_m(t-u)}E(u)\,du\right]^* \quad\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

* This method and formula were given by the author in a paper published in Teknisk Tidskrift, Elektroteknik, No. 4—5, Stockholm 1914.

If $p=o$ is a root of $h\ (p)=o$ we get instead the following expansion in partial fractions:

$$\frac{g\,(p)}{h\,(p)}=\frac{g\,(o)}{h'\,(o)}\cdot\frac{1}{p}+\left[\frac{g'\,(o)}{h'\,(o)}-\frac{1}{2}\,g\,(o)\frac{h''\,(o)}{[h'\,(o)]^2}+\right.$$

$$+\sum\frac{1}{p_m}\frac{g\,(p_m)}{h'\,(p_m)}\cdot\frac{p}{p-p_m}$$

In this case the constant term A_o will change, and we get a new term

$$\frac{g\,(o)}{h'\,(o)}\int_o^t E\,(u)\,du$$

Although the formulæ given above can be used when the E. M. F. is of arbitrary form, when it is of a special kind we may expect to obtain formulæ which are more easily dealt with and discussed.

Such a case will occur when the E. M. F. is periodic and a period is composed of segments identical in form with segments of curves produceable analytically according to the formula

$$\frac{\varphi\,(p)}{\Psi\,(p)}\cdot E_o,$$

where E_o is a constant and $\varphi\ (p)$ and $\Psi\ (p)$ are integral functions of p.

We will see later what expressions are obtained for φ and Ψ in different cases.

Our operator solution

$$y=\frac{g\,(p)}{h\,(p)}\cdot E$$

can then be written instead

$$y=\frac{g\,(p)}{h\,(p)}\cdot\frac{\varphi\,(p)}{\Psi\,(p)}\cdot E_o,$$

and our problem will thus be brought directly back to Heaviside's expansion theorem with a constant E. M. F.

If, on the other hand, Heaviside's expansion theorem is applied to the expression

$$E=\frac{\varphi\,(p)}{\Psi\,(p)}\cdot E_o.$$

we obtain E expanded in a Fourier's series.

We have further assumed the E. M. F. to be periodic. A half period may then consist of several arcs joined together. We assume these to be of forms $f_1\ (t)$, $f_2\ (t)$ and $f_3\ (t)$ respectively. For $t=o$, these functions give the ordinates at which the three curve segments begin. The lengths (or more correctly times) representing the durations of the various curve segments we will call τ_1, τ_2, and τ_3. Our curve is thus composed of three curves. The first is zero when $t=o$, then $f_1\ (t)$ when $t=\tau_1$, and after that zero; the second must be zero when $t=\tau_1$, then $f_2\ (t)$ during the time from $t=\tau_1$ to $t=\tau_1+\tau_2$, and after that zero. In the same way the third will be zero for all values of t except those lying between $t=\tau_1+\tau_2$ and $t=\tau_1+\tau_2+\tau_3$, when it must have the same value as $f_3\ (t)$. We will first take the middle section and find an operator expression for it.

We have

$$e^{-p\tau}\,f(t)=f(t-\tau)$$

which is found directly by expanding the exponential expression in a Taylor's series. $f\ (t)$ is assumed to be zero for all negative values of t. The curve $f\ (t-\tau)$ will then be zero until the time $t=\tau$, after which it will pass through the same values as $f\ (t)$ from $t=o$. The operator $e^{-p\tau}$ consequently shifts the curve $f\ (t)$ a distance τ towards greater values of t.

If instead of $f\ (t)$ we take the curve $f\ (t+\tau)$ and assume as before that this is zero for negative values of t, we see that from $t=o$ we obtain that part of the curve $f\ (t)$ beginning at $t=\tau$. If this curve is shifted a distance τ towards greater values of t, we get the same values as $f\ (t)$ when t is greater than τ and zero when t is less than τ. If we subtract this curve from $f\ (t)$ we get finally a curve which is zero until $t=o$, then $f\ (t)$ until $t=\tau$, and then again zero. This section of curve is therefore represented by the formula

$$f(t)-e^{-pt}f(t+\tau).$$

The second portion of our complex curve is of this form. We have only to replace $f\ (t)$ by $f_2\ (t)$ and τ by τ_2. But this section must be shifted so as to begin at time $t=\tau_1$.

We can therefore write this curve segment in the form

$$e^{-p\tau_2}\,[f_2\,(t)-e^{-p\,\tau_2}\,f_2\,(t+\tau_2)]$$

Arguing in this way we see that the half period as a whole will be represented by the expression

$$f_1(t)-e^{-p\tau_1}f_1\,(t+\tau_1)+e^{-p\tau_1}[f_2\,(t)-e^{-p\tau_2}\,f_2\,(t+\tau_2)]$$

$$+\,e^{-p(\tau_1+\tau_2)}\,[f_3\,(t)-e^{-p\tau_3}\,f_3\,(t+\tau_3)]$$

We now assume that we can write

$$f_1\,(t)=\frac{\varphi_1\,(p)}{\Psi_1\,(p)}\cdot E_o$$

and introduce

$$f_1\ (t+\tau)=\frac{\overline{\varphi}_1\ (p)}{\overline{\Psi}_1(p)}\cdot E_o$$

and the corresponding terms for $f_2\ (t+\tau)$ and $f_3\ (t+\tau)$.

$\overline{\varphi}_m\ (p)$ and $\overline{\Psi}_m\ (p)$ are then functions of τ. The first half (or whole) period can then be written

$$\left\{\frac{\varphi_1(p)}{\Psi_1(p)}-e^{-p\tau_1}\cdot\frac{\overline{\varphi}_1(p)}{\overline{\Psi}_1(p)}+e^{-p\tau_1}\cdot\left[\frac{\varphi_2(p)}{\Psi_2(p)}-e^{-p\tau_2}\frac{\overline{\varphi}_2(p)}{\overline{\Psi}_2(p)}\right]\right.$$

$$\left.+e^{-p(\tau_1+\tau_2)}\cdot\left[\frac{\varphi(p)}{\Psi(p)}-e^{-p\tau_3}\frac{\overline{\varphi}_3(p)}{\overline{\Psi}_3(p)}\right.\right. E_o \ldots\ldots\ldots (3)$$

The second half-period is built up in the same way; it follows on the first after a time τ and must therefore be furnished with a factor $e^{-p\tau}$, where τ is the time when the second half-period begins. In this way the whole period is formed and we may put this section of curve in the form

$$\frac{\varphi(p)}{\Psi(p)}\cdot E_o$$

If the length of the full period is τ, we may obviously write the E. M. F. E in the following form

$$\frac{\varphi(p)}{\Psi(p)}\cdot E_o+e^{-p\tau}\frac{\varphi(p)}{\Psi(p)}\cdot E_o+e^{-2p\tau}\cdot\frac{\varphi(p)}{\Psi(p)}\cdot E_o+\ldots$$

$$=\frac{1}{1-e^{-p\tau}}\cdot\frac{\varphi(p)}{\Psi(p)}\cdot E_o$$

We will now apply the above method to some examples.

1. For a constant E. M. F. we obviously have $\varphi\ (p)$ and $\Psi\ (p)$ both unity.

2. For an E. M. F. increasing rectilinearly, i. e. of the form

$$E=a+b\,t$$

we get the expression

$$\frac{\varphi(p)}{\Psi(p)}=a+\frac{b}{p}$$

or

$$\varphi(p)=a\,p+b$$

$$\Psi(p)+p.$$

If b is negative the E. M. F. will be decreasing.

3. For a sinusoidal E. M. F. of the form

$$E=E_o\sin(\omega\,t+a)$$

we have

$$E=\frac{p^2\sin a+\omega p\cos a}{p^2+\omega^2}\cdot E_o$$

or

$$\varphi(p)=p^2\sin a+\omega\,p\cos a$$

$$\Psi(p)=p^2+\omega^2$$

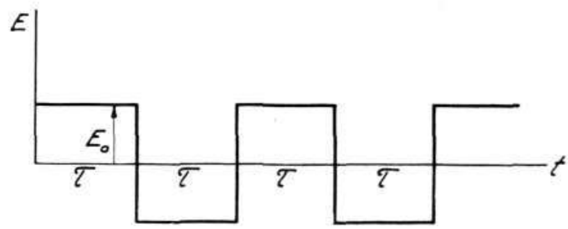
4. For a damped sinusoidal E. M. F. of the form

$$E=E_o\,e^{-\beta t}\sin(\omega t+a)$$

we have

$$E=p\,\frac{(p+\beta)\sin a+\omega\cos a}{(p+\beta)^2+\omega^2}\cdot E_o$$

5. If we have an E. M. F. of the form given below



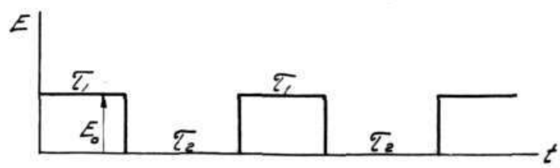
we can immediately write

$$E=E_o-2\,e^{-p\tau}E_o+2\,e^{-2p\tau}E_o-\ldots$$

or

$$E=\frac{1-e^{-p\tau}}{1+e^{-p\tau}}\cdot E_o$$

6. If again the E. M. F. is of this type



we get

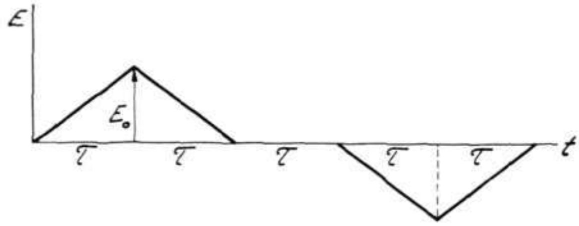
$$E=E_o-e^{-p\tau_1}E_o+e^{-p(\tau_1+\tau_2)}E_o-e^{-p\,2\tau_1+\tau_2}E_o$$

$$+e^{-p\,2(\tau_1+\tau_2)}E_o-\ldots$$

$$=\frac{1}{1-e^{-p(\tau_1+\tau_2)}}\cdot E_o-\frac{e^{-p\tau_1}}{1-e^{-p(\tau_1+\tau_2)}}\cdot E_o$$

$$=\frac{1-e^{-p\tau_1}}{1-e^{-p(\tau_1+\tau_2)}}\cdot E_o$$

7. If the E. M. F. is like the figure below



we can apply our general rule. The curve $f_1\ (t)$ is $\frac{E_o}{\tau} \cdot t$ and the curve $f_2\ (t) = E_o - \frac{E_o}{\tau} \cdot t$.

According to our scheme we will then get for the half-period

$$\frac{E_o}{\tau} t - e^{-p\tau} \frac{E_o}{\tau} (t+\tau)$$

$$+ e^{-p\tau} \left[E_o - \frac{E_o}{\tau} t - e^{-p\tau} \left(E_o - \frac{E_o}{\tau} (t+\tau) \right) \right]$$

$$= \left\{ 1 - 2e^{-p\tau} + e^{-2p\tau} \right\} \frac{E_o}{p\tau} = \frac{(1-e^{-p\tau})^2}{p\tau} \cdot E_o = \xi(p) \cdot E_o.$$

The whole series will then be, as we see from the figure,

$$\xi(p)\, E_o - e^{-3p\tau}\, \xi(p)\, E_o + e^{-6p\tau}\, \xi(p) \cdot E_o - \dots$$

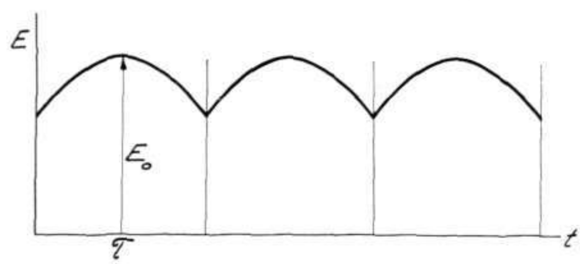
or

$$\frac{\xi(p)}{1+e^{-3p\tau}} \cdot E.$$

Substitution of $\xi\ (p)$ gives us finally

$$E = \frac{(1-e^{-p\tau})^2}{1+e^{-3p\tau}} \cdot \frac{1}{p\tau} \cdot E_o$$

8. As another example we will take the case in which our E. M. F. consists of a series of sine arcs, stretching equal distances on either side of the maxima. The E. M. F. is accordingly of the type shown in the next figure:



Then

$$f(t) = E_o \sin (\omega t + a)$$

if τ is the length of a period we have

$$\omega\tau = \pi - 2\,a$$

For $f\ (t+\tau)$ we get the expression

$$E_o \sin (\omega t + \omega\tau + a) = -E_o \sin (\omega t - a)$$

and

$$E_o \sin (\omega t + a) = \frac{p^2 \sin a + \omega p \cos a}{p^2+\omega^2} \cdot E_o.$$

According to our scheme, then, for the whole period we get

$$\left[\frac{p^2 \sin a + \omega p \cos a}{p^2+\omega^2} + \right.$$

$$\left. + e^{-p\tau} \frac{-p^2 \sin a + \omega p \cos a}{p^2+\omega^2} \right] \cdot E_o = \xi(p) \cdot E_o.$$

For the whole series of periods we get

$$\xi(p) \cdot E_o + e^{-p\tau}\, \xi(p)\, E_o + e^{-2p\tau} \cdot \xi(p)\, E_o + \dots$$

or

$$\frac{1}{1-e^{-p\tau}}\, \xi(p) \cdot E_o$$

or, if we substitute the value of $\xi\ (p)$,

$$E = \left[\frac{p^2 \sin a}{p^2+\omega^2} + \frac{1+e^{-p\tau}}{1-e^{-p\tau}} \cdot \frac{\omega p \cos a}{p^2+\omega^2} \right] E_o.$$

$$\omega\tau = \pi - 2\,a$$

9. For a short rectangular impulse we have the expression

$$E = E_o - e^{-p\tau} \cdot E_o$$

If we now make τ approach zero and at the same time E_o increase, but keeping $\tau\, E_o$ constant, we get an impulse. By expanding the exponential and taking the limits we find we can write an impulse

$$E = p \cdot \tau \cdot E_o = p \cdot K.$$

A series of equal impulses succeeding one another at intervals of τ is given by the formula

$$E = \frac{p}{1-e^{-p\tau}} \cdot K.$$

10. As a further example we will assume the E. M. F. to consist of a portion τ of an aperiodic impulse of the form

$$t e^{-\beta t} E_o$$

and that this impulse is repeated at times $t=\tau$, $2\ \tau$, $3\ \tau, \dots$

If we put $t+\tau$ instead of t we get

$$f(t+\tau) = (t+\tau)\, e^{-\beta(t+\tau)} =$$

$$= e^{-\beta\tau} \left[t e^{-\beta t} + \tau e^{-\beta t} \right] \cdot E_o$$

In operator form this expression becomes

$$\frac{\overline{\varphi}\ (p)}{\Psi\ (p)} \cdot E_o = e^{-\beta\tau} \left[\frac{p}{(p+\beta)^2} + \tau \frac{p}{p+\beta} \right] \cdot E_o.$$

Further

$$\frac{\varphi\ (p)}{\Psi\ (p)} \cdot E_o = \frac{p}{(p+\beta)^2} \cdot E_o$$

Whence, for an impulse of the length τ,

$$\left\{\frac{p}{(p+\beta)^2} - e^{-(p+\beta)\tau}\left[\frac{p}{(p+\beta)^2} + \frac{p\tau}{p+\beta}\right]\right\} \cdot E_o$$

or

$$\frac{p}{(p+\beta)^2} \cdot \left\{1 - e^{-(p+\beta)\tau}\,(1+\overline{p+\beta}\cdot\tau)\right\} \cdot E_o,$$

Thus for the series of impulses as a whole we get

$$\frac{p}{(p+\beta)^2} \cdot \frac{1 - e^{-(p+\beta)\tau}\,(1+\overline{p+\beta}\cdot\tau)}{1 - e^{-p\tau}} \cdot E_o$$

which is thus the operator expression for an E. M. F. consisting of a series of aperiodic impulses.

The denominator has a double root $p = -\beta$. But this root is also a double root of the numerator and can therefore not be used in an expansion according to the expansion theorem.

We also have the roots p_m, obtained from the formula

$$p_m \tau = 2\,\pi\, mj$$

where m can assume any positive or negative integral value or zero. The root corresponding to $m = o$ or $p = o$ is, however, also a root of the numerator, and cannot therefore be considered either.

Since we now have the expression for a variable E. M. F. in vector form with a constant operand, Heaviside's expansion theorem enables us to obtain directly the expansion of the E. M. F. in a Fourier series.

But here a remark is necessary. The operator expression obtained represents an E. M. F. which is zero until $t = o$ and then assumes a given series of values.

In integral form it is represented by

$$\lim_{R \to \infty} \frac{1}{2\,\pi j} \int_{\supset} \frac{e^{pt}}{p} \frac{\varphi\ (p)}{\Psi\ (p)} E_o \, d\,p^*$$

This is an integral along a semi-circle of radius R over the right half-plane and the radius R approaches infinity. If this integral is convergent for positive values of t, the corresponding integral over the left-hand plane will be zero and hence *for positive values of t* the above integral can be completed to give a circle about the origin with a radius approaching infinity. According to Cauchy's theorem this integral now gives Heaviside's expansion theorem, which consequently gives the correct value only for positive values of t. If the E. M. F. is periodic, we will for negative values of t get the periodic continuation of the original curve for positive values of t. It is, then, this continuous periodic curve —continuous from $-\infty$ to $+\infty$— that we are expanding in a Fourier series by means of the expansion theorem.

It should, however, be noted that if in an operator expression

$$y = \frac{g\ (p)}{h\ (p)} \cdot E\ (t)$$

we replace $E(t)$ in the manner indicated above by an expression

$$E(t) = \frac{\varphi\ (p)}{\Psi\ (p)} \cdot E_o$$

we still obtain the correct value of y if we first expand $E\ (t)$ according to Heaviside's expansion theorem in a Fourier series, and then introduce this series in the formula for y. The solution y will here correspond to an E. M. F. which is zero until $t = o$ and then equal to the Fourier series, which in its turn, for positive values of t, gives the original curve.

As the E. M. F. is assumed to be periodic, the division gives a constant and a sum of sine terms whose frequencies are integral multiples of the fundamental frequency. The roots of the denominator of the operator expression must consequently, when the former is made zero, be purely imaginary. Thus no real roots will exist. To make the calculation of the terms of the expansion easier, we will employ the rule given below.

We assume our operator formula to be of the following form

$$E\ (t) = \frac{\varphi\ (p)}{\Psi\ (p)} \cdot E_o$$

where $\varphi\ (p)$ and $\Psi\ (p)$ are integral functions of p.

We assume further that p_m is a complex root with the coefficient of j positive. The conjugate root we will represent by $\bar{p}_m$.

* An exposition of the operator calculus based on this integral will be published shortly.

Heaviside's expansion theorem now gives us the following expansion

$$E(t) = \frac{\varphi(o)}{\Psi(o)} \cdot E_o + \sum \left[\frac{1}{p_m} \frac{\varphi(p_m)}{\Psi(p_m)} e^{p_m t} + \frac{1}{\bar{p}_m} \cdot \frac{\varphi(\bar{p}_m)}{\Psi'(\bar{p}_m)} e^{\bar{p}_m t} \right] E_o$$

The two terms in the brackets together give a sine oscillation of angular frequency equal to the imaginary coefficient in p_m. We assume

$$p_m = -\beta_m + j\,\omega_m,$$

that is, the angular frequency is ω_m.

The two terms in the brackets are now conjugate to one another. If we call the first y_m and make

$$y_m = A_m + j\,B_m$$

the second will be

$$\bar{y}_m = A_m - j\,B_m$$

whence

$$y_m + \bar{y}_m = 2\,A_m$$

or $y_m + \bar{y}_m$ is the coefficient of j in the expression

$$Y_m = 2\,j\,y_m$$

Y_m is thus the complex vector of the solution. We thus have the rule:

We select the solution y_m which according to the expansion theorem corresponds to the root $p_m = -\beta_m + j\omega_m$, where ω_m is positive, and multiply by 2 j. The coefficient of the imaginary part will then give the solution corresponding to one pair of complex roots.

We will apply this to one of the foregoing examples, no. 8.

We have the operator expression

$$E(t) = \frac{p^2 \sin a}{p^2 + \omega^2} + \frac{1 + e^{-p\tau}}{1 - e^{-p\tau}} \cdot \frac{\omega p \cos a}{p^2 + \omega^2} \cdot E_o$$

with

$$\omega\tau = \pi - 2\,a.$$

The first term gives us directly

$$\sin a \cdot E_o \cos \omega\, t.$$

To determine the second term we have to find the roots when the denominator is made zero, i. e. the equation

$$(1 - e^{-p\tau})(p^2 + \omega^2) = 0$$

This equation will have roots p_m given by

$$p_m = \begin{cases} j \cdot \dfrac{2\pi}{\tau} m = j m \cdot \dfrac{2\pi}{\pi - 2a} \cdot \omega;\ m = 1, 2, \ldots \\ j\omega \end{cases}$$

Although $p = o$ is a root of the denominator, it is also a root of the numerator, and the whole expression will be finite when $p = o$. This root can therefore not be used.

For a root of the first kind we now get, using Heaviside's expansion theorem and the rule given above,

$$Y_m = 2j \cdot \frac{1}{p_m} \cdot \frac{1 + e^{-p_m \tau}}{\tau e^{-p_m \tau}} \cdot \frac{\omega p_m \cos a}{p_m^{\,2} + \omega^2} \cdot E_o\, e^{p_m t}$$

or after substitution

$$Y_m = -\frac{4j}{\pi - 2a} \cdot \frac{E_o \cos a}{m^2 \left(\dfrac{2\pi}{\pi - 2a}\right)^2 - 1} e^{jm \frac{2\pi}{\pi - 2a} \omega t}$$

The coefficient of the imaginary part of this expression will be

$$-\frac{4}{\pi - 2a} \cdot \frac{E_o \cos a}{m^2 \left(\dfrac{2\pi}{\pi - 2a}\right)^2 - 1} \cdot \cos \left\{ m \cdot \frac{2\pi}{\pi - 2a} \omega t \right\}$$

The other root $p = j\omega$ gives us

$$Y = 2j \cdot \frac{1 + e^{-j\omega\tau}}{1 - e^{-j\omega\tau}} \cdot \frac{\omega \cos a}{2j\omega} E_o\, e^{j\omega t}$$

$$= -j \operatorname{ctg} \frac{\omega\tau}{2} \cos a \cdot e^{j\omega t}$$

The corresponding solution will thus be

$$-\operatorname{ctg} \frac{\omega\tau}{2} \cos a \cdot E_o \cos \omega t = -E_o \sin a \cos \omega t.$$

$$\frac{\varphi(o)}{\Psi(o)} E_o \text{ gives } \frac{2 \cos a}{\pi - 2a} E_o$$

We thus get

$$E(t) = \frac{2 \cos a}{\pi - 2a} E_o - \sum_{m = 1, 2, \ldots} \frac{4}{\pi - 2a} \frac{1}{m^2 \left(\dfrac{2\pi}{\pi - 2a}\right)^2 - 1} E_o \cos a \cdot \cos \left\{ m \cdot \frac{2\pi}{\pi - 2a} \omega t \right\}$$

It might be convenient here to see if the expansion theorem gives the right value when the E. M. F. has the form of a finite impulse. The E. M. F. may then, as we have shown, be written in the form

$$E(t) \left[\frac{\varphi(p)}{\Psi(p)} - e^{-p\tau} \frac{\bar{\varphi}(p)}{\bar{\Psi}(p)} \right] E_o$$

and

$$y = \frac{g(p)}{h(p)} \cdot E(t)$$

$$= \left[\frac{g(p)}{h(p)} \cdot \frac{\varphi(p)}{\Psi(p)} - e^{-p\tau} \cdot \frac{g(p)}{h(p)} \cdot \frac{\bar{\varphi}(p)}{\bar{\Psi}(p)}\right] E_o$$

The first term is obtained in the usual way. The second is given, according to the above, by the semi-circle integral:

$$\lim_{R \to \infty} \frac{E_o}{2\pi j} \int_{\supset} \frac{e^{p(t-\tau)}}{p} \frac{g(p)}{h(p)} \cdot \frac{\bar{\varphi}(p)}{\bar{\Psi}(p)} dp.$$

But this integral is zero for all values of t less than τ, which corresponds to the physical conditions. If, on the other hand, the integral is taken over the left half-plane, it will not be zero until t is greater than τ.

If, therefore, we integrate over the closed circle—which will give us the same result as the expansion theorem—the integral will not be a solution to our problem unless $t > \tau$.

The expansion theorem, that is, only gives the right solution for values of t greater than τ. For values less than τ we must make the latter part of y zero.

An exception to this is when the denominator $\Psi(p)$ contains a term with the factor $e^{-p\tau}$, as in this case the integral over the left half-plane, when R approaches infinity, will also be zero for values of t between $t=o$ and $t=\tau$. In this case the expansion theorem can be used and will give the solution for all positive values of t.

Reduction to a Constant of a Variable E. M. F. with the Assistance of the Heaviside Operator Calculus. — R. D. D.

A General Method for Determining Oscillations in a Network with Small Losses.

Communication from the Research and Development Department.

By *H. Pleijel.*

It is frequently necessary to determine voltages and currents in the various parts of a network consisting of directly or indirectly coupled oscillating electric circuits with small losses.

In many cases it is possible to regard the network as a quadripole and use the theory of wave filters, but on the other hand there are cases where it will be both simpler and clearer to make a direct calculation without the introduction of such quantities as the characteristic and the wave length constant. This is particularly true of closed networks.

It will therefore be of some interest to find a method whereby we can determine the frequencies, dampings, amplitudes and phase angles of the electrical oscillations arising in an arbitrarily composed network when an E. M. F. is connected. The method we intend to use is analogous to that previously employed by the author for computing filter circuits.[1]

The E. M. F. connected may be constant, an impulse, or a periodic function of the time. To obtain more general formulæ, the E. M. F. may be written in operator form. We introduce

$$E(t) = \frac{\varphi(p)}{\Psi(p)} \cdot E_o,$$

where E_o is a constant, and $\varphi(p)$ and $\Psi(p)$ are integral functions of p. In another paper the author showed how the functions $\varphi(p)$ and $\Psi(p)$ can be determined in various cases. This paper has now been extended to provide a basis for the following method.

We will use Heaviside's operator method and therefore introduce the symbol $p = \frac{d}{dt}$ regarding p as an algebraic quantity throughout the argument.

[1] The L. M. Ericsson Review, No. 7—9, 1931.

The network is assumed to be made up of conductors containing resistances, inductances and capacities coupled in series; we also assume the existence of mutual inductances between some of the conductors in the network.

The operator expression for the impedance of one of the conductors in the network we will represent by z_m', where the index m is the number of the conductor. The impedance z_m' may now be written

$$z'_m = r_m + p\,l_m + \frac{1}{p\,c_m},$$

where r_m is the resistance of the conductor, including also the series resistance corresponding to the condenser losses and the resistance of the inductance l_m.

Similarly, we introduce an impedance for a mutual inductance, putting

$$z_m' = r_m + p\,l_m$$

l_m is here a mutual inductance and r_m the resistance corresponding to the losses in the iron core which promotes the inductive coupling of two conductors.

Having introduced these operator expressions for the impedances, we determine the current or voltage at a point in the network by means of Kirchhoff's Laws, regarding p as an algebraic quantity in our calculations. We naturally make allowance here for the mutual inductances as well.

If E is the E. M. F. and y the current or voltage to be determined, we get by elimination an operator solution of the following form:

$$y = \frac{g(z_1'\, z'_2 \ldots)}{h(z'_1\, z'_1 \ldots)} \cdot E$$

If—as we shall assume in the following—the network consists of discrete capacities and inductances, g and h will be homogeneous polynomials in z_1', z_2'.... If y is a voltage, g and h

will be of the same degree; but if y is a current, h will be of one degree higher than g.

As we have said before, we assume that all resistances are small compared to the corresponding reactance operators. We further introduce the expression

$$z_m = p l_m + \frac{1}{p c_m}$$

or

$$z_m' = r_m + z_m$$

Natural Frequencies and dampings in the Network.

According to the Heaviside expansion theorem, the network will have a certain number of natural frequencies, which are obtained by finding the roots of the equation

$$h\ (z_1'\ z_2' \ldots) = 0,$$

regarding this as an equation in p.

We represent these roots by p_m', putting

$$p_m' = -\beta_m' + j\,\omega_m'$$

ω_m' will then give us the frequency of the natural oscillation and β_m' its damping constant.

If we make all resistances zero, we get instead the equation

$$h\ (z_1\ z_2 \ldots) = 0$$

The roots of this equation we represent by p_m; as a rule they are purely imaginary.

We introduce the constant β_m, defined by the equation

$$p_m' = -\beta_m + p_m = -\beta + j\omega_m$$

For the sake of brevity we introduce further the expression

$$a_1 \frac{d}{dz_1} + a_2 \frac{d}{dz_2} + \ldots = a \triangledown$$

As all r_m are small we may, by Taylor's theorem, write:

$$h\,(z'_1\, z'_2 \ldots) = h\,(z'_1\, z'_2 \ldots) + r \triangledown \cdot h\,(z'_1\, z'_2 \ldots)$$

In the following we will represent $h\ (z_1, z_2 \ldots)$ and $g\ (z_1, z_2 \ldots)$ by h and g respectively.

If we introduce

$$p = p_m' = -\beta_m + p_m$$

in the above equation, the left-hand side will vanish, giving us

$$0 = [h + r \triangledown \cdot h]_{p=p'_m}$$

But on the other hand we may write

$$[h]_{p=p'_m} = [h]_{p=p_m} - \beta_m \left[\frac{dh}{dz_1}\frac{dz_1}{dp} + \frac{dh}{dz_2} \cdot \frac{dh}{dz_2} \cdot \frac{dz_2}{dp} + \ldots\right]_{p=p_m}$$

and as

$$[h]_{p=p_m} = 0$$

$$[h)_{p=p_m'} = -\beta_m \left[\frac{dz}{dp} \triangledown \cdot h\right]_{p=p_m}$$

If we introduce this expression for $[h]_{p=pm}$, and only keep terms of the first order we get

$$0 = -\beta_m \left[\frac{dz}{dp} \triangledown \cdot h\right]_{p=p_m} + [r \triangledown \cdot h]_{p=p_m}$$

or

$$\beta_m = \left[\frac{r \triangledown \cdot h}{\frac{dz}{dp} \triangledown \cdot h}\right]_{p=p_m}$$

We will now transform the denominator by taking advantage of the fact that $h\ (z_1\ z_2 \ldots)$ is homogeneous in $z_1, z_2 \ldots$ If n is the degree of h we have, according to Euler's theorem,

$$z_1 \frac{dh}{dz_1} + z_2 \frac{dh}{dz_2} + \ldots = n\,h$$

If here we introduce $p = p_m$, the right-hand side will vanish, and we can then write the left-hand side

$$\left(p l_1 + \frac{1}{pc_1}\right)\frac{dh}{dz_1} + \left(p l_2 + \frac{1}{pc_2}\right)\frac{dh}{dz_2} + \ldots,$$

whence the equation

$$\left[\left(l_1 + \frac{1}{p^2 c_1}\right)\frac{dh}{dz_1} + \left(l_2 + \frac{1}{p^2 c_1}\right)\frac{dh}{dz_2} + \ldots\right]_{p=p_m} = 0$$

But on the other hand we have

$$\left[\frac{dz}{dp} \triangledown \cdot h\right]_{p=p_m} = \left[\left(l_1 - \frac{1}{p^2 c_1}\right)\frac{dh}{dz_1} + \left(l_2 - \frac{1}{p^2 c_2}\right)\frac{dh}{dz_2} + \ldots\right]_{p=p_m}$$

By addition we get

$$\left[\frac{dz}{dp} \triangledown \cdot h\right]_{p=p_m} = 2\left[l_1 \frac{dh}{dz_1} + l_2 \frac{dh}{dz_2} + \ldots\right]_{p=p_m} = 2[l \triangledown \cdot h]_{p=p_m}$$

The formula for β_m can therefore be written

$$\beta_m = \frac{1}{2}\left[\frac{r \triangledown \cdot h}{l \triangledown \cdot h}\right]_{p=p_m} \ldots\ldots\ldots\ldots (1)$$

The roots p_m were obtained from the equation

$$h = 0 \quad \ldots\ldots\ldots\ldots \quad (2)$$

The expression obtained for β_m shows that β_m is real. If, as we have assumed, the roots of (2) are purely imaginary, we get

$$p_m = \pm j\,\omega'_m = \pm j\,\omega_m$$

and

$$\beta_m = \beta_m' = \frac{1}{2}\left[\frac{r \nabla \cdot h}{l \nabla \cdot h}\right]_{p=p_m}$$

$p_m = j\omega_m$ thus gives us the angular frequency and β_m the corresponding network damping constant.

(1) and (2) thus give us all the natural frequencies of the system and their respective damping constants.

If $\frac{r}{l}$ is the same for all the conductors, including mutual inductances, all the natural frequencies will have the same damping constant, which will be $\frac{r}{2l}$.

In practice $\frac{r}{l}$ may be the same for all the inductances in the network, and likewise $\frac{a}{c}$, where a is the leakage conductance of a condenser, the same for all the condensers.

We will show that all the natural frequencies of the network will always have the same damping constant, and that this will be the simple expression

$$\beta = \frac{1}{2}\left(\frac{r}{l} + \frac{a}{c}\right)$$

We introduce the symbols

$$\frac{r}{l} = f'$$

$$\frac{a}{c} = f''$$

The resistance operator of a condenser with a leakance a_m and a capacity c_m is now, it will be remembered, equal to

$$-\frac{a_m}{p^2 c_m^2}$$

As the impedance operator of a condenser with leakage is

$$\frac{1}{a_m + pc_m}$$

If we expand this expression in powers of a_m, we get

$$\frac{1}{a_m + pc_m} = \frac{1}{pc_m} \cdot \frac{1}{1 + \frac{a_m}{pc_m}} = \frac{1}{pc_m}\left\{1 - \frac{a_m}{pc_m} + \ldots\right\} =$$

$$= \frac{1}{pc_m} - \frac{a_m}{p^2 c^2_m} + \ldots$$

If we take only the first two terms, we see that the condenser with leakage may be regarded as a condenser of capacity c_m in series with a resistance $-\frac{a_m}{p^2 c^2_m}$.

If the resistance of the inductance of the conductor is called r_m', we find that the resistance r_m of the conductor will be given by:

$$r_m = r'_m - \frac{a_m}{p^2 c^2_m}.$$

Our expression $r \nabla \cdot h$ can then be written

$$r \nabla \cdot h = \sum \left(r_m' - \frac{a_m}{p^2 c^2_m}\right) \frac{\delta h}{\delta z_m}$$

or, if we introduce f' and f'', where $f' = \frac{r_m'}{l_m}$ and $f'' = \frac{a_m}{c_m}$,

$$r \nabla \cdot h = \sum \left(f' \cdot l_m \frac{\delta h}{\delta z_m} - f'' \cdot \frac{1}{p^2 c_m} \frac{\delta h}{\delta z_m}\right) =$$

$$= f' \cdot l \nabla \cdot h - f'' \sum \frac{1}{p^2 c_m} \frac{\delta h}{\delta c_m}$$

But we have already found that if p is a root of the equation $h\ (z_1\ z_2 \ldots) = 0$, we have

$$-\sum \frac{1}{p^2 c_m} \cdot \frac{\delta h}{\delta z_m} = l \nabla \cdot h$$

We thus get

$$r \nabla \cdot h = (f' + f'')\, l \nabla \cdot h$$

and consequently according to (1)

$$\beta_m = \frac{1}{2}(f' + f'') = \frac{1}{2}\left(\frac{r'}{l} + \frac{a}{c}\right)$$

Amplitude and Phase Angle of the Natural Oscillation.

We started from the operator solution

$$y = \frac{g\,(z'_1\, z'_2 \ldots)}{h\,(z'_1\, z'_2 \ldots)} \cdot E(t)$$

As has been mentioned above, we showed in another paper that, whatever the composition of $E(t)$, we can in general write

$$E(t) = \frac{\varphi\,(p)}{\Psi(p)} \cdot E_o.$$

where E_o is a constant and $\varphi\ (p)$ and $\Psi\ (p)$ integral functions of p. This is always the case if $E(t)$ is composed of curve segments that can be expressed in this way as φ and Ψ polynominals in p.

For a sinusoidal E. M. F., then, we have

$$E(t) = E_o \sin(\omega_o t + a_o) = E_o \cdot \frac{p^2 \sin a_o + p\omega_o \cos a_o}{p^2 + \omega^2_o} \cdot 1$$

For further details we refer to the paper mentioned above.

Our operator solution can thus be written

$$y = \frac{g(z'_1 z'_2 \ldots)}{h(z'_2 z'_2 \ldots)} \cdot \frac{\varphi(p)}{\Psi(p)} \cdot E_o$$

where E_o is a constant.

The natural frequencies of the network are now given by the roots of the equation

$$h(z_1 z_2 \ldots) = 0$$

If p'_m is one of these roots, Heaviside's expansion theorem will give us the corresponding solution:

$$y_m = \frac{1}{p'_m} \left[\frac{g(z'_1 z'_2 \ldots)}{\frac{d}{dp} h(z'_1 z'_2 \ldots)} \right]_{p=p'_m} \cdot \frac{\varphi(p'_m)}{\Psi(p'_m)} \cdot e^{p'_m t}$$

In the expression we may substitute $z_1', z_2' \ldots$ for z_1', z_2' respectively, and p_m for p_m', where p_m is the corresponding root of $h(z_1 z_2 \ldots) = 0$.

The only uncertainty here is with regard to $\Psi(p_m')$. If p_m makes $\Psi(p)$ exactly or nearly zero, we cannot directly replace p_m' by p_m in this function. We now have

$$p_m' = -\beta_m + p_m$$

and can therefore write

$$\Psi(p'_m) = \Psi(p_m) - \beta_m \Psi'(p_m)$$

y_m may then be written

$$y_m = \frac{1}{p_m} \left[\frac{g}{\frac{\delta h}{\delta p}} \right]_{p=p_m} \cdot \frac{\varphi(p_m)}{\Psi(p_m) - \beta_m \Psi'(p_m)} e^{-\beta_m t} \cdot E_o \cdot e^{p_m t}$$

But we have already found that

$$\left[\frac{dh}{dp}\right]_{p=p_m} = \left[\frac{dz}{dp} \nabla \cdot h\right]_{p=p_m} = 2\,[l \nabla \cdot h]_{p=p_m}$$

If this expression is introduced, we can finally write

$$y_m = \tfrac{1}{2} \left[\frac{g}{l \nabla \cdot h}\right]_{p=p_m} \cdot \frac{1}{p_m} \frac{\varphi(p_m}{\Psi(p_m) - \beta_m \Psi'(p_m)} \cdot$$
$$\cdot e^{-\beta_m t} \cdot E_o \cdot e^{p_m t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Y_m now gives the particular solution corresponding to the root p_m. As we see, we get a damped sine oscillation of frequency ω_m, where $p_m = j\omega_m$.

We have, however, another such damped sine oscillation with the same β_m and ω_m, namely the one corresponding to the root $\bar{p}_m$ conjugate to p_m, given by

$$\bar{p}_m = -\beta_m - j\omega_m$$

The corresponding y_m is obtained directly from equation (3), if in that we replace j by $-j$ throughout. The expression y_m thus obtained is therefore conjugate to y_m. The particular solution which gives the oscillation of damping β_m and natural frequency ω_m will thus be

$$\bar{y}_m + y_m.$$

If we now have

$$y_m = A + j\,B$$

we get

$$\bar{y}_m = A - j\,B$$

and

$$y_m + \bar{y}_m = 2\,A$$

If we introduce as a "complex vector" $Y_m = 2\,j\,y_m$, we see that

$$Y_m = 2\,j\,A - 2\,B.$$

We see accordingly that $\bar{y}_m + y_m$ *will be equal to the coefficient of j in the expression* $2\,j\,y_m$.

In the language of A. C. theory, $\bar{y}_m + y_m$ is the instantaneous value of the vector $Y_m = 2\,j\,y_m$.

We introduce the vector Y_m in our calculations and so get

$$Y_m = \frac{1}{2} \left[\frac{g}{l \nabla \cdot h}\right]_{p=p_m} \cdot \frac{2j}{p_m} \frac{\varphi(p_m)}{\Psi(p_m) - \beta'_m \Psi'(p_m)} \cdot$$
$$\cdot e^{-\beta_m t} E_o \cdot e^{j\omega_m t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

The coefficient of j in Y_m will, according to the above, give us the damped sine term corresponding to the natural frequency ω_m.

Amplitude and Phase Angle of the forced Oscillations.

Above we have determined the natural frequencies of the network. Apart from these we have the oscillations corresponding to the frequencies of which $E(t)$ is composed.

These are obtained from the roots of the equation

$$\Psi(p) = 0$$

where $\Psi(p)$ is the denominator in the equation

$$E(t) = \frac{\varphi(p)}{\Psi(p)} \cdot E_o$$

The roots are purely imaginary if $E(t)$ is periodic. If p_o is one root, the corresponding superimposed frequency ω_o will in that case be given by the equation

$$p_o = j\,\omega_o$$

In the following we will only determine the oscillation corresponding to p_o. The others are obtained by substituting the values of the other roots in the equations.

According to Heaviside's expansion theorem we now have

$$y = \left[\frac{g\,(z'_1\,z'_2\ldots\ldots)}{h\,(z'_1\,z'_2\ldots\ldots)}\right]_{p=p_o}\cdot\frac{1}{p_o}\cdot\frac{\varphi\,(p_o)}{\Psi'(p_o)}\cdot E_o\,e^{p_o t}$$

Here we can replace z_1', $z_2'\ldots\ldots$ by z_1, $z_2\ldots\ldots$ respectively, unless p_o lies close to a root of the equation $h\ (z_1\,z_2\ldots\ldots)=0$.

To include this case in our final formula we expand $h\ (z_1'\,z_2'\ldots\ldots)$.

We have

$$h\,(z'_1\,z'_2\ldots\ldots) = h\,(z'_1\,z'_2\ldots\ldots) + r\,\nabla\cdot h$$

or

$$h\,(z'_1\,z'_2\ldots\ldots) = h\,(z'_1\,z'_2\ldots\ldots) + \frac{1}{2}\,\frac{r\nabla\cdot h}{l\,\nabla\cdot h}\cdot 2\,l\,\nabla\cdot h$$

We have found that if in $\frac{1}{2}\,\frac{r\nabla\cdot h}{l\,\nabla\cdot h}$ we substitute $p=p_m$, this expression gives the damping β_m for the natural oscillation corresponding to the root p_m. We may therefore say the above is a general expression for the damping, and substitute

$$\beta = \frac{1}{2}\cdot\frac{r\,\nabla\cdot h}{l\,\nabla\cdot h}$$

The value of β obtained when we make $p=p_o$ we will denote by β_o.

The vector Y_o will then take the form

$$Y_o = \left[\frac{g}{h+\beta\cdot 2l\nabla\cdot h}\right]_{p=p_o}\cdot\frac{2j}{p_o}\,\frac{\varphi\,(p_o)}{\Psi'(p_o)}\,E_o\,e^{p_o t}\ \ldots\ (5)$$

If p_o is not equal to p_m, we may neglect, in the denominators of (4) and (5), the terms in which β is a factor.

The Oscillations in the Neighbourhood of a Resonance Point.

Resonance occurs when p_o coincides with a root p_m. But we may also get resonance if p_o coincides with a root of the equation $g\ (z_1\,z_2\ldots\ldots)=0$.

We will see first what formulæ we get in the first case.

In equation (4), $\Psi\ (p_m)$ will be nearly zero. We can then write, with close approximation,

$$\Psi\,(p_m) = \Psi\,(p_o) + (p_m - p_o)\,\Psi'\,(p_o) = (p_m - p_o)\,\Psi'\,(p_o).$$

In the same way we can write in (5)

$$h\,(p_o) = h\,(p_m) + (p_o - p_m)\left[\frac{\delta h}{\delta p}\right]_{p=p_m} =$$
$$= (p_o - p_m)\cdot 2\,[l\,\nabla\cdot h]_{p=p_m}$$

As p_o and p_m are very nearly equal, we may put

$$\Psi\,(p_m) = (p_m - p_o)\,\Psi'\,(p_m)$$

and

$$h\,(p_o) = (p_o - p_m)\cdot 2\,[l\,\nabla\cdot h]_{p=p_o}$$

If we substitute the above expression, we can write:

$$Y_m = \left[\frac{g}{2\cdot l\,\nabla\cdot h}\right]_{p=p_m}\cdot$$
$$\cdot\,\frac{2j}{p_m}\cdot\frac{\varphi\,(p_m)}{(p_m - p_o - \beta_m)\,\Psi'\,(p_m)}\,e^{-\beta_m t}\cdot E_o\,e^{j\omega_m t}$$

$$Y_o = -\left[\frac{g}{2\,l\nabla\cdot h}\right]_{p=p_m}\cdot\frac{2j}{p_o}\cdot\frac{\varphi\,(p_o)}{(p_m - p_o - \beta_o)\,\Psi'\,(p_m)}\,E_o e^{j\omega_o t}$$

In these expressions p_o and p_m can be interchanged everywhere except in $p_m - p_o$, so that for the neighbourhood of the resonance point we can write

$$Y_m = -\frac{1}{2}\left[\frac{g}{l\,\nabla\cdot h}\right]_{p=p_m}\cdot\frac{2j}{p_m}\cdot$$
$$\cdot\,\frac{\varphi\,(p_o)}{(p_o + \beta_m - p_m)\,\Psi'\,(p_o)}\cdot E_o\cdot e^{-\beta_m t}\cdot e^{j\omega_m t}$$

$$Y_o = \frac{1}{2}\left[\frac{g}{l\nabla\cdot h}\right]_{p=p_m}\cdot\frac{2j}{p_m}\cdot\frac{\varphi\,(p_o)}{(p_o + \beta_m - p_m)\,\Psi'\,(p_o)}\,E_o\cdot e^{j\omega_o t}$$

Thus if we neglect the term $e^{-\beta_m t}$ we get the same amplitude and phase angle for $-Y_m$ and Y_o when a forced frequency and one of the natural frequencies, ω_m, of the network are close together. Thus for small values of t the two oscillations are opposite in phase and the same in amplitude. In time Y_m will be damped, and at the same time the two vectors will separate at a relative angular velocity of ω_m and ω_o.

The ratio of the amplitude of the oscillation to its *maximum* amplitude will, for both the natural frequency and the superimposed frequency, be

$$\frac{\beta_m}{p_o - \beta_m - p_m}$$

If here we substitute

$$p_o = j\,\omega_o$$
$$p_m = j\omega_m$$

we find the ratio of amplitude to maximum amplitude becomes

$$\frac{\beta_m}{\sqrt{\beta_m{}^2 + (\omega_m - \omega_o)^2}}$$

At the actual point of resonance we have $p_o = p_m$. The two natural oscillations will then together form an oscillation of the form

$$Y_o + Y_m = \frac{1}{2}\left[\frac{g}{l\bigtriangledown \cdot h}\right]_{p=p_m} \cdot \frac{2\,j}{p_m} \cdot \frac{\varphi\,(p_m)}{\beta_m \Psi'(p_m)} \cdot$$
$$\{1 - e^{-\beta_m t}\} \cdot e^{j\omega_m t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

We will now go on to consider the case when p_o is nearly equal to a root of the equation $g\ (z_1{}'\ z_2{}' \ldots.) = 0$.

If we represent this root by $p_s{}'$, we can put

$$p_s{}' = -\beta_s + p_s$$

where p_s is a root of the equation

$$g\ (z_1\ z_2 \ldots.) = 0$$

By the same method as we have already used to determine β_m, we get here the formula

$$\beta_s = \tfrac{1}{2}\left[\frac{r\bigtriangledown \cdot g}{l\bigtriangledown \cdot g}\right]_{p=p_s.}$$

β_s is real. If, therefore, the root p_s is purely imaginary, we have divided $p_s{}'$ into its real and imaginary parts.

We can then write

$$g\,(z'_1\,z'_2 \ldots.) = g\,(z_1\,z_2 \ldots.) + r\bigtriangledown \cdot g\,(z_1\,z_2 \ldots.).$$

When p_o is in the neighbourhood of p_s we may then write

$$[g\,(z_1\,z_2 \ldots.)]_{p=p_o} = [g\,(z_1 z_2 \ldots.)]_{p=p_s}$$
$$+ (p_o - p_s)\left[\frac{\delta g}{\delta p}\right]_{p=p_s} = (p_o - p_s)\,(2 \cdot l\bigtriangledown \cdot g)_{p=p_s}$$

(since g is homogeneous in $z_1\,z_2 \ldots.$, and $p = p_s$ is a root of $g = 0$).

We may then write

$$[g\,(z'_1 z'_2 \ldots.)]_{p=p_o} = [r\bigtriangledown \cdot g]_{p=p_o} + (p_o - p_s)\,2 \cdot [l\bigtriangledown \cdot g]_{p=p_s} =$$
$$= \frac{1}{2}\left[\frac{r\bigtriangledown \cdot g}{l\bigtriangledown \cdot g}\right]_{p=p_o} \cdot 2 \cdot [l\bigtriangledown \cdot g]_{p=p_s} + (p_o - p_s)\,2 \cdot [l\bigtriangledown \cdot g]_{p=p_s}$$

or when p_o and p_s are nearly equal

$$[g\,(z'_1 z'_2 \ldots)]_{p=p_o} = [\beta_s + p_o - p_s] \cdot 2\,[l\bigtriangledown \cdot g]_{p=p_o}$$

When p_o approximates to a root p_s we thus get

$$Y_m = \frac{1}{2}\left[\frac{g}{l\bigtriangledown \cdot h}\right]_{p=p_m} \cdot \frac{2\,j}{p_m} \cdot \frac{\varphi\,(p_m)}{\Psi'(p^m)} \cdot E_o\, e^{-\beta_m t} \cdot e^{j\omega_m t} \quad (7)$$

$$Y_o = 2\left[\frac{l\bigtriangledown \cdot g}{h}\right]_{p=p_o} \cdot \frac{2j}{p_o}(\beta_s + p_o - p_s) \cdot \frac{\varphi\,(p_o)}{\Psi'(p_o)} E_o\, e^{j\omega_o t} \quad (8)$$

We see that with this kind of resonance the natural oscillations of the system are not affected. But, on the other hand, the superimposed oscillation corresponding to a root p_s will be affected strongly, as, for $p_o = p$, Y_o will be very nearly zero.

Recapitulation of the Formulæ Obtained.

Before going on to apply the foregoing to special examples, we will recapitulate our formulæ.

We have assumed the operator solution to be of the form

$$y = \frac{g\,(z'_1 \ldots.)}{h\,(z'_1 \ldots.)} \cdot \frac{\varphi\,(p)}{\Psi'(p)} \cdot E_o$$

If here we make $p = o$, we get any constant term there may be.

For the acting E. M. F. $E(t)$ we have substituted

$$E = \frac{\varphi\,(p)}{\Psi'(p)} \cdot E_o$$

Further, for the impedance of a conductor forming part of the system we have introduced the term

$$z_m = r_m + p\,l_m + \frac{1}{p\,c_m};\ z_m{}' = r_m + z_m$$

We have also used the following abreviations

$$g\ (z_1\ z_2 \ldots.) = g$$
$$h\ (z_1\ z_2 \ldots.) = h$$

The natural frequencies ω_m in the system are then given by the formula

$$h\ (z_1\ z_2 \ldots.) = 0$$

and the damping corresponding to a root $p_m = j\omega_m$ of this equation will be

$$\beta_m = \frac{1}{2}\left[\frac{r\bigtriangledown \cdot h}{l\bigtriangledown \cdot h}\right]_{p=p_m}$$

If $p_o = j\,\omega_o$ is one of the frequencies in the E. M. F., i. e. a root of the equation

$$\Psi\,(p) = 0$$

and if this frequency is not near any of the natural frequencies, $p_m = j\,\omega_m$, of the system, the stationary vector will be

$$Y_o = \left[\frac{g}{h}\right]_{p=p_o} \cdot \frac{2}{\omega_o} \cdot \frac{\varphi(p_o)}{\Psi'(p_o)} E_o\, e^{j\omega_o t} \;\ldots\ldots\ldots\ldots\ldots \;(9)$$

and one of the vectors of the natural frequencies

$$Y_m = \left[\frac{g}{l\nabla \cdot h}\right]_{p=p_m} \cdot \frac{1}{\omega_m} \cdot \frac{\varphi(p_o)}{\Psi(p_o)} \cdot E_o \cdot e^{-\beta_m t} e^{j\omega_m t} \;(10)$$

If the frequency ω_o is nearly equal to a certain ω_m, the vectors of these frequencies will instead be of the form

$$\left.\begin{aligned} Y_o = \left[\frac{g}{l\nabla \cdot h}\right]_{p=p_m} \cdot \frac{1}{\omega_m} \cdot \frac{1}{\beta_m + j(\omega_o - \omega_m)} \cdot \\ \cdot \frac{\varphi(p_o)}{\Psi'(p_o)} \cdot E_o \cdot e^{j\omega_o t} \end{aligned}\right\} (11)$$

$$\left.\begin{aligned} Y_m = -\left[\frac{g}{l\nabla \cdot h}\right]_{p=p_m} \cdot \frac{1}{\omega_m} \cdot \frac{1}{\beta_m + j(\omega_o - \omega_m)} \cdot \\ \cdot \frac{\varphi(p_o)}{\Psi'(p_o)} \cdot E_o \cdot e^{-\beta_m t} \cdot e^{j\omega_m t} \end{aligned}\right\} (12)$$

If ω_o is in the neighbourhood of a root

$$p = j\,\omega_s$$

of the equation

$$g\,(z_1\, z_2 \ldots) = 0$$

the formulæ for the natural oscillations Y_m of the system will be the same as when there is no resonance. But the stationary oscillation will be made much weaker. For the stationary vector we get the expression

$$y_o = 4\left[\frac{l\nabla \cdot g}{h}\right]_{p=p_s} \cdot \frac{\beta_s + j(\omega_o - \omega_s)}{\omega_s} \cdot \frac{\varphi(p_o)}{\Psi'(p_o)} \cdot E_o e^{j\omega_o t} \;(13)$$

Here we have

$$\beta_s = \frac{1}{2}\left[\frac{r\nabla \cdot g}{l\nabla \cdot g}\right]_{p=p_s}$$

The expressions $\frac{g}{h}, \frac{g}{l\nabla \cdot h}, \frac{l\nabla \cdot g}{h}$ in the above formulæ depend on the nature of the network and are homogeneous functions of $z_1, z_2 \ldots$; they are therefore either real or purely imaginary. If y is the strength of a current, $\frac{g}{h}$ will be purely imaginary and the other two expressions real. But if y is a voltage, g will be real and the last two expressions imaginary. Thus, if in computing the amplitude we retain the expressions with their signs, we must for a current introduce the phase angle $-\frac{\pi}{2}$ for $\frac{g}{h}$, and for a voltage the phase angle $\frac{\pi}{2}$ for $\frac{g}{l\nabla \cdot h}$ and $-\frac{\pi}{2}$ for $\frac{l\nabla \cdot g}{h}$. In any case it is easy to see what phase angles these expressions bring. For the sake of simplicity, however, we assume that we have determined these angles.

The expressions $\frac{\varphi(p)}{\Psi(p)}$ and $\frac{\varphi(p)}{\Psi'(p)}$ on the other hand belong to the E. M. F. and have nothing to do with the nature of the network.

If the E. M. F. is constant, we have $\varphi(p) = \Psi(p) = 1$. Here we only get natural oscillations in the system and therefore have

$$y_m = \left[\frac{g}{l\nabla \cdot h}\right]_{p=p_m} \cdot \frac{1}{\omega_m} E_o\, e^{-\beta_m t} \cdot e^{j\omega_m t} \;\ldots\ldots\; (14)$$

In this case the solution may be written

$$y = y_o + \sum_m \left[\frac{g}{l\nabla \cdot h}\right]_{p=p_m} \cdot \frac{1}{\omega_m} \cdot E_o\, e^{-\beta t_m} \cdot \\ \cdot \sin(\omega_m t + u),$$

where u is zero if y is a current, and $\frac{\pi}{2}$ if y is a voltage. y_o is the value assumed by the operator expression for y when p is made zero.

If the E. M. F. is a single impulse of size E_o, we get $\varphi(p) = p$ and $\Psi(p) = 1$.

The conditions will then be the same as above, except that y_o will vanish, ω_m will disappear from the denominator, and the phase angle u will be increased by $\frac{\pi}{2}$.

Of special interest is the case of a simple sinusoidal E. M. F. We will therefore consider this in more detail. We assume $E(t)$ to be of the form

$$E(t) = E_o \sin(\omega_o t + a_o)$$

A simple calculation then gives us

$$\varphi(p) = p^2 \sin a_o + p\,\omega_o \cos a_o$$

$$\Psi(p) = p^2 + \omega_o^2$$

$$\Psi'(p) = 2p.$$

We thus get

$$\frac{\varphi(p)}{\Psi(p)} = p \cdot \frac{[\omega_o \cos a_o + p \sin a_o]}{p^2 + \omega_o^2}$$

$$\frac{\varphi(p)}{\Psi'(p)} = \frac{1}{2}[\omega_o \cos a_o + p \sin a_o]$$

Whence

$$\frac{\varphi(p_o)}{\Psi'(p_o)} = \tfrac{1}{2}\,\omega_o e^{j a_o}$$

We further introduce the symbols

$$\sqrt{\omega_o^2 \cos^2 \alpha_o + \omega_m^2 \sin \alpha_o} = K_m$$

$$\frac{\omega_m}{\omega_o} \cdot \operatorname{tg} \alpha_o = \operatorname{tg} \alpha_m$$

With these we can now write

$$\frac{\varphi\ (p_m)}{\Psi(p_m)} = \frac{j\omega_m}{\omega_o^2 - \omega_m^2} \cdot K_m e^{j\alpha_m}$$

$$\frac{\varphi\ (p_m)}{\Psi'(p_m)} = \tfrac{1}{2} K_m\ e^{j\alpha_m}$$

Finally we have the expressions

$$\beta_m + j\ (\omega_o - \omega_m)$$

and

$$\beta_s + j\ (\omega_o - \omega_s).$$

For these two we may substitute the following

$$\sqrt{\beta_m^2 + (\omega_o - \omega_m)^2} \cdot e^{j\xi_m}$$

$$\sqrt{\beta_s^2 + (\omega_o - \omega_s)^2} \cdot e^{j\xi_s}$$

where

$$\operatorname{tg} \xi_m = \frac{\omega_o - \omega_m}{\beta_m}$$

$$\operatorname{tg} \xi_s = \frac{\omega_o - \omega_s}{\beta_s}$$

We see that at resonance ξ_m and ξ_s will both be zero, and that as ω_o increases through ω_m, these angles change from nearly $-\frac{\pi}{2}$ through zero to approximately $\frac{\pi}{2}$.

With the symbols now introduced we can put down the instantaneous solutions directly.

1. *When ω_o does not lie in the neighbourhood of any value of ω_m.*

$$y = y_o + \sum_m \left[\frac{g}{l\bigtriangledown \cdot h}\right]_{p=p_m} \cdot \frac{K_m}{\omega_o^2 - \omega_m^2} \cdot E_o\, e^{-\beta_m t}$$

$$\cdot \sin\left(\omega_m t + \alpha_m + u + \frac{\pi}{2}\right) +$$

$$+ \left[\frac{g}{h}\right]_{p=p_o} \cdot E_o \sin(\omega_o t + \alpha_o + u) \ldots\ldots\ldots\ldots\ldots \quad (16)$$

2. *When ω_o and ω_m are close to one another.*

For the stationary term and the natural oscillation which has very nearly the same angular frequency as the superimposed frequency together, we get the expression

$$\left[\frac{g}{l\bigtriangledown \cdot h}\right]_{p=p_m} \cdot \frac{1}{2} \frac{E_o}{\sqrt{\beta_m^2 + (\omega_o - \omega_m)^2}} \cdot [e^{-\beta_m t}$$

$$\sin(\omega_m t + \alpha_m + u - \xi_m) - \sin(\omega_o t + \alpha_o + u + \xi_m)] \quad (17)$$

At resonance, $\omega_o = \omega_m$, $\alpha_m = \alpha_o$, and ξ is zero.

3. *When ω_o is close to an ω_s.*

The natural oscillations will be the same as under (1) above. The stationary oscillation, however, will assume the form

$$y = 2\left[\frac{l\bigtriangledown \cdot g}{h}\right]_{p=p_s} \cdot \sqrt{\beta_s^2 + (\omega_o - \omega_s)^2} \cdot$$

$$\cdot E_o \sin(\omega_o t + \alpha_o + u + \xi_s) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

The angles u are, as we have said, either zero or $\pm\frac{\pi}{2}$ according to the expression obtained for the factor depending on the network.

We have now obtained definite formulæ for determining the oscillations in a network. To show how a calculation is made we will determine $\frac{g}{h}, \frac{g}{l\bigtriangledown \cdot h}$ etc. in a couple of definite cases.

1) *An E. M. F. is closed through an inductance connected in series with another inductance shunted by a condenser.*

The arrangement is thus as shown in fig. 1.

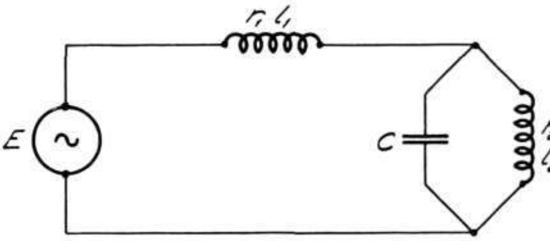


Fig. 1.

Here we have three impedances, viz.

$$z_1' = r_1 + pl_1 = r_1 + z_1$$

$$z_1' = r_2 + pl_2 = r_2 + z_2$$

$$z'_3 = \frac{1}{pc} = z_3$$

The current passing through the coil z_2 will be

$$i_2 = \frac{z_3'}{z_1' z_2' + z_3(z_1' + z_2')} \cdot E$$

Thus

$$g = z_3$$

$$h = z_1\ z_2 + z_3\ (z_1 + z_2)$$

The equation

$$h = 0$$

gives us

$$l_1 l_2 p^2 + \frac{1}{pc}[pl_1 + pl_2] = 0$$

or

$$p^2 = -\left[\frac{1}{c\,l_1} + \frac{1}{c\,l_2}\right]$$

Further we have

$$h = l_1 l_2 \left\{p^2 + \frac{1}{c\,l_1} + \frac{1}{c\,l_2}\right\}$$

$$l \nabla \cdot h = l_1 (z_2 + z_3) + l_2 (z_1 + z_3) =$$

$$= \frac{l_1 l_2}{p}\left[2\,p^2 + \frac{1}{c\,l_1} + \frac{1}{c\,l_2}\right]$$

$$r \nabla \cdot h = r_1 (z_2 + z_3) + r_2 (z_1 + z_3) =$$

$$= \frac{l_1 l_2}{p}\left[\frac{r_1}{l_1}\left(p^2 + \frac{1}{c\,l_2}\right) + \frac{r_2}{l_2}\left(p^2 + \frac{1}{c\,l_1}\right)\right]$$

We have here only one root, p^2, and therefore only one natural frequency. For the sake of brevity we introduce

$$\frac{1}{c\,l_1} = \omega_1{}^2;\ \frac{1}{c\,l_2} = \omega_2{}^2$$

Whence, if the natural frequency is represented by ω', we get

$$\omega'^2 = \omega_1{}^2 + \omega_2{}^2 = -p_1{}^2$$

If we introduce this value of the root we get

$$r \nabla \cdot h = -\frac{l_1 l_2}{p}\left[\frac{r_1}{l_1}\,\omega_1{}^2 + \frac{r_2}{l_2}\,\omega_2{}^2\right]$$

$$l \nabla \cdot h = -\frac{l_1 l_2}{p}\left[\omega_1{}^2 + \omega_2{}^2\right]$$

and the damping β' of the natural frequency

$$\beta = \frac{1}{2}\,\frac{r_1}{l_2}\,\frac{\omega_1{}^2}{\omega_1{}^2 + \omega_2{}^2} + \frac{1}{2}\,\frac{r_2}{l_2}\,\frac{\omega_2{}^2}{\omega_1{}^2 + \omega_2{}^2}$$

Further we get

$$\frac{g}{h} = \frac{1}{p\,l_1} \cdot \frac{\omega_2{}^2}{p^2 + \omega_1{}^2 + \omega_2{}^2}$$

$$\frac{g}{l \nabla \cdot h} = \frac{1}{l_1} \cdot 2\,\frac{\omega_2{}^2}{p^2 + \omega_1{}^2 + \omega_2{}^2}$$

or

$$\left[\frac{g}{h}\right]_{p=p_o} = \frac{1}{j\,\omega_o l_1} \cdot \frac{\omega_2{}^2}{\omega'^2 - \omega_o{}^2}$$

$$\left[\frac{g}{l \nabla \cdot h}\right]_{p=p_1} = -\frac{1}{l_1} \cdot \frac{\omega_2{}^2}{\omega'^2}$$

If we have a sinusoidal E. M. F. we get no constant term, as $\varphi\ (p)$ becomes zero when $p = o$. If we denote the angle a_m by a', that is

$$\text{tg}\ a' = \frac{\omega'}{\omega_o}\ \text{tg}\ a,$$

the angle ξ_m by ξ and K_m by K,

$$\text{tg}\ \xi = \frac{\omega_o - \omega'}{\beta}$$

$$K = \sqrt{\omega_o{}^2 \cos^2 a_o + \omega'^2 \sin^2 a_o}$$

we get the solution

$$i_2 = \frac{1}{l_1} \cdot \frac{\omega_2{}^2}{\omega'^2} \cdot \frac{K}{\omega'^2 - \omega_o{}^2} \cdot E_o\,e^{-\beta' t} \cos\,(\omega t + a') -$$

$$- \frac{1}{\omega_o l_1} \cdot \frac{\omega_2{}^2}{\omega'^2 - \omega_o{}^2} \cdot E_o \cos\,(\omega_o t + a_o)$$

This formula applies when we are far from resonance. Near the resonance point we have instead

$$i_2 = \frac{1}{2} \cdot \frac{l}{l_1} \cdot \frac{\omega_2{}^2}{\omega'^2} \cdot \frac{E_o}{\sqrt{\beta'^2 + (\omega_o - \omega')^2}}\ [\sin\,(\omega_o t + a_o - \xi) -$$

$$- e^{-\beta' t} \cdot K \cdot \sin\,(\omega' t + a' - \xi)]$$

We will take another example.

An E. M. F. is closed through an oscillating circuit inductively coupled to another oscillating circuit.

The arrangement is shown in fig. 2.

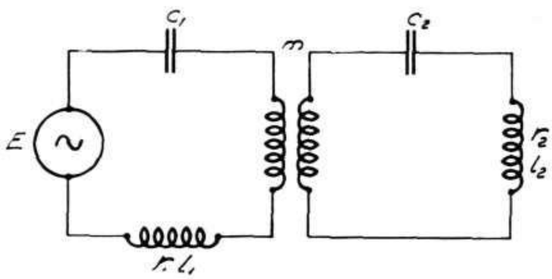


Fig. 2.

We will determine the current in the primary circuit.

We introduce the following

$$z_1 = p l_1 + \frac{1}{p c_1} = l_1 \cdot \frac{p^2 + \omega_1{}^2}{p}$$

$$z_2 = p l_2 + \frac{1}{p c_2} = l_2 \cdot \frac{p^2 + \omega_2{}^2}{p}$$

$$z_3 = pm$$

thus

$$\omega_1{}^2 = \frac{1}{c_1 l_1};\ \omega_2{}^2 = \frac{1}{c_2 l_2}$$

If we neglect the resistances we have the pair of equations:

$$\left\{\begin{array}{l} E_o = z_1 i_1 + z_3 i_2 \\ \ \ 0 = z_3 i_1 + z_2 i_2 \end{array}\right\}$$

whence

$$i_1 = \frac{z_2}{z_1 z_2 - z_3^2} \cdot E.$$

We thus have here

$$g = z_2$$
$$h = z_1 z_2 - z_3^2$$

We introduce

$$\varepsilon^2 = \frac{m^2}{l_1 l_2}$$

The equation

$$h = 0$$

can then be written

$$(p^2 + \omega_1^2)(p^2 + \omega_2^2) - \varepsilon^2 p^4 = 0$$

If, therefore, we represent the natural frequencies by ω' and ω'', these will be given by the equation

$$(\omega_1^2 - \omega^2) \cdot (\omega_2^2 - \omega^2) - \varepsilon^2 \omega^4 = 0$$

This equation shows that ω' and ω'' fall one on each side of the values between ω_1 and ω_2. Thus one, e. g. ω', will be less than either of the natural frequencies of the circuit, and the other, ω'', greater than either.

We now have

$$r \bigtriangledown \cdot h = r_1 z_2 + r_2 z_1$$
$$l \bigtriangledown \cdot h = l_1 z_2 + l_2 z_1 - m \cdot 2 z_3$$
$$r \bigtriangledown \cdot g = r_2$$
$$l \bigtriangledown \cdot g = l_2$$

We thus get the dampings

$$\beta' = \frac{1}{2}\left[\frac{r \bigtriangledown \cdot h}{l \bigtriangledown \cdot h}\right]_{p=p_1} = \frac{1}{2}\,\frac{r_1 z_2 + r_2 z_1}{l_1 z_2 + l_2 z_1 - m \cdot 2 z_3}$$
$$= \frac{1}{2}\,\frac{r_1 l_2 (p^2 + \omega_2^2) + r_2 l_1 (p^2 + \omega_1^2)}{l_1 l_2 [2 p^2 + \omega_1^2 + \omega_2^2 - 2 \varepsilon^2 p^2]}$$

or

$$\beta' = \frac{\frac{r_1}{2 l_1}(\omega_2^2 - \omega'^2) + \frac{r_2}{2 l_2}(\omega_1^2 - \omega'^2)}{\omega_1^2 + \omega_2^2 - 2(1 - \varepsilon^2)\omega'^2}$$

β'' is obtained by replacing ω' and by ω''.

The equation

$$g = 0$$

has one root p_s

$$p_s^2 + \omega_2^2 = 0$$

We thus get the corresponding ω_s:

$$\omega_s = \omega_2$$

and the damping β_s:

$$\beta_s = \frac{r_2}{2 l_2} = \beta_2$$

Further we have

$$\frac{g}{h} = \frac{(p^2 + \omega_2^2)\, p}{l_1 [p^4 (1 - \varepsilon^2) + p^2 (\omega_1^2 + \omega_2^2) + \omega_1^2 \omega_2^2]}$$

$$\frac{g}{l \bigtriangledown \cdot h} = \frac{p^2 + \omega_2^2}{l_1 [2 p^2 (1 - \varepsilon^2) + \omega_1^2 + \omega_2^2]}$$

$$\frac{l \bigtriangledown \cdot g}{h} = \frac{p^2}{l_1 [p^4 (1 - \varepsilon^2) + p^2 (\omega_1^2 + \omega_2^2) + \omega_1^2 \omega_2^2]}$$

By substitution in these expressions we get

$$\left[\frac{g}{h}\right]_{p=p_o} = \frac{j\omega_o (\omega_2^2 - \omega_o^2)}{l_1 [\omega_o^4 (1 - \varepsilon^2) - \omega_o^2 (\omega_1^2 + \omega_2^2) + \omega_1^2 \omega_2^2]};$$

the angle u is thus here equal to $\frac{\pi}{2}$.

$$\left[\frac{g}{l \bigtriangledown \cdot h}\right]_{p=p_1} = \frac{\omega_2^2 - \omega'^2}{l_1 [\omega_1^2 + \omega_2^2 - 2\omega'^2 (1 - \varepsilon^2)]};$$

the angle u is thus zero.

$$\left[\frac{l \bigtriangledown \cdot g}{h}\right]_{p=p_s} = \frac{1}{l_1 \varepsilon^2 \omega_2^2}$$

the angle $u = o$.

All we have to do now is to introduce these quantities in our general solutions (9), (10), (11), (12) and (13) or, if the E. M. F. is constant, in (15) and, if it is sinusoidal, in (16), (17), and (18).

As a check we will, however, make the calculations on the assumption that $\omega_1 = \omega_2 = \omega$.

The equation $h = o$ will then give us

$$\omega' = \frac{\omega}{\sqrt{1 + \varepsilon}}$$

$$\omega'' = \frac{\omega}{\sqrt{1 - \varepsilon}}$$

Further we have

$$\omega_s = \omega$$

$$\left[\frac{g}{h}\right]_{p=p_o} = \frac{j\omega_o (\omega^2 - \omega_o^2)}{l_1 [\omega_o^2 - (1 + \varepsilon)\omega^2][\omega_o^2 - (1 - \varepsilon)\omega^2]}$$

The above formulæ give us

$$\left[\frac{g}{l \bigtriangledown \cdot h}\right]_{p=p_1} = \frac{1}{2 l_1 (1 + \varepsilon)}$$

$$\left[\frac{g}{l \bigtriangledown \cdot h}\right]_{p=p_2} = \frac{1}{2 l_1 (1 - \varepsilon)}$$

$$\left[\frac{l \bigtriangledown \cdot g}{h}\right]_{p=p_s} = \frac{1}{l_1 \varepsilon^2 \omega^2}$$

$$\beta' = \left(\frac{r_1}{2\,l_1} + \frac{r_2}{2\,l_2}\right) \cdot \frac{1}{2\,(1+\varepsilon)}$$

$$\beta'' = \left(\frac{r_1}{2\,l_1} + \frac{r_2}{2\,l_2}\right) \cdot \frac{1}{2\,(1-\varepsilon)}$$

Thus for a constant E. M. F. we get the solution

$$i_1 = \frac{E_o}{2\,\omega l_1}\left[\frac{e^{-\beta' t}}{\sqrt{1+\varepsilon}} \sin \omega' t + \frac{e^{-\beta'' t}}{\sqrt{1-\varepsilon}} \sin \omega'' t\right]$$

For a sinusoidal E. M. F. of the type

$$E = E_o \sin\,(\omega_o t + a_o)$$

we get, when away from the resonance point

$$i_1 = \frac{1}{2\,l_1\,(1+\varepsilon)} \cdot \frac{K'}{\omega_o^2 - \omega'^2} \cdot E_o \cdot e^{-\beta' t} \cos\,(\omega' t + a') +$$

$$+ \frac{1}{2\,l_1\,(1-\varepsilon)} \cdot \frac{K''}{\omega_o^2 - \omega''^2} \cdot E_o \cdot e^{-\beta'' t} \cos\,(\omega'' t + a'') +$$

$$+ \frac{\omega_o\,(\omega^2 - \omega_o^2)}{l_1\,[\omega_o^2 - (1+\varepsilon)\,\omega^2]\,[\omega_o^2 - (1-\varepsilon)\,\omega^2]} \cdot E_o \cos(\omega_o t + a_o)$$

$$K' = \sqrt{\omega_o^2 \cos^2 a_o + \omega'^2 \sin^2 a_o}$$

$$K'' = \sqrt{\omega_o^2 \cos^2 a_o + \omega''^2 \sin^2 a_o}$$

$$\operatorname{tg} a' = \frac{\omega'}{\omega_o} \operatorname{tg} a_o$$

$$\operatorname{tg} a'' = \frac{\omega''}{\omega_o} \operatorname{tg} a_o$$

Near the resonance point $\omega_o = \omega'$ we get

$$i_1 = \frac{1}{4\,l_1\,(1+\varepsilon)} \cdot \frac{E_o}{\sqrt{\beta'^2 + (\omega_o - \omega')^2}} \cdot$$

$$\cdot \left[e^{-\beta' t} \sin\,(\omega' t + a' - \xi') - \sin\,(\omega_o t + a_o - \xi')\right]$$

$$+ \frac{1}{2\,l_1\,(1-\varepsilon)} \cdot \frac{K''}{\omega_o^2 - \omega''^2} \cdot E_o \cdot e^{-\beta'' t} \cos\,(\omega'' t + a'')$$

$$\operatorname{tg} \xi' = \frac{\omega_o - \omega'}{\beta'}$$

Finally, close to where $\omega_o = \omega$ we get

$$i_1 = \frac{1}{2\,l_1\,(1+\varepsilon)} \cdot \frac{K'}{\omega_o^2 - \omega'^2} \cdot E_o \cdot e^{-\beta' t} \cos\,(\omega' t + a')$$

$$+ \frac{1}{2\,l_1\,(1-\varepsilon)} \cdot \frac{K''}{\omega_o^2 - \omega''^2} \cdot E_o \cdot e^{-\beta'' t} \cos\,(\omega'' t + a'')$$

$$+ \frac{2}{l_1\,\varepsilon^2\,\omega^2} \cdot \sqrt{\beta^2 + (\omega_o - \omega)^2} \cdot E_o \sin\,(\omega_o t + a_o + \xi)$$

where

$$\beta = \frac{r_2}{2\,l_2}$$

$$\operatorname{tg} \xi = \frac{\omega_o - \omega}{\beta}$$

EXCERPT
SR 51 E

WORKING RELIABILITY AND MAINTENANCE IN THE L. M. ERICSSON AUTOMATIC TELEPHONE SYSTEM

Telefonaktiebolaget
L. M. Ericsson
Communication from the
Research and Development Department
Kungsgatan 33, Stockholm

SR 51 E (VII. 32)
500

Working Reliability and Maintenance in the L. M. Ericsson Automatic Telephone System.

By *A. Lignell.*

On January 1st 1932 the Telephone System in Stockholm proper comprised 115 208 subscribers' lines, with 138 758 instruments, and of these 63 870 lines and 72 134 instruments were connected to automatic exchanges.

At the beginning of March this year about 20 000 subscribers' lines will be switched over to the new "Södra Vasa" automatic exchange, fitted for 30 000 numbers, and in April 1933 about 17 000 suscribers will be switched to the "Östermalm" exchange, which will be fitted for 25 000 numbers and will replace the present two manual exchanges in Östermalm.

The "Norr", with just over 10 000 subscribers, will then be the only manual exchange left, and will be changed to automatic as soon as possible. In this case the delay must be ascribed entirely to the building question not being settled yet.

During the period of transition to automatic working, traffic from manual to automatic exchanges is directed via digit key strips centralized in the Norr exchange, and from automatic to manual exchanges via Call Display positions.

The conversion to automatic working in the four automatic exchanges so far taken into use —with a present aggregate capacity of 91 000 numbers and 63 870 connected subscribers' lines —has been carried out most satisfactorily, work throughout the network continuing as usual without being disturbed by the change-over.

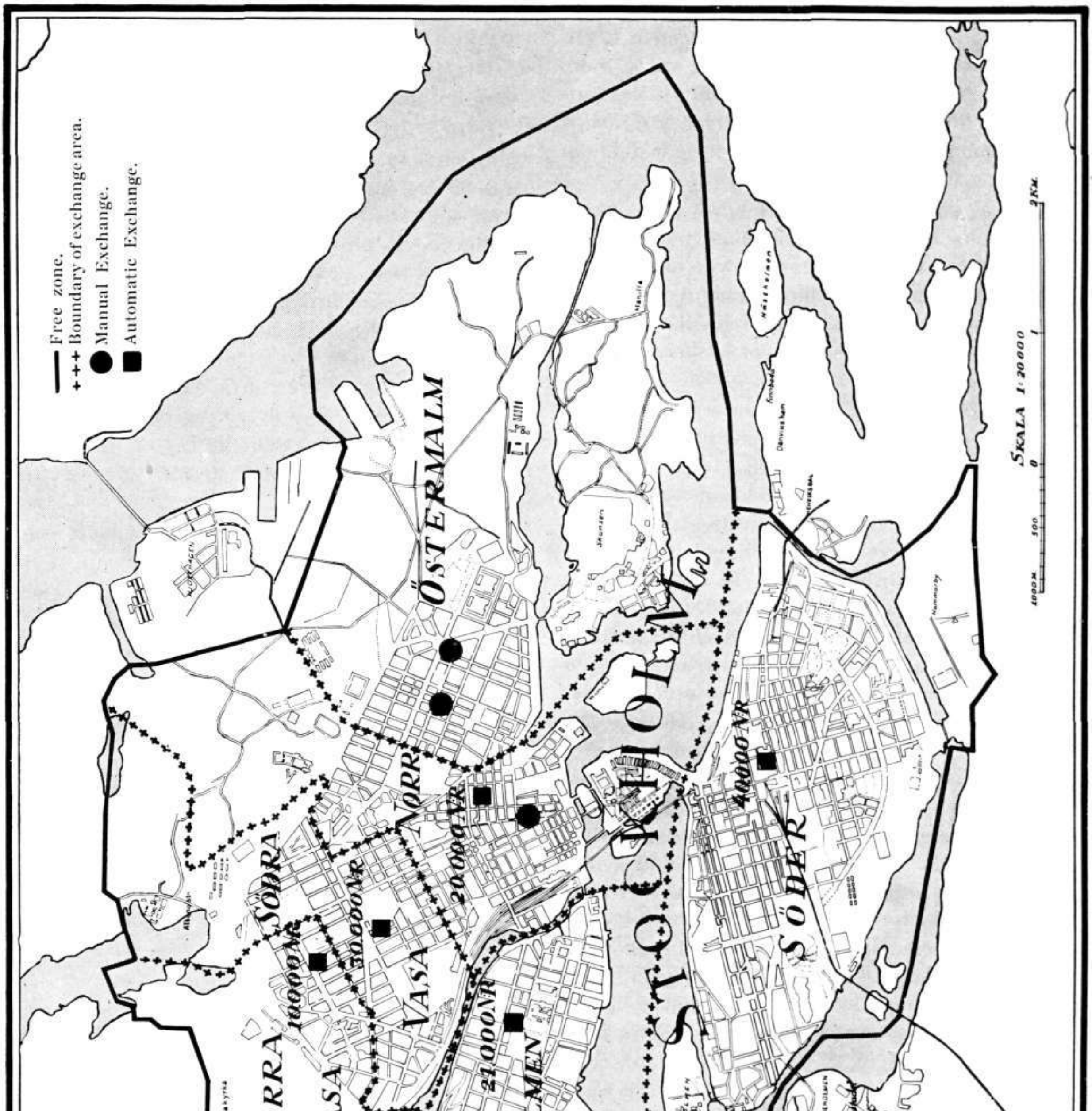
The first automatic exchange, "Norra Vasa", now equipped for 10 000 numbers, was opened in January 1924. This exchange was allowed to run for a little over 4 years before the next one, "Kungsholmen", was taken into use in June 1928, equipped for 21 000 numbers.

In the interval, the reliability of the system was subjected to extremely careful control, and the results were entirely good. "Centralen", fitted for 20 000 numbers, followed in January 1929, and "Söder", with 40 000 numbers, in April 1931.

The automatization of the telephones in the environs of Stockholm is at present being planned, the area suggested being that lying within a radius of about 15 km. of Stockholm (62 exchanges with 27 800 subscribers' lines); the intentions is to include these subscribers, via 52 tellite exchanges, in the Stockholm series of 6-figure numbers.

The control of reliability in manual telephone systems has always been concentrated on the actual traffic, as this is the only way to judge the real efficiency of the working, and what share the operators, the public, and the equipment have in the undisturbed running of the traffic. From the very first this control has also been in use in the automatic exchanges. It is partly continuous, a checking of the calls made by subscribers, and partly, parallel with this, occasional, cutting in on a connexion whenever a special register lamp lights up. This happens when the register is held too long or when a fault, whether caused by a subscriber or by the technical devices, occurs in the connexion. In such a case it is necessary to find out if the fault is due to wrong manipulation by the subscriber, to the automatic devices, or to other technical deficiencies. If the subscriber has made a mistake, he is put right, otherwise the connexion is locked and the fault traced and remedied. The same method is used for the continuous control. A detailed description of the way this control is arranged was given in the L. M. Ericsson Review No. 4—6, 1929, in the article "The Continued Automatization of the Stockholm Telephone Net".

As, however, from the point of view of maintenance, it was desirable to have unmistakable evidence of the advantage of continuous control in



Working Reliability and Maintenance in the L. M. Ericsson Automatic Telephone System.

the automatic system, in which the control of the operators necessary in the manual system is eliminated, the control staff of "Norra Vasa" were ordered, from April 17th 1930 until further notice, not to report to the Central Fault Office—for tracing and reparing—any technical faults occurring in the course of the control.

The reporting of faults was, however, resumed as early as June 10th of the same year, as this period of less than two months was sufficient to prove the great importance, from the point of view of maintenance also, of reliability control.

The result of this experiment is shown in diagram I.

The highest, lowest, and average percentage of losses per month during the period Jan. 1st 1929 to April 30th 1930, i. e. during 16 months, are indicated by the dotted lines and amount to 0.21, 0.07 and 0.16 per cent. respectively. It should be noted that this percentage of losses includes calls lost not only through faults in the automatic system of this exchange, but also through faults in other exchanges (manual and automatic), as well as any faults in lines or instruments of a nature permitting connexion to the exchange register. The curve shows a loss of 0.11 per cent. at the time when the reporting of faults stopped. When the reporting was resumed, this percentage had gone up to 0.99 and continued to rise for some time, until increased use of the red register lamps and the resulting sorting out and remedying of the faults effected a gradual return to normal conditions.

The loss percentage due to technical faults rose during the month of May, among 6 339 controlled calls, to 0.38, and during June (4 608 calls) to 0.85, but dropped during July, when faults were again reported, to 0.25 per cent. in 4 387 controlled calls. The average for the rest of the year, August to December, was 0.17 per cent. in 26 163 controlled calls.

Complaints from subscribers also increased considerably while the fault reports were inhibited. From January 1st to April 30th their monthly average was 0.74 per 10 000 calls. The highest proportion of complaints in a month during this time was 0.79 per 10 000 calls, the lowest 0.66.

With the cessation of the fault reports the number of complaints per 10 000 calls rose, as is shown by diagram II, and was in the middle of June about 40 per cent. higher than the January to April average.

The 1929 figures, on the other hand, are the same as this last average.

During 1931 the average number of subscribers connected to the 4 automatic exchanges in use was 60 794 out of a total capacity of 91 000 line numbers. The total number of faults in the automatic system put right during the year in these exchanges was 3 537. Of these 1 025, or 29 per cent., were remedied through the action of the reliability control, which will suffice to prove its importance. Not only does it afford a true picture of the working of the traffic but—provided the control is extended to every register—it also allows, in conjunction with the use of the special register lamps mentioned above, an effective cleaning up of faults in the plant.

Table 1 shows the result of the reliability control during 1931 at the "Centralen" automatic exchange, built for 20 000 numbers and opened in January 1929.

During 1931 this exchange had on an average

14 944 subscribers' lines connected, with a call frequency of
11.20 calls per subscriber per day (8 a.m. — 9 p.m.), and
1.30 calls per subscriber per busy hour, with a maximum of
1.66 calls per subscriber per busy hour;
35 706 controlled calls, of which

	98.71 %	were faultless (8.36 % engaged and 6.97 % no reply)
	1.12 %	were faults due to the subscriber
	0.03 %	were faults due to operators in manual exchanges
	0.14 %	were faults due to the technical equipment
Total	100.00 %	

Of the technical faults

	0.03 %	were traced to own exchange and remedied,
	0.01 %	were traced to another automatic exchange and remedied,
	0.10 %	were traced to another, manual, exchange
Total	0.14 %	

As it may be of interest to see the corresponding results for 1931 for our oldest automatic exchange, "Norra Vasa", which has been working 8 years, these are given in Table 2.

This exchange had an average of

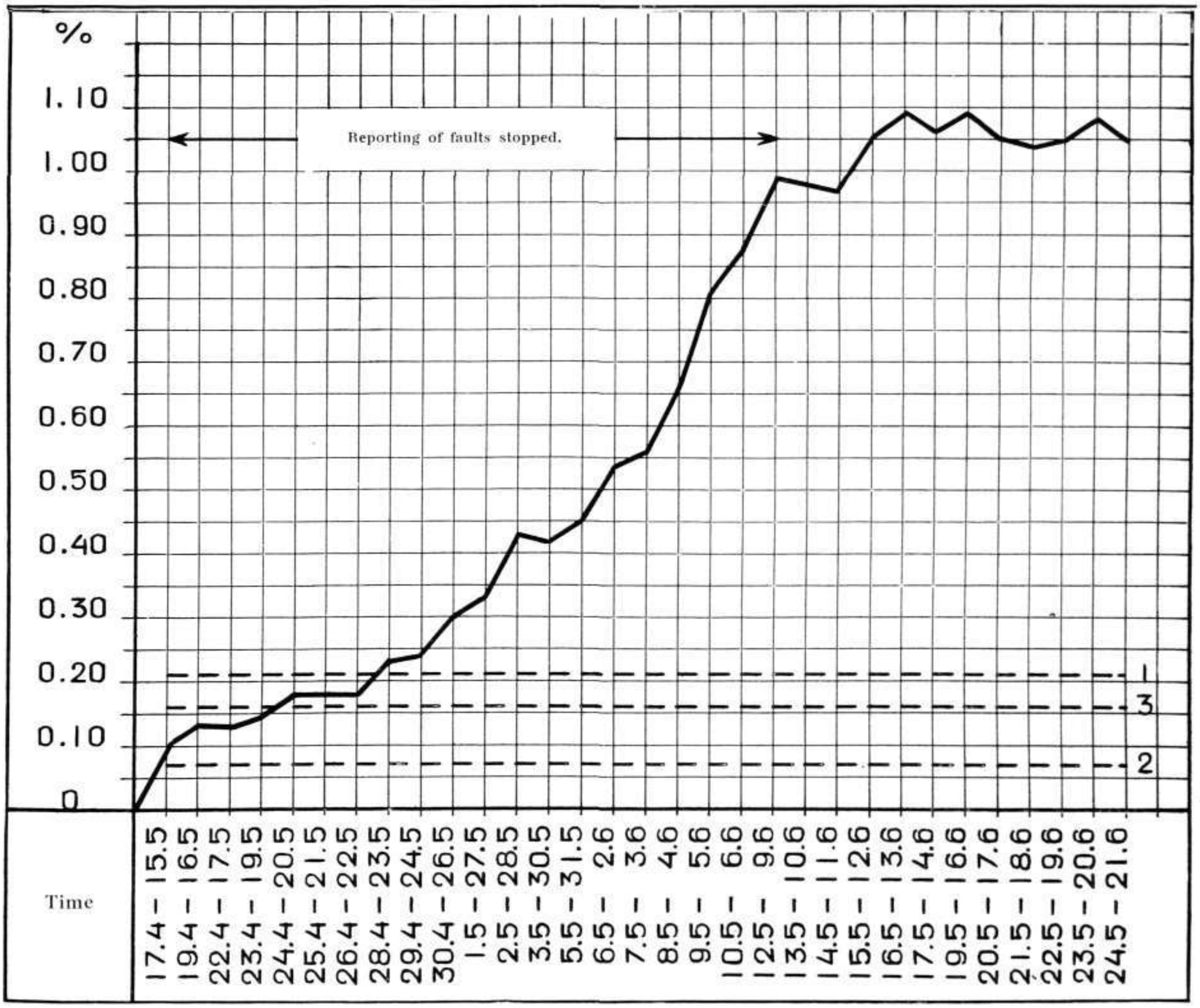
7 762 subscribers' lines connected during the year, with a call frequency of
5.60 calls per subscriber per day (8 a.m. — 9 p.m.) and

DIAGRAM I.

Norra Vasa Automatic Exchange 1930.

The curve shows the percentage of losses caused by technical faults during given periods of time (According to the reliability control.)

The reporting of faults from the traffic control to the Central Fault Office was stopped between 17/4—9/6.



1. Highest precentage of loss per month from 1/1 29 to 30/4 30
3. Average „ „ „ „ „ „ „ „ „ „ „
2. Lowest „ „ „ „ „ „ „ „ „ „ „

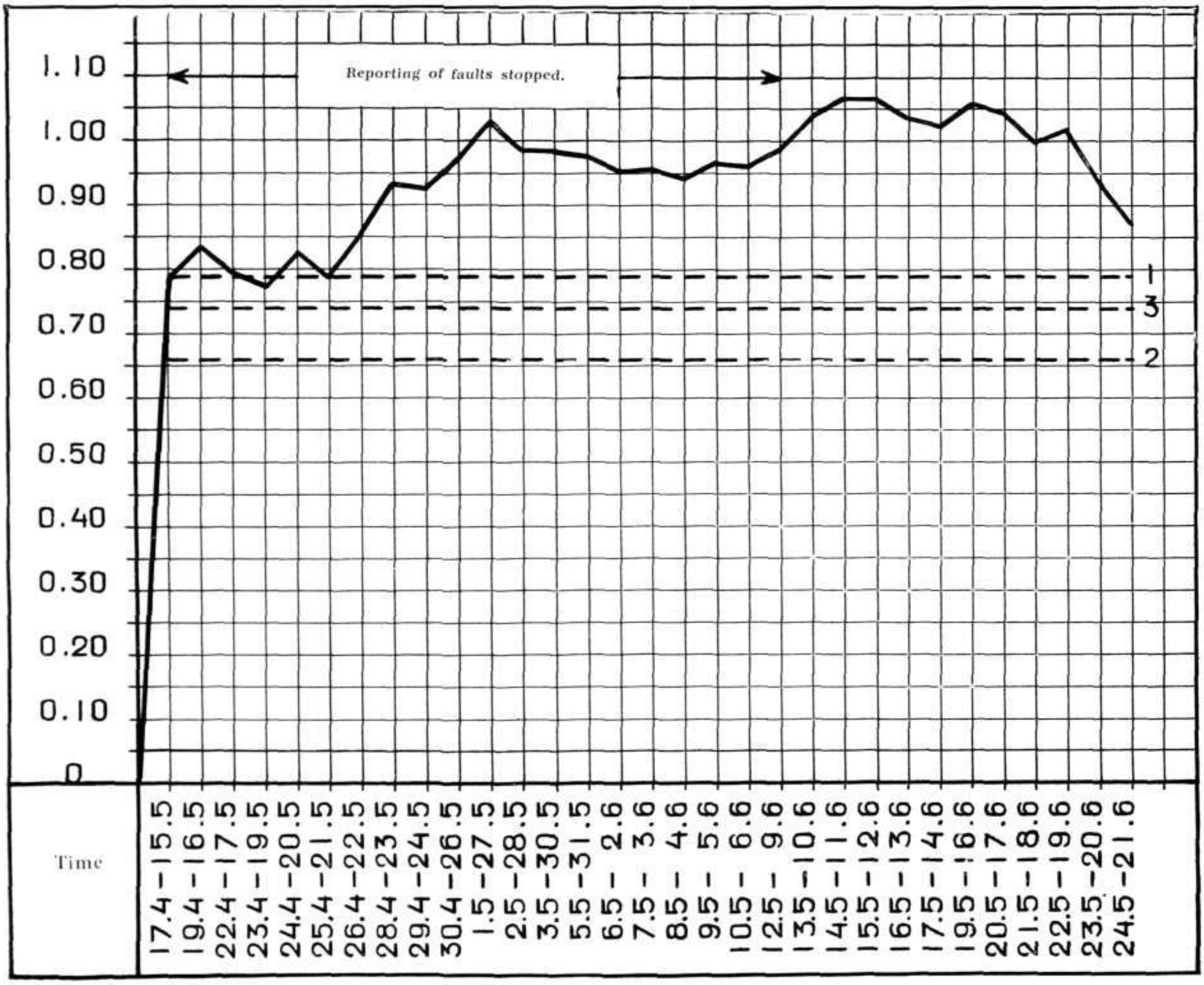
DIAGRAM II.

Norra Vasa Automatic Exchange 1930.

Number of complaints per 10 000 calls.

The curve shows the number of complaints per 10 000 calls from subscribers in Norra Vasa during given periods of time.

The reporting of faults from the traffic control to the Central Fault Office was stopped between 17/4—9/6.



1. Highest number of complaints per 10 000 calls between 1/1 30 and 30/4 30
3. Average " " " " " " " " " " " "
2. Lowest " " " " " " " " " " " "

Table 1.

Reliability Control.

AUTOMATIC EXCHANGE: "CENTRALEN". STOCKHOLM TELEPHONE EXCHANGE 1931.

Month		Total number of controlled calls	Switch-over to white register lamp. Faultless: Calls put through	Faultless: Changed, Vakant, or cut off Nos.	Faultless: No reply	Faultless: Engaged	Total faultless	Total faults by subscriber	Total faults by operators	Faults in technical equipment: Total	of which located in: own exchange	another exchange	lines	instruments	Total traced	Total not traced
APRIL	Number	2 619	2 111	23	202	265	2 601	14	2	2	1	—	—	—	1	1
	%	—	80,60	0,88	7,71	10,12	99,31	0,53	0,08	0,08	0,04	—	—	—	0,04	0,04
MAY	Number	3 100	2 557	7	224	270	3 058	34	—	8	2	1	—	—	3	5
	%	—	82,48	0,23	7,23	8,71	98,65	1,10	—	0,25	0,06	0,03	—	—	0,09	0,16
JUNE	Number	3 467	2 897	10	268	260	3 435	26	1	5	2	—	—	—	2	3
	%	—	83,56	0,29	7,73	7,50	99,08	0,75	0,03	0,14	0,06	—	—	—	0,06	0,08
JULY	Number	5 047	4 127	23	444	389	4 983	55	—	9	1	1	—	—	2	7
	%	—	81,77	0,45	8,80	7,71	98,73	1,09	—	0,18	0,02	0,02	—	—	0,04	0,14
AUGUST	Number	4 907	4 111	20	355	370	4 856	48	1	2	1	—	—	—	1	1
	%	—	83,78	0,41	7,23	7,54	98,96	0,98	0,02	0,04	0,02	—	—	—	0,02	0,02
SEPTEMBER	Number	5 542	4 618	21	355	468	5 462	74	1	5	3	—	—	—	3	2
	%	—	83,33	0,38	6,41	8,44	98,56	1,33	0,02	0,09	0,05	—	—	—	0,05	0,04
OCTOBER	Number	4 453	3 647	28	288	412	4 375	70	—	8	2	1	—	—	3	5
	%	—	81,90	0,63	6,47	9,25	98,25	1,57	—	0,18	0,05	0,02	—	—	0,07	0,11
NOVEMBER	Number	3 374	2 839	19	177	293	3 328	41	3	2	—	—	—	—	—	2
	%	—	84,14	0,56	5,25	8,68	98,63	1,22	0,09	0,06	—	—	—	—	—	0,06
DECEMBER	Number	3 197	2 699	13	178	258	3 148	39	1	9	1	—	—	—	1	8
	%	—	84,42	0,41	5,57	8,07	98,47	1,22	0,03	0,28	0,03	—	—	—	0,03	0,25
TOTAL	Number	35 706	29 606	164	2 491	2 985	35 246	401	9	50	13	3	—	—	16	34
	%	—	82,92	0,46	6,97	8,36	98,71	1,12	0,03	0,14	0,03	0,01	—	—	0,04	0,10

0.60	calls per subscriber per busy hour, with a maximum of
0.84	calls per subscriber per busy hour.
62.834	controlled calls, of which
96.37 %	were faultless (8.63% engaged and 8.15% no reply)
3.45 %	were faults comitted by subscribers (This high figure was due to the majority of telephones being residential and much used by the children of the families).
0.05 %	were faults due to operators in manual exchanges
0.13 %	were faults in the technical equipment
Total 100.00 %	

Of the technical faults

0.06 %	were traced to own exchange and remedied,
0.02 %	were traced to another automatic exchange and remedied,
0.05 %	were traced to another, manual, exchange
Total 0.13 %	

The losses in the traffic from these two exchanges on account of technical faults originating *not only in the exchange equipment but partly* also, as mentioned above, in subscribers' lines and subscribers' instruments, thus amounted to 133 in 98 540 controlled calls, or 0.135 per cent. i. e. 13.5 lost calls in 10 000. It should be observed, that in the Ericsson system there are no additional losses caused by there being too few connecting devices, as this only lengthens the time taken for getting the connexion. The reliability can thus be regarded as particularly good.

Maintenance.

To illustrate the maintenance costs, tables are attached giving these for 1931 at the 4 automatic exchanges then in use, of which

"Centralen"	has been working since	January	1929
"Norra Vasa"	„ „ „ „	„	1924
"Kungsholmen"	„ „ „ „	June	1928
"Söder"	„ „ „ fully „	July	1931

These tables show that "Centralen", which has the highest call frequency, employs 2.23 male and 0.42 female working hours per subscriber per year for the upkeep of not only the actual automatic machinery but also the trunk junction board—which is also used for inquiries and information about vacant numbers—and the main distribution frame. Work in the Central Fault Office and power plant of the exchange, night duty, and deputizing for holiday-making or sick members of the staff are also included in these working hours.

For "Kungsholmen" the corresponding figures are 1.73 and 0.35, and for "Norra Vasa" 2.51 male and 0.30 female working hours.

That the "Norra Vasa" figures are slightly *higher is not due to a higher frequency of faults* —that is actually lower in this exchange—but to the smaller number of subscribers and the necessity of keeping a certain minimum staff on permanent watch duty.

The number of faults *remedied in the automatic* equipment per 10 000 calls was

at "Centralen" 0.30,
at "Kungsholmen" 0.41,
at "Söder" .... 0.34, and
at "Norra Vasa" .... 0.15.

The lower figure in the last exchange is the result of manual B-boards still being used for the traffic from manual exchanges.

The number of faults per day in the automatic equipment was

at "Centralen" 4.32
at "Kungsholmen" 2.60
at "Söder" 4.33

For the main distribution frames, power plants, and fault bureaux, and for fault-repairing and maintenance—that is to say for work outside the selector rooms as well—in the 4 automatic exchanges at present working, with an aggregate capacity of 91 000 line numbers and an average *of 60 794 connected subscribers*, 42 male and 8 female workers were required during 1931, or 0.69 male and 0.13 female workers per 1 000 subscribers per year.

A few words may be needed in explanation of *the fact that the upkeep of trunk junction boards* is included in the maintenance costs of automatic exchanges. There are several reasons why in Sweden we have gone in for manual junction boards for trunk traffic instead of using the auto*matic equipment for connecting the subscribers* in trunk calls.

For one thing, this method makes it possible to leave the subscriber free for local traffic during the time that his line is blocked for trunk calls, *allowing the trunk operator to put off the breaking*, if this is necessary, until the trunk call is connected; for another, short-distance trunk calls can be broken for the more difficult long-distance

Table 2.

Reliability Control.

AUTOMATIC EXCHANCE: "NORRA VASA." STOCKHOLM TELEPHONE EXCHANGE 1931.

Month		Total number of controlled calls	Faultless					Total faults by subscriber	Total faults by operators	Faults in technical equipment						
			Calls put through	Changed. Vacant, or cut off Nos.	No reply	Engaged	Total faultless			Total	of which located in					Total not traced
											own exchange	another exchange	lines	instru-ments	Total traced	
JANUARY	Number	5 318	4 321	34	304	456	5 115	190	6	7	4	—	—	1	5	2
	%	—	81,25	0,64	5,72	8,57	96,18	3,57	0,11	0,14	0,08	—	—	0,02	0,10	0,04
FEBRUARY	Number	5 439	4 404	21	347	468	5 240	185	3	11	3	6	1	1	11	—
	%	—	80,97	0,39	6,38	8,61	96,35	3,40	0,05	0,20	0,05	0,11	0,02	0,02	0,20	—
MARCH	Number	5 148	4 183	23	335	421	4 962	175	2	9	5	2	—	—	7	2
	%	—	81,26	0,43	6,51	8,18	96,38	3,40	0,04	0,18	0,10	0,04	—	—	0,14	0,04
APRIL	Number	4 380	3 464	24	344	399	4 231	141	4	4	3	—	—	—	3	1
	%	—	79,09	0,55	7,85	9,11	96,60	3,22	0,09	0,09	0,07	—	—	—	0,07	0,02
MAY	Number	4 855	3 877	22	379	436	4 714	136	2	3	2	—	—	—	2	1
	%	—	79,86	0,45	7,81	8,98	97,10	2,80	0,04	0,06	0,04	—	—	—	0,04	0,02
JUNE	Number	4 933	3 798	20	460	464	4 742	175	7	9	2	—	—	—	2	7
	%	—	77,00	0,40	9,32	9,41	96,13	3,55	0,14	0,18	0,04	—	—	—	0,04	0,14
JULY	Number	6 364	4 820	19	774	519	6 132	226	—	6	2	—	—	—	2	4
	%	—	75,74	0,30	12,16	8,16	96,36	3,55	—	0,09	0,03	—	—	—	0,03	0,06
AUGUST	Number	5 636	4 233	20	673	498	5 424	203	—	9	5	1	—	—	6	3
	%	—	75,11	0,35	11,94	8,84	96,24	3,60	—	0,16	0,09	0,02	—	—	0,11	0,05
SEPTEMBER	Number	5 714	4 458	11	504	508	5 481	226	—	7	1	—	—	—	1	6
	%	—	78,02	0,19	8,82	8,89	95,92	3,96	—	0,12	0,02	—	—	—	0,02	0,10
OCTOBER	Number	5 554	4 414	36	412	477	5 339	210	3	2	—	—	—	—	—	2
	%	—	79,47	0,65	7,42	8,59	96,13	3,78	0,05	0,04	—	—	—	—	—	0,04
NOVEMBER	Number	5 145	4 148	38	347	426	4 959	176	2	8	5	—	—	—	5	3
	%	—	80,62	0,74	6,74	8,28	96,38	3,42	0,04	0,16	0,10	—	—	—	0,10	0,06
DECEMBER	Number	4 348	3 587	35	246	348	4 216	121	3	8	4	1	—	—	5	3
	%	—	82,50	0,81	5,66	8,00	96,97	2,78	0,07	0,18	0,09	0,02	—	—	0,11	0,07
TOTAL	Number	62 834	49 707	303	5 125	5 420	60 555	2 164	32	83	36	10	1	2	49	34
	%	—	79,11	0,48	8,15	8,63	96,37	3,45	0,05	0,13	0,06	0,02	0,00	0,00	0,08	0,05

Table 3.

Maintenance costs in "Centralen" Automatic Exchange during 1931.

The exchange equipped for 20 000.

Average number of subscribers' lines connected during the year	14 944
Number of outgoing calls during the year	51 872 910 [1])
" " " " per subscriber per year	3 471
" " " " " " " weekday from 8 a.m.—9 p.m.	11.2
" " " " " " " busy hour	1.3 (max. 1.66)

Cost of labour, including night duty, holidays, and sick leave				Cost of materials		Total maintenance cost	Average per connected subscriber per year				
No. of staff for		Hours	Kr.	In the automatic equipment Kr.	Outside the automatic equipment Kr.	Kr.	Hours	Cost of labour Kr.	Cost of material: In the automatic equipment Kr.	Cost of material: Outside the automatic equipm. Kr.	Total maintenance cost Kr.
Main distribution frames and Fault Office	3	7 350	12 242	—	—	—	0,49	0,82	—	—	—
Selector rooms (connecting devices, junction-boards, exchange equipment for subscribers' lines, and power plants)	10	25 958	43 163	—	—	—	1,74	2,89	—	—	—
Cleaning of the automatic devices (female labour)	3	6 321	6 190	—	—	—	0,42	0,41	—	—	—
Charring and floors (female labour)	2	2 493	1 994	—	—	—	0,17	0,13	—	—	—
Total	18	42 122	63 589	5 435	8 789	77 813	2,82	4,25	0,36	0,59	5,20

Faults remedied during the year in	Number	Per subscribers' line connected	Per 10 000 calls
1. automatic equipment	1579	0,10	0,30
2. exchange equipment outside the automatic system [2])	1568	0,10	—
Total	3147	0,20	

[1]) **In addition there were 4 696 996 connexions** made on the trunk junction board, of which 3 433 099 were for trunk calls and 157 391 were for telephoned telegrams and the rest for "Förmedlings" Bureau, Telephone queue, Supervisor's desk, and information about changed and vacant numbers.

[2]) Main distribution frames, current protection, multiple and service equipment of the trunk junction board.

Table 4.

Maintenance Costs in "Kungsholmen" Automatic Exchange during 1931.

The exchange equipped for 21 000

Average number of subscribers' lines connected during the year	12 834
Number of outgoing calls during the year	23 325 540 [1])
" " " " per subscriber per year	1 817
" " " " " " " weekday from 8 a.m.—9 p.m.	5.5
" " " " " " " busy hour	0.56 (max. 0.84)

Cost of labour, including night duty, holidays, and sick leave.				Cost of materials		Total maintenance cost	Average per connected subscriber per year				
									Cost of material		
No. of staff for		Hours	Kr.	In the automatic equipment Kr.	Outside the automatic equipment Kr.	Kr.	Hours	Cost of labour Kr.	In the automatic equipment Kr.	Outside the automatic equipm. Kr.	Total maintenance cost Kr.
Main distribution frames and Fault Office	2	4 785	8 689	—	—	—	0,37	0,68	—	—	—
Selector rooms (connecting devices junction-boards, exchange equipment for subscribers' lines, and power plants)	7	17 452	29 595	—	—	—	1,36	2,31	—	—	—
Cleaning of the automatic devices (female labour)	2	4 491	4 238	—	—	—	0,35	0,33	—	—	—
Charring and floors (female labour)	2	3 100	2 520	—	—	—	0,24	0,19	—	—	—
Total	13	29 828	45 042	2 342	1 300	48 684	2,32	3,51	0,18	0,10	3,79

Faults remedied during the year in	Number	Per subscribers' line connected	Per 10 000 calls
1. automatic equipment	947	0,07	0,41
2. exchange equipment outside the automatic system[1])	729	0,06	—
Total	1676	0,13	

[1]) **In addition there were 1 245 064 connexions** made on the trunk junction board,
of which 838 259 were for trunk calls
and 45 212 were for telephoned telegrams and the rest for "Förmedlings" Bureau, Telephone queue, Supervisor's desk, and information about changed and vacant numbers.

[11]) Main distribution frames, current protection, multiple and service equipment of the trunk junction board.

Table 5.

Maintenance Costs in "Norra Vasa" Automatic Exchange during 1931.

The exchange equipped for 10 000

Average number of subscribers' lines connected during the year	7 762
Number of outgoing calls during the year	14 483 260 [1]
" " " " per subscriber per year	1 866
" " " " " " " weekday from 8 a.m.—9 p.m.	5.6
" " " " " " " busy hour	0.6 (max. 0.84)

Cost of labour, including night duty, holidays, and sick leave				Cost of materials		Total maintenance cost	Average per connected subscriber per year				
No. of staff for		Hours	Kr.	In the automatic equipment Kr.	Outside the automatic equipment Kr.	Kr.	Hours	Cost of labour Kr.	Cost of material: In the automatic equipment Kr.	Cost of material: Outside the automatic equipm. Kr.	Total maintenance cost Kr.
Main distribution frames and Fault Office; Selector rooms (connecting devices junction-boards, exchange equipment for subscribers' lines, and power plants)	8	19 453	33 704	—	—	—	2,51	4,34	—	—	—
Cleaning of the automatic devices (female labour)	1	2 322	2 309	—	—	—	0,30	0,29	—	—	—
Charring and floors (female labour)	1	855	684	—	—	—	0,11	0,9	—	—	—
Total	10	22 637	36 697	2 227	1 393	40 317	2,92	4,72	0,29	0,18	5,19

Faults remedied during the year in	Number	Per subscribers' line connected	Per 10 000 calls
1. automatic equipment..	215	0,03	0,15
2. exchange equipment outside the automatic system [2]	495	0,06	—
Total	710	0,09	

1) **In addition there were 486 961 connexions** made on the trunk junction board, of which 661 426 were for trunk calls and 37 180 were for telephoned telegrams and the rest for "Förmedlings" Bureau, Telephone queue, Supervisor's desk, and information about changed and vacant numbers.

2) Main distribution frames, current protection, multiples, and service equipment of trunk junction and local switchboards. Incoming traffic from manual exchanges is dealt with from B-positions.

Table 6.

Maintenance in "Söder" Automatic Exchange during 1931.

Exchange in full work on July 1st 1931, with a capacity of 40 000.

Average number of subscribers' lines connected during the six months	25 254
Number of outgoing calls during *six months*	23 254 120
„ „ „ „ per subscriber per weekday 8 a.m.—9 p.m.	5.6
„ „ „ „ „ subscriber per busy hour	0.56 (max. 0.69)

Number of staff for		Number of faults remedied 1/7—31/12 in	Number	Per subscribers' line connected per year	Per 10000 calls
Main distribution frames and Fault Central Office	2	1. Automatic equipment	796	0.06	0.34
Selector rooms (connecting devices, junction-boards, exchange equipment for subscribers' lines, and power plant)	10	2. Exchange equipment outside the automatic system. [1])	1358	0.11	—
Cleaning of the automatic devices (female labour)	2				
Charring and floors (female labour) ..	2				
Total	16	Total	2154	0.17	

[1]) Main distribution frames, current protection, multiple, and service equipment of the trunk switch-board.

calls, e. g. inter-continental calls. Further, it is possible with the junction board occasionall to have trunk calls referred to another number when this is desired. The trunk junction board also gives information about numbers cut off, changes of numbers, and vacant numbers.

The use of the trunk switch-board multiple for information purposes is also of great advantage from the point of view of both service and saving of staff.

The maintenance costs are undoubtedly very low, and are but slightly higher than the corresponding costs in a modern manual exchange. Our most up-to-date manual exchange, "Östermalm Riks", which is equipped for 10 000 numbers and had an average of 6 820 subscribers in 1931, the call frequency was 9 calls per subscriber per day (8 a. m.—to 9 p. m.) and the number of calls per subscriber per busy hour 1.0.

In 1931 the cost of work per subscriber

per year was	kr. 2: 93
the cost of materials per subscriber per year was	„ 1: 57
making a total cost per subscriber per year of	kr. 4: 50

Table 7.

Exchange	Average number of subscribers during the year	Number of calls: Per subscriber per weekday 8 a.m.—9 p.m.	Number of calls: Per subscriber per busy hour	Current	Number of kwh per subscriber per year	Per subscriber per year Kr.	Remarks
"Centralen" ...... (automatic exch.)	14 944	11,2	1.3	48 Volts	3.75	0.188	Battery charge
				220 „	**1.50**	**0.120**	For the bay motors
					5.25	0.31	
"Kungsholmen" .. (automatic exch.)	12 834	5,5	0,56	48 „	2.61	0.13	Battery charge
				220 „	**1.32**	**0.11**	For bay motors
					3.93	0.24	
"Östermalm Riks" (manual exchange)	6 820	9,0	1,0	24 „	2.28	0.114	Battery charge

As we see, the cost is higher than in the automatic "Kungsholmen" exchange (kr. 3:79), but lower than in "Centralen" (kr. 5:20).

The frequency at the Östermalm exchange is, however, higher than at Kungsholmen, namely:

9, against 5.5, calls per subscriber per day (8 a.m.—9 p.m.)
1, against 0.56, calls per subscriber per busy hour,

while the same comparison with "Centralen" gives

9, against 11.2, calls per subscriber per day (8 a.m.—9 p.m.)
1, against 1.3, calls per subscriber per busy hour.

The current consumption for 1931 in the automatic exchanges "Centralen" and "Kungsholmen" and in the manual exchange "Östermalm Riks" is given in Table 7 above.

The current consumed by the motors is here shown separate from the rest of the consumption and, as we see, the amount required for power driving is very small.

"Centralen" uses 1.5 kWh per subscriber per year Kr 0:12
"Kungsholmen" „ 1.32 „ „ „ „ „ „ 0:11

The costs of maintenance and working in the L. M. Ericsson automatic telephone system with 500-line selectors can thus without exaggeration be termed *sensationally low*, while at the same time the remarkably small number of faults in the automatic equipment conduces to first class reliability in traffic.

Telephone Exchanges on the L. M. Ericsson Automatic System.

Communication from the Head Office of the Ericsson Concern.

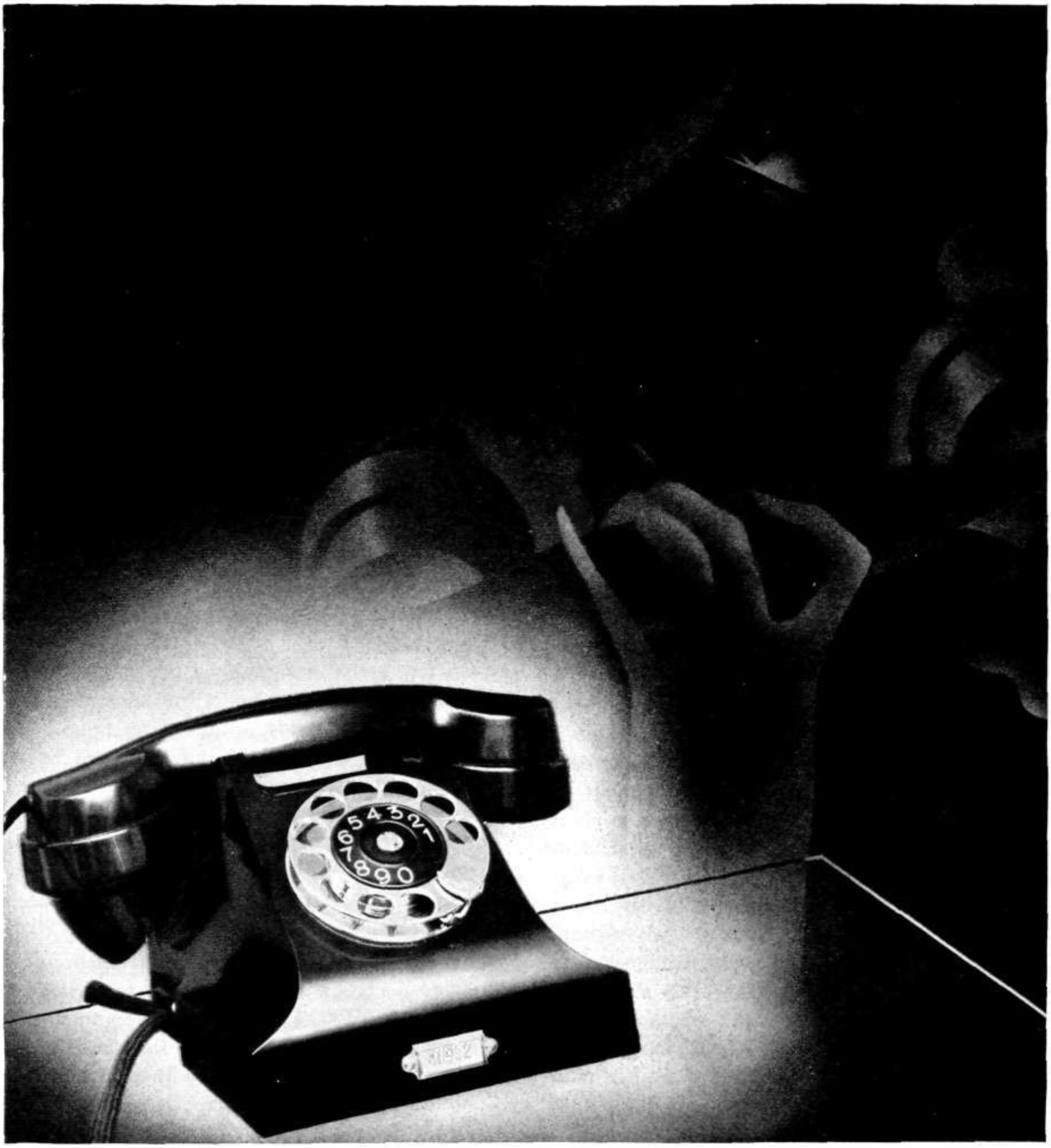
Since the publication—with the L. M. Ericsson Review No. 10-12, 1931—of the list of automatic exchanges working or under construction on Feb. 1st. 1932, the following have been taken into use:

In *Argentina:* in Victoria,
„ Villaguay.
In *Italy:* in Asti,
„ Bari,
„ Barletta,
„ Brindisi,
„ Foggia,
„ Molfetta,
„ Naples, the Bagnoli exchange,
„ Venice, the Mestre exchange.
In *Mexico:* in Mexico City, the Piedad exchange,
„ Merida,
„ Toluca.
In *New Zealand:* in Whangarei.

The exchanges
In *Argentina:* in Gualeguaychú and Concepción del Uruguay have been enlarged by 500 lines to 2 000 lines each.
In *Italy:* in Trapani has been enlarged by 500 lines to 1 000 lines.

The new Ericsson telephone set, 1931 model.

Telephone Exchanges on the L. M. Ericsson Automatic System.


1234567890

A Theorem of Reciprocity in Wireless Telegraphy.

Communication from the Research and Development Department.

By Professor *H. Pleijel.*

In a previous paper[1] the author formulated and proved the general theorem of reciprocity obtaining between EMF and current between any two points in a medium. There, the EMF was of arbitrary form, which has not been the case in the theorems of reciprocity previously put forward. In these, the EMF has either been assumed to be constant or of the form $e^{j\omega t}$, which latter represents a simple stationary sinusoidal EMF. (An exception to this is the proof given in 1890 by A. Vaschy[2] and in 1917 by the present author[3].) I demonstrated that the general theorem of reciprocity could be expressed as follows:

There is a linear operator relation between the current vector at a point B and the vector, at a point A, of the EMF producing the current. If the EMF and current are exchanged, another linear operator relation is obtained which is the conjugate of the former.

One consequence of this general proposition is that two equal EMFs at the points *A* and *B*, having given directions in space, will produce at *B* and *A* respectively currents whose projections on to the directions of the EMFs at these points are equal. This thesis of reciprocity is valid *for any arbitrary form of EMF*. With regard to the constants, the dielectric constant, permeability, and conductivity of the medium, the only assumption is that they are independent of the electric and magnetic field strengths. But these may vary from point to point in the medium, and on certain surfaces in jumps. Finally it has been assumed that the field strengths diminish in infinity more rapidly than $\frac{1}{r}$. This condition is always fulfilled in finite systems, as these fields diminish in infinity as $\frac{e^{-p\frac{r}{u}}}{r}$, where p is the operator $\frac{\delta}{\delta t}$ and u is a constant, corresponding to the velocity of electromagnetic propagation. In the proof we assumed a start from a state of rest at the time $t=0$. The stationary conditions therefore follow the law of reciprocity, in so far as they are the final states of non-stationary conditions which are obtained from one in which all the currents and charges are zero. This theorem can be proved directly from the above by replacing the E. M. F:s by field vectors. Here, however, we will choose a more direct method for future use.

On the above assumptions the same theorem of reciprocity is now valid for current vectors and the electric fields produced by them. This theorem supplements the one proposed before, and is of some importance, particularly in wireless telegraphy, as the current in an aerial produces an electric field which appears in another aerial as an EMF. The field will then be determined on the assumption that the second aerial does not exist.

For the better definition of our theorem we imagine a certain region of space to contain all the active EMFs. The currents in conductors in this region are assumed to be the cause of the electromagnetic field, and are therefore known. Conductivity within the space is assumed to be nil, but the medium in it has a particular dielectric constant, and so we get an electric field in the medium. The relation of current to field strength in the region is, however, left undetermined.

The region may consist of a system of conductors or part of such a system.

[1]) Ingeniörvetenskapsakademiens handlingar Nr. 68, 1927. (Proceedings of the Swedish Academy of Engineering Science, No. 68, 1927).

[2]) *A. Vaschy:* Traité d'Eléctricité et de Magnetisme, 202 Paris 1890.

[3]) Allmänna egenskaper hos ett system parallella ledningar med variabla konstanter (General properties of a system of parallel lines with variable constants) Kungl. Tekniska Högskolan, Stockholm 1917.

According to the law of continuity, certain true charges situated within the space or on its surface correspond to the given current. We choose, however, not the actual charges but only those corresponding to the given current. If ϱ is the volume density in the region and σ the surface density of the charge, I the current vector and I_N the normal component of the current vector at the surface of the region, we have the relations:

$$-\frac{\delta \varrho}{\delta t} = \operatorname{div} I$$

$$\frac{\delta \sigma}{\delta t} = I_N$$

As we start from a state of rest at $t=0$, ϱ and σ are completely determined by the given current vector. If, for example, the region consists of a linear conductor element with a given current, we get opposite charges at the ends of the element, and current and charges between them form what Heaviside called a rational current element and Hertz an oscillating dipole.

The given currents and attendant charges now determine the electromagnetic field quite unambiguously, as we now shall see.

As we have mentioned above, we make the conductivity in the given region nil, but assume a certain dielectric constant and permeability. In the remainder of space, both in the dielectric and in the conductors, we assume a certain conductivity. The given current vector in the region selected we call I. I is thus nil outside this region.

The same is the case with ϱ and σ. We may imagine the space outside the region to have a dielectric constant equal to $\varepsilon + \frac{\lambda}{4\pi P}$; ϱ and σ will then be "actual" charges in the system.

The field equations determining the electromagnetic field may be summed up as follows:

$$\left\{ \begin{aligned} 4\pi I + 4\pi\lambda E + \varepsilon \frac{\delta E}{\delta t} &= \operatorname{rot} H \\ \mu \frac{\delta H}{\delta t} &= -\operatorname{rot} E \end{aligned} \right\}$$

We assume that we have two solution systems E and H fulfilling the given conditions. If the vectorial field of differences (E', H') between these two solutions is formed, this differential field will satisfy the equation system:

$$\left\{ \begin{aligned} 4\pi\lambda E' + \varepsilon \frac{\delta E'}{\delta t} &= \operatorname{rot} H' \\ \mu \frac{\delta H'}{\delta t} &= -\operatorname{rot} E' \end{aligned} \right\}$$

The tangential components of E' and H' are continuous at a surface separating different media, because this is the case in the two solutions corresponding to the components. As we have the same ϱ and σ in the two systems of solutions, both current and charge will be zero in the field (E', H').

Multiplying the two equations by E' and H' respectively, and adding, we get:

$$4\pi\lambda E'^2 + \frac{1}{2}\varepsilon \frac{\delta E'^2}{\delta t} + \frac{1}{2}\mu \frac{\delta H'^2}{\delta t} = -\operatorname{div}[E' + H']$$

If we multiply these expressions by the volume element $\delta\tau$ and integrate over the whole of space, and remembering that $\int [E' \times H']_N$ extends over a sphere of infinite radius will be nil, we get the relation:

$$\int 4\pi\lambda E'^2 \,\delta\tau + \frac{\delta}{\delta t}\int \frac{1}{2}\varepsilon E'^2 \,\delta\tau + \frac{\delta}{\delta t}\int \frac{1}{2}\mu H'^2 \,\delta\tau = 0$$

If we integrate this expression with respect to time from $t=0$, to $t=t$, since E' and H' are nil when $t=0$, we get:

$$\int_0^t \delta t \int 4\pi\lambda E'^2 \,\delta\tau + \int \frac{1}{2}\varepsilon E'^2 \,\delta\tau + \int \frac{1}{2}\mu H'^2 \,\delta\tau = 0$$

All the elements of the above expression are positive. For this to be an equality, both E' and H' must be zero at every point of space. That is, $E' = H' = 0$. The two systems thus coincide and we only have one solution.

We now designate the components of E and I along the three axes x, y, and z at right angles to one another by E_1, E_2, E_3, and I_1, I_2, I_3.

By the principle of superposition, each component of I will produce its own field, and the components of the total field will be obtained by addition.

According to the rules for Heaviside's operator calculus we will now imagine the solution of the equation system in operator form, and introduce $\frac{\delta}{\delta t}$ as a constant in the equations, which will give the solution as a function of p and the coordinates. According to the rules of the operator calculus we may then pass on to a solution in terms of t and the coordinates. For general investigations the solution in operator form is most suitable. From this the real solution can be obtained in various forms according to circumstances. It is possible to solve it by series of powers of the time, by wave systems, by integrals according to Cauchy's method, or by Heaviside's expansion theorem. The operator solution forms, so to speak, the kernel of various forms of expression.

We now assume that the current vector I is given in an unit volume round a point $A\,(x', y', z')$ and that the electric field is determined at a point $P\,(x, y, z)$. According to the above we then have, according to the principle of superposition, the relation between E and I expressed in linear form:

$$E_1 = k_{11}\, I_1 + k_{12}\, I_2 + k_{13}\, I_3$$
$$E_2 = k_{21}\, I_1 + k_{22}\, I_2 + k_{23}\, I_3$$
$$E_3 = k_{31}\, I_1 + k_{32}\, I_2 + k_{33}\, I_3$$

The current vector I need not be fixed in space, and the same applies to the vector E of the electric field. It is further clear that, even if the vector I has a fixed position in space, this need not be the case with the vector E. In this respect we may refer to the field produced in the earth by the current in a double pole above ground. As we know, the field in the earth will be of varying direction.

The coefficients k are operators, and therefore functions of p and of the coordinates of the points where I and E are applied.

The values obtained for the components E_1, E_2, and E_3 satisfy the differential equations when p in the latter and in the coefficients k is regarded as a constant. If we go on to a solution in terms of t and the coordinates by the methods of the operator calculus, this solution system becomes the solution of the original differential equations when p in these is replaced by $\frac{\delta}{\delta t}$. I shall return to this question later, as the validity of the reciprocity theorem has been criticized on the grounds that, as the equations have been deduced on the assumption that p is a constant, the final solution will not satisfy the original differential equations.

We now assume that we have two regions in which the current vector is given, and that these two regions have no common portion. The current vector in one region, which we will call A, is assumed to be I' and in the other region, which we call B, I''. According to the law of superposition the field at any given point is now the geometric sum of the fields produced by I' and I''. The former field we will call E', the latter E''. Between the vectors of the two systems the following differential equations apply:

$$\left\{\begin{array}{r} 4\pi I' + (4\pi\lambda + p\,\varepsilon)\, E' = \operatorname{rot} H' \\ \mu p\; H' = -\operatorname{rot} E' \end{array}\right\} \begin{array}{r} E'' \\ -H'' \end{array}$$

and

$$\left\{\begin{array}{r} 4\pi I'' + (4\pi\lambda + p\,\varepsilon)\, E'' = \operatorname{rot} H'' \\ \mu p\; H'' = -\operatorname{rot} E'' \end{array}\right\} \begin{array}{r} -E' \\ H' \end{array}$$

λ is zero in both regions, A and B.

We multiply the equations by the vectors on the right-hand side and add all the equations so obtained, regarding p as an algebraic quantity. We get:

$$4\pi I'\; E'' + 4\pi I''\, E' = E'' \operatorname{rot} H' + H'' \operatorname{rot} E'$$
$$- E' \operatorname{rot} H'' - H' \operatorname{rot} E'' = \operatorname{div}\,[E' \times H'']$$
$$- \operatorname{div}\,[E'' \times H'\,].$$

The terms on the left-hand side are zero everywhere except in the regions A and B. All vectors are finite, and the tangential components of E', E'', H', and H'' are continuous at all separating surfaces, and hence the normal components of $[E' \times H'']$ and $[E'' \times H']$ will also be continuous at such surfaces.

The vectors E', H', E'' and H'' are assumed to diminish with the distance r according to a law by which the integrals of the normal components of $[E' \times H'']$ and $[E'' \times H']$ over the surface of an infinite sphere become zero. (This is always the case in finite systems, as E and H diminish as $\frac{e^{-\frac{r}{u}}}{r}$, where u is the velocity of electromagnetic propagation.)

If, therefore, we multiply the above equation by the volume element $\delta\tau$ and integrate over the whole of space, we obtain the relation:

$$\int I'\; E''\; \delta\tau = \int I''\; E'\; \delta\tau,$$

where the first integral is extended over region A and the second over region B.

Assuming that the two regions A and B are limited to a volume element $\delta\tau$, we finally get for the points A and B

$$I'\; E'' = I''\; E'.$$

As mentioned above, the linear relations between E' and I' and between E'' and I'' may be written:

$$\begin{cases} E_1' = k_{11}' I_1' + k_{12}' I_2' + k_{13}' I_3' \\ E_2' = k_{21}' I_1' + k_{22}' I_2' + k_{32}' I_3' \\ E_3' = k_{31}' I_1' + k_{32}' I_2' + k_{33}' I_3' \end{cases}$$

$$\begin{cases} E_1'' = k_{11}'' I_1'' + k_{12}'' I_2'' + k_{13}'' I_3'' \\ E_2'' = k_{21}'' I_1'' + k_{22}'' I_2'' + k_{23}'' I_3'' \\ E_3'' = k_{31}'' I_1'' + k_{32}'' I_2'' + k_{33}'' I_3'' \end{cases}$$

Two coefficients k_{rs}' and k_{rs}'' interchange if the coordinates of A and B change places.

Whatever the size and direction of the vectors I' and I'', the equation will, according to the above, apply to these operator relations:

$$I' E'' = I'' E'$$

We may now write:

$$E_r' = \sum_s k_{rs}' I_s'$$

whence:

$$I'' E' = \sum_r I_r'' E_r' = \sum_r \sum_s k_{rs}' I_r'' I_s'$$

In the same way we get:

$$I' E'' = \sum_r \sum_s k_{rs}'' I_r' I_s''$$

or if we change indices:

$$I' E'' = \sum_r \sum_s k_{sr}'' I_r'' I_s'$$

In order to make the relation given above valid for all values of I' and I'', we must have:

$$k_{rs}' = k_{sr}''$$

The two determinants of our linear forms must therefore be conjugate to one another. The general expression for the law of reciprocity between current and electric field strength at two points A and B will thus be as follows.

The linear equation system in operator form between the electric field strength at a point B and a current vector at A will, when current vector and field strength change places, change over into the conjugate linear equation system.

The three components of the current may in that case be arbitrary functions of the time.

We can therefore write:

$$E_r' = \sum_s k_{rs} I_s'$$

$$E_r'' = \sum_s k_{sr} I_s'$$

The above form of the law of reciprocity applies to operator expressions. We will now find the expression for the law of reciprocity between the instantaneous current vector and electric field strength. We then assume the current vectors I' and I'' to be always equal in size and changing direction in the same way. To define more accurately what is meant, we imagine two systems of coordinates fixed in space in different positions, with origins at A and B. In these two systems of coordinates I' and I'' are assumed both to have the same components along the axes at every moment. We will make the system with origin at A our main one. The components of the current along the axes we will call I_1, I_2 and I_3. The components of I'' along the main axes may now be written:

$$I_s'' = \sum_s a_{ts} I_t$$

where a_{ts} is the cosine of the angle between the t-axis of the B-system and the s-axis of the main system.

If the currents are of simple sinusoidal form and the condition is stationary, we find, if the instantaneous values of current and electric field strength are exchanged for the corresponding complex vectors, that p is a constant, $j\omega$, where ω is the frequency. In this case, then, the relation:

$$I' E'' = I'' E'$$

is valid for the complex vectors.

If I' and I'' are equal, we find that the complex vectors of E' and E'' have equal projections on the directions I'' and I' respectively. The same will then be the case with the projections of the instantaneous values of E' and E'', provided that I' and I'' have definite directions in space.

Will this relation also be valid for arbitrary equal current vectors I' and I''? We will examine this more closely.

We have now:

$$E' I'' = \sum_r \sum_t E_r' \cdot a_{tr} I_t$$

or, if we introduce the expression for E_r'

$$E' I'' = \sum_r \sum_t \sum_s a_{tr} I_t \cdot k_{rs} I_s$$

k_{rs} is an operator function acting on I_s.

In the same way we get:

$$E'' I' = \sum_r I_r' E_r'' = \sum_r \sum_s I_r \cdot k_{sr} I_s''$$

$$= \sum_r \sum_s \sum_t I_r k_{sr} \cdot a_{ts} I_t$$

or, if we exchange the indices r and s,

$$E'' I' = \sum_r \sum_t \sum_s I_s k_{rs} a_{tr} I_t = \sum_r \sum_t \sum_s a_{tr} I_s k_{rs} I_t$$

To make these two expressions equal, it is necessary that

$$I_t k_{rs} I_s = I_s k_{rs} I_t$$

If the values of the components of I are arbitrary, the above relation is only valid if I_t and I_s are in a constant proportion. But the fact of a constant proportion between the components is equivalent to their resultant having a fixed direction in space.

It is also conceivable that the components might be of such special form that the above equations apply. Such a case occurs when the current components are of the form $A_s e^{j\omega t}$, where A is a complex number. As we have mentioned before, p will then be a constant equal to $j\omega$, and I_s and I_t may therefore change places without altering the expression. The components I_s are then complex vectors. For the relation to be valid even for the simple sinusoidal components corresponding to these vectors it is, however, essential that the components should be of the same phase or that they should be components of a sinusoidal vector. We thus come back to the general case.

The reciprocity theorem may therefore be expressed as follows: *if the two current vectors I' and I'' have fixed directions in space and are always equal in magnitude, the projection of E'' on I' at any instant will be equal to the projection of E' on I''.*

As mentioned at the beginning of this paper, the author has already advanced a corresponding general theorem of reciprocity for EMF:s and the currents produced by them. That paper was published in Ingeniörsvetenskapsakademiens Handlingar, No. 68, 1927, under the title "Two reciprocal theorems in electricity".

In a paper entitled "Het reciprociteitstheorema in de electriciteit", Tijdschrift van het nederlandsch radiogenootschap 5, 69—84, July 1931, J. W. Alexander has criticized my theorem of reciprocity. As Mr. Alexander's argument seems to turn on a misunderstanding of the methods of the operator calculus used for this problem, which may itself be due to the account being rather condensed, it will be convenient to reply to his criticism at this point, especially as these general reciprocity theorems are very important, and valid for EMF:s of any given form and for the variable state of the currents. The laws of reciprocity are thus of very general application.

We will confine our attention to the theorem as applied to EMF:s and the currents produced by them. As all the reciprocity theorems are formulated in the same way and only differ in using different kinds of vectors, it is possible to apply immediately to other similar theorems what is pointed out below regarding the first of them.

We will now make I^1 and I^{11} the current vectors at the points B and A in the field. These vectors are caused by the vectors of EMF:s $\mathcal{E}'$ and $\mathcal{E}''$ placed at A and B respectively. The electric field vectors corresponding to the currents are represented by E' and E'', and the magnetic vectors by H' and H''.

According to Alexander, this theorem cannot be correct because the solutions do not satisfy the conditions:

$$E'' \frac{\delta E'}{\delta t} = E' \frac{\delta E''}{\delta t}$$

$$E'' \frac{\delta \mathcal{E}'}{\delta t} = \mathcal{E}' \frac{\delta E''}{\delta t}$$

$$E' \frac{\delta \mathcal{E}''}{\delta t} = \mathcal{E}'' \frac{\delta E'}{\delta t}$$

$$H' \frac{\delta H''}{\delta t} = H'' \frac{\delta H'}{\delta t}$$

E' and E'' are here the strengths at a given point P of the electric fields due to $\mathcal{E}'$ and u″ situated at A and B respectively.

It is quite true that the above equations are not satisfied by the solutions obtained *after* conversion from the operator form to the form of a function of t. But then there is no utter reason why the solutions should satisfy these conditions. On the contrary, they cannot a priori be expected to satisfy them, for the simple reason that they are solutions of Maxwell's differential equations, so that they are very unlikely to be able to satisfy any extra conditions. These four equations can only be satisfied when $\mathcal{E}'$ and $\mathcal{E}''$ are constant and we choose the stationary solution, or when we want the stationary solution for an EMF of the form $Ae^{j\omega t}$, i. e. for complex EMF:s. These equations are not satisfied with real EMF:s or with variable conditions. The same is the case with the relation $I' \mathcal{E}' = I'' \mathcal{E}'$, as we will show below for a simple case.

But this has nothing to do with the validity of the reciprocity theorem. The author has not used these four equations in the proof of the theorem, but only the fact that these equations (the first and last in particular) are satisfied if $\frac{\delta}{\delta t} = p$ is assumed to be an algebraic quantity.

The relation

$$I' \mathcal{E}'' = I'' \mathcal{E}'$$

is therefore not valid except when $p = \frac{\delta}{\delta t}$ in the operator solution is assumed to be an algebraic quantity. This was expressly pointed out in my paper and will be illustrated below by a simple example.

To make these equations clear, we will give again briefly the main features of the proof of the reciprocity theorem.

Henceforth we designate the EMF:s E' and E''.

We assume Maxwell's equations to be solved *on the condition that* $\frac{\delta}{\delta t}, \frac{\delta^2}{\delta t^2}$ *and* p, p^2 *are taken as algebraic quantities.*

For I' and I'' at the points B and A respectively the solutions will then be of the form:

$$I' = k'(p) \cdot E'$$
$$I'' = k''(p) \cdot E'',$$

where $k'(p)$ and $k''(p)$ are linear algebraic forms of the components of I', E' and E'' and I''. This is obvious from the linear nature of the equations (the principle of superposition).

If p is still taken as an algebraic quantity, *but only on this assumption,* we find that the relation

$$I' E'' = I'' E'$$

holds good and that therefore $k'(p)$ and $k''(p)$ must be conjugate linear forms.

So far we have not employed the operator calculus, but only the relations subsisting if p is taken as an algebraic quantity.

According to the fundamental principle of the operator calculus the following applies: If the members of a system of differential equations *are linear in* $p, p^2, p^3, \frac{1}{p}, \frac{1}{p^2}$ *and in the dependent variables* (in this case E, H, and I), the solution obtained when p is taken as an algebraic quantity will still hold if p is replaced by the operator $\frac{\delta}{\delta t}$ both in the equations and in the solution. This is based finally on the fact that the operators $p, p^2, \frac{1}{p}$ .... follow the same mathematical laws as the corresponding algebraic quantities.

If, therefore, we regard p as an operator, and complete the operations, the expressions

$$I' = k'(p) \cdot E'$$
$$I'' = k''(p) \cdot E''$$

will be solutions of Maxwell's equations. But we have previously found that k' and k'' are conjugate linear forms. The general reciprocity theorem can therefore be expressed as follows: *the currents in B and A are connected with the EMF:s in A and B by linear forms, the coefficients of which are functions of the operator p such that the two linear forms are conjugate to one another.*

According to the above it is essential to the proof that there should be a linear relation between the current vector I and the vector of the EMF E.

The equation

$$I' E'' = I'' E'$$

and the four equations quoted by Alexander are not linear and are therefore not satisfied except in the very special case mentioned before.

The above reasoning will be illustrated by a simple example

We assume E' and E'' to be in the same plane and at right angles to one another.

We assume further that this plane divides the medium symmetrically in respect to its properties.

On account of the symmetry, I' and I'' will then be in the same plane.

We choose the direction of E' as X-axis and that of E'' as Y-axis, and distinguish the components in these two directions by the suffixes 1 and 2 respectively.

If we take p as an algebraic quantity, we get solutions of the form:

$$\left\{\begin{array}{l} I_1' = k_{11}' E_1' \\ I_2' = k_{21}' E_1' \end{array}\right\}$$
$$\left\{\begin{array}{l} I_1'' = k_{12}'' E_2'' \\ I_2'' = k_{22}'' E_2'' \end{array}\right\}$$

The relation

$$I' E'' = I'' E'$$

will in this case be

$$I_2' E_2'' = I_1'' E_1'$$

or $$k_{21}' E_1' E_2'' = k_{12}'' E_2'' E_1'$$

or $$k_{21}' = k_{12}'' = k_{12}$$

The two linear forms are thus conjugate, and the solutions can be written in the form:

$$\left\{\begin{array}{l} I_1' = k_{11} E' \\ I_2' = k_{12} E' \end{array}\right\}$$
$$\left\{\begin{array}{l} I_1'' = k_{12} E'' \\ I_2'' = k_{22} E'' \end{array}\right\}$$

We now choose the EMF:s to be simple sine functions of the same angular frequency, and then determine the stationary solution without the use of the operator calculus.

We thus assume

$$E' = A \sin(\omega t + \alpha)$$
$$E'' = B \sin(\omega t + \beta)$$

According to classical method we exchange these EMF:s for the corresponding complex expressions

$$\overline{E}' = A e^{j(\omega t + \alpha)}$$
$$\overline{E}'' = B e^{j(\omega t + \beta)}$$

and seek solutions I_2' and I_1'', of the form

$$C e^{j(\omega t + j)}$$

Maxwell's differential equations being linear in the field strengths the factor $e^{j\omega t}$ will disappear and $\frac{\delta}{\delta t}$ will everywhere be replaced by $j\omega$. We consequently come back again to introducing p equal to a constant (in this case $j\omega$) and to constant EMF:s $A e^{j\alpha}$ and $B e^{j\beta}$.

In the operator calculus E' and E'' are regarded as constant and p as an algebraic quantity. The procedure will thus be exactly the same in both methods, and the result must therefore also be the same.

If in the stationary case we get the solutions:

$$\overline{I}_2' = k_{21}(j\omega) \cdot \overline{E}'$$
$$\overline{I}_1'' = k_{12}(j\omega) \cdot \overline{E}''$$

then we get the operator expressions:

$$I_2' = k_{21}(p) \cdot E'$$
$$I_1'' = k_{12}(p) \cdot E'',$$

where the functions k_{21} and k_{12} are the same functions of $j\omega$ in the former case as they are of p in the latter.

For exponential expressions we have:

$$\overline{I}_2' \cdot \overline{E}'' = \overline{I}_1'' \cdot \overline{E}''$$

If in these we substitute the solutions, we get:

$$k_{21}(j\omega) = k_{12}(j\omega)$$

We therefore also have:

$$k_{21}(p) = k_{12}(p).$$

The reciprocity theorem advanced by the author is thus satisfied, the two linear forms giving I' and I'' being conjugate to one another.

We now find the solutions corresponding to the sinusoidal EMF:s. These, it will be remembered, will be obtained as the coefficients of j in the exponential solutions. We introduce

$$k_{12}(j\omega) = K e^{j\varphi},$$

where K is real. As we have terms including both $\frac{\delta^2}{\delta t^2}$ and $\frac{\delta}{\delta t}$, μ will in general not be zero

Our solutions will then be:

$$I_2' = KA \sin(\omega t + \alpha + \varphi)$$
$$I_1'' = KB \sin(\omega t + \beta + \varphi)$$

Further

$$E' = A \sin(\omega t + \alpha)$$
$$E'' = B \sin(\omega t + \beta)$$

whence

$$I_2' E'' = KAB \cdot \sin(\omega t + \alpha + \varphi) \cdot \sin(\omega t + \beta)$$

and

$$I_1'' E' = KAB \cdot \sin(\omega t + \beta + \varphi) \cdot \sin(\omega t + \alpha)$$

Consequently:

$$I_2' E'' - I_1'' E' = KAB \cdot [\cos(\omega t + \alpha)\sin(\omega t + \beta) - \cos(\omega t + \beta)\sin(\omega t + \alpha)] \sin\varphi$$
$$= KAB \cdot \sin(\beta - \alpha) \cdot \sin\varphi.$$

Although the general theorem of reciprocity is valid, the condition $I' E'' = I'' E'$ is not satisfied even in this simple case. It is easy to see that the equations proposed by Alexander are not satisfied either.

An arbitrary EMF can, we know, be represented as a sum (integral) of simple sine functions, and the general solution will be obtained as the sum of the *stationary* solutions of the various component sine functions.

If we have now got the operator solution

$$I = k(p) \cdot E,$$

where E is of the form:

$$E = \Sigma A_m \sin(\omega_m t + \alpha_m)$$

(the summation sign can be exchanged for an integral sign), the operator calculus gives the *stationary* solution in the form

$$I = \Sigma K_m A_m \sin(\omega_m t + \alpha_m + \varphi_m),$$

where

$$k(j\omega_m) = K_m e^{j\varphi_m}$$

that is, the same solution as we obtain by the classical method, when we work term by term. The reciprocity theorem in its general form is thus satisfied, but neither the relations given by Alexander nor the relation $I' E'' = I'' E'$ will be valid.

Elimination of Hum in Mains-operated Radio Receivers by Means of Compensation Methods.

Communication from the Research and Development Department.

By *E. Löfgren.*

I. INTRODUCTION.

The operating power for wireless receiving sets being now generally taken from the electric light mains, many valuable practical advantages have been gained over the old battery sets, but at the same time the design of the receivers have become more complicated on account of special technical problems involved, the most important of these being the elimination of disturbing hum from the mains.

The feed potentials obtained from the electric light mains, whether straight from D. C. mains or from A. C. mains through an intermediate rectifier, are rarely if ever as pure, *i. e.* free from superimposed A. C. components or ripple, as is required for the anode and grid voltages of the valves. In generator-fed D. C. mains irregularities of voltage are generally caused by the generators themselves (commutator and slot ripple), but sometimes also by motors or other machines connected to the mains. In this case there are usually many different disturbing frequencies, scattered irregularly over a comparatively wide range, from a few hundreds to some thousands of cycles per second; the R. M. S value of all the superimposed A. C. voltages only amounts to a few tenths of a volt. Conditions are much worse with D. C. mains fed from rectifiers. Here a dominant fundamental frequency arises, generally about 300 cycles per second, and the R. M. S. value of the A. C. voltage is of the order of 10 volts. With direct A. C. supply, on the other hand, the voltage pulsations from the rectifier of the receiver itself have a fundamental frequency of 50 or 100 cycles per second according as the rectifier is of the half-wave or full-wave type, and the R. M. S. voltage of the pulsations amounts to tens of volts.

With so differing external conditions there will naturally be much variation in the devices required for eliminating disturbances from the A. C. components in the feed circuits. Theoretically, this problem presents no serious difficulty. Filters have long been used which transmit direct current with slight loss of voltage, but attenuate the superimposed alternating current to some suitable level at which it is no longer disturbing. The use of such hum filters or smoothing circuits for wireless receivers is almost as old as receivers themselves, and may still be considered the basic method of eliminating mains hum. Besides this, however, many other methods have been suggested with the object of either completely dispensing with smoothing or else, as is more usual, making it possible to employ much simpler and cheaper smoothing units, the problem of eliminating mains hum being just as much economic as technical. In the usual all-mains receivers the smoothing devices represent a fairly considerable portion of the cost, although they do not properly contribute to the performance of the receiver, but merely serve the negative purpose of eliminating undesirable disturbances. It is therefore very tempting to devise some means of attaining the same result in a simpler way. Most attempts in this direction have been based on the fundamental idea of introducing artificially, in addition to the original disturbing A. C. voltage, another voltage of the same character but in the opposite direction, and adjusting it to exactly neutralize the effect of the former. Although the idea itself seems simple, there are a great many ways of realizing it in practice. All such arrangements are here brought together under the common designation of *compensation methods.*

The object of the present paper is to give a general account of the more important devices embodying the compensation principle. As far as the author is aware the subject has as yet attracted but superficial attention in technical literature, and for this reason the references given are chiefly Patent Specifications.

II. SMOOTHING THE FEED CURRENT BY COMPENSATION METHODS.

1. Impedance Circuits.

The circuit[1] shown in fig. 1 is an obvious way of adapting the compensation principle. The pulsating direct current is fed in between points *1* and *2*, and between points *3* and *4* a more or less pure direct current is taken out. This is accomplished by inducing in the transformer winding L_2 an E. M. F. equal in magnitude but opposite in phase to the A. C. voltage between points *1* and *2*. The condition necessary for this, provided the capacity C is large, will be

$$L_1 + L_{11} = M.$$

This "compensation transformer" at first glance looks rather attractive, but a closer examination shows that it works like one of the ordinary filter circuits. The equivalent T network is given in fig. 2, from which we can also see the part played by the choke coil L_{11} in fig. 1. If this is left out, the above condition may still be obtained, but then $L_1 = M = L_2$, *i. e.* $L_2 - M = 0$, which means that the right-hand choke arm in fig. 2 will disappear.

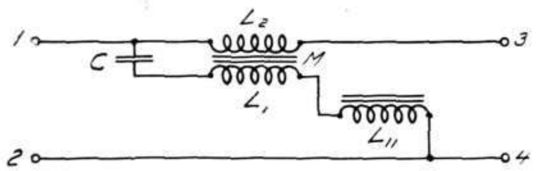


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Fig. 1. Elimination of ripple by a kind of compensation transformer.

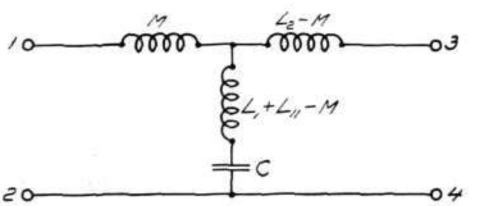

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Fig. 2. Equivalent T network of the circuit arrangement of fig. 1.

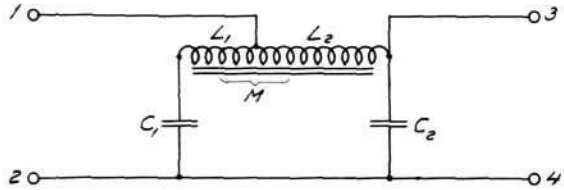

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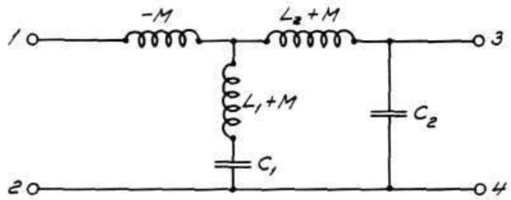
Fig. 3. Arrangement for suppressing ripple of one dominant frequency.



R 4130

Fig. 4. Equivalent T network of the arrangement of fig. 3.

A similar circuit[2] is shown in fig. 3. Here again the idea is to let the current, by means of an arm $L_1 - C_1$ in parallel with the input terminals $1-2$, induce an E. M. F. in the series arm L_2 which just cancels out the incoming A. C. voltage. The relation between the directions of winding of L_1 and L_2 is here opposite to that in fig. 1, and therefore the phase of the current must be shifted 180° by means of condenser C_1, the capacity of which is given by the expression:

$$C_1 = \frac{1}{\omega^2 (L_1 + M)}.$$

The completeness of elimination of the A. C. component here varies widely with frequency. The equivalent T network in fig. 4 also reveals the character of the circuit as a band-pass filter.

Other arrangements for the same purpose, composed of resistances, capacities, inductances, and mutual inductances, can be reduced to equivalent filter circuits in the same way.

Only in the case of a single frequency will it be possible by means of pure impedances to obtain compensation in the proper sense, namely by a bridge arrangement of some kind. But the fact that in reality the voltage pulsations are never purely sinusoidal naturally detracts from the practical value of this method.

[1] Cf. British patent 323 862. *N. V. Philips' Gloeilampenfabrieken.*

[2] *B. F. Miessner:* Proc. I. R. E. vol. 18, p. 160, Jan. 1930.

2. Separate Smoothing Valve.

The special properties of thermionic valves offer other possibilities for compensating the pulsations of a direct current. Several different methods suggest themselves. The simplest way is to make use of the saturation phenomenon by feeding the current consuming apparatus through a thermionic valve working with full saturation current. Two other methods[3] of greater interest are illustrated in figs. 5 and 6. The current consuming apparatus, *A*, is in one case connected in series in the anode circuit of the valve, in the other in parallel across the valve. In the first case the problem is obviously how to keep the anode current constant, *i. e.* free from superimposed alternating current; in the second the same

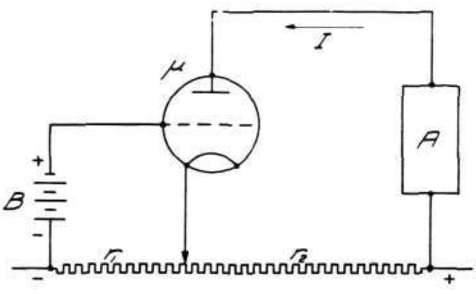


Fig. 5. Smoothing valve with compensated anode current.

must hold true for the anode voltage. With the circuits shown either condition may be satisfied if the anode, cathode and grid of the valve are connected to suitable points on a voltage divider connected across the current supply. These phenomena, on which many of the compensation methods described below are based, may be explained in the following way.

We will first consider the circuit shown in fig. 5. If we neglect the direct current and only take into account the superimposed A. C. component, we find that, in accordance with well-known properties of thermionic valves, the current in the anode circuit is produced partly by the portion of the total A. C. voltage (E) across the resistance r_2, and partly by an E. M. F. of opposite direction, caused by the A. C. voltage across the resistance r_1 applied to the grid. If these two are equal and thus neutralize each other, the anode alternating current will be compensated. The following equation will then apply:

$$\frac{r_2}{r_1+r_2}E=\mu\frac{r_1}{r_1+r_2}E,$$

$$r_2:r_1=\mu:1.$$

If there were no grid battery *B*, this would mean that the D. C. voltages at the anode and grid were also in the same ratio μ:1, and hence that the lower bend of the valve characteristic, where the anode direct current is very small, would be used. If the apparatus *A* requires more current, this may be obtained by counterbalancing part of the excessive negative grid bias by means of a grid battery *B*.

In a circuit like that of fig. 6 a constant anode voltage must instead be produced. This is accomplished by artificially *increasing* the ripple current in the anode circuit so much as to make the A. C. voltage drop in the resistance **r** just balance the whole A. C. voltage of the current supply. The internal A. C. voltage drop in the valve will then be balanced by an E. M. F. acting in the same direction as the current and produced by the A. C. grid voltage, which for this purpose is taken from a point on the positive side of the cathode. Using the same symbols as before, and further ϱ for the internal resistance of the valve, we may express the above argument as follows:

$$E=rI,$$

$$\mu\frac{r_1}{r_1+r_2}E=\varrho I.$$

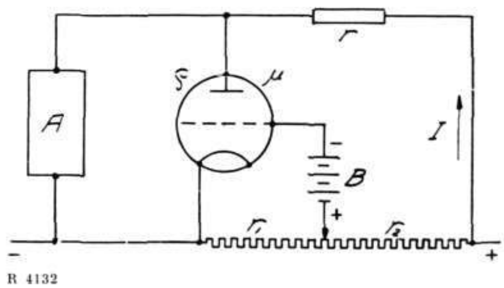


Fig. 6. Smoothing valve with compensated anode voltage.

Division of the two equations gives

$$\frac{r_1}{r_1+r_2}=\frac{\varrho}{\mu r}=\frac{1}{sr},$$

where *s* is the mutual conductance of the valve.

The battery *B* also in this case supplies suitable grid bias to give the desired anode current and

[3] German patent 330 275. *H. G. Möller.* — British patent 279 214. *W. J. Brown and Metropolitan-Vickers Electrical Co., Ltd.*

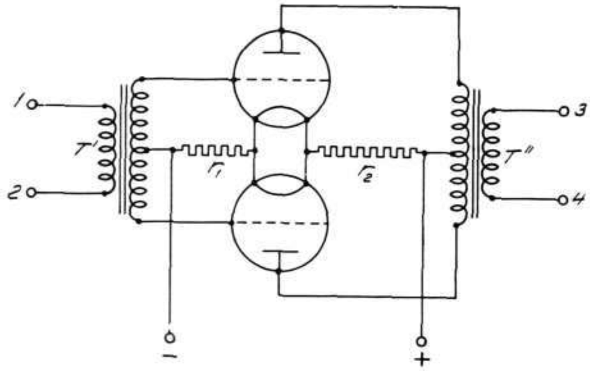
anode voltage. To make the D. C. voltage loss in the resistance r as small as possible, we should obviously make $r_2=0$, *i. e.* the grid should, as regards alternating current, be directly connected to the positive terminal of the current supply, and the voltage divider may be omitted. The resistance r is then made adjustable and should for compensation be set at the value $1/s$.

The two circuits shown in figs. 5 and 6, like other circuits with a separate smoothing valve[4], have not been used in practice in radio receivers for the simple reason that, on account of the extra valve, they are not economical. Nevertheless they are of great interest from the point of view that they give an idea of the possibilities of using thermionic valves for other than the conventional purposes. They also illustrate clearly just those properties of thermionic valves which have since been used in a more practical way in modern compensation circuits.

III. COMPENSATION METHODS FOR SINGLE VALVE STAGES.

1. Balanced Circuits.

The distinctive feature of most compensation circuits now in use is that the valves of the receiver itself are so arranged that while performing their functions as amplifiers or detector they also compensate the mains hum. The rough outline of this idea was known rather early, before the era of broadcasting proper. Thus a Swedish Patent Specification[5] of 1917 describes a balanced circuit intended to make the ripple voltages impressed on two symmetrically arranged valves or electrode systems neutralize one another. This circuit differed from the later commonly used push-pull arrangement (fig. 7) only in that, according to the practice of that time, it worked with positive grid bias. The signal current is fed in and taken out by differential transformers in directly opposite phases in the two valves, while variations in the feed current, whether coming in on grids, anodes or filaments, are of like phase



R 4133 Fig. 7. Balanced amplifier circuit.

and therefore neutralize one another in the two oppositely wound halves of the output transformer. The balancing principle might, of course, be used in a multi-valve apparatus, but the whole set of valves would then be duplicated, which will hardly be an economical way just to eliminate the mains hum. For the last valve it is a different matter. On one hand we are here concerned with a relatively strong direct current and the cost of filtration grows with the current; on the other hand the function of the last valve differs fundamentally from that of the preceding amplifying valves in that useful energy is delivered by it, whereas the others are practically unloaded. If two valves are used instead of one, the available output will be doubled for the same electrode voltages. Although two separate valves are usually more expensive than a single one of double the power, this is made up for by the possibility of using a smaller and cheaper output transformer, since this will be relieved of the D. C. magnetization. There are other advantages too in the balanced circuit shown in fig. 7, but this falls outside our subject.

The accuracy of the compensation will of course depend on how nearly equal the two valves or electrode systems are made. Ordinary commercial valves generally show such large individual differences as to make matching desirable. Another possibility is to introduce some correction for the asymmetry of the valves. We can, for instance, use different taps on the voltage divider for the grid voltages of the two valves[6], or insert a small adjustable resistance into one of the anode circuits[7].

The simplicity which characterizes the balancing principle has tempted many inventors

[4] U. S. patent 1 832 646. *B. F. Miessner, Radio Corporation of America.*

[5] Swedish patent 44 872. *O. R. Gertz.*

[6] British patent 341 472. *International General Electric Co., Inc.*

[7] British patent 324 464. *N. V. Philips' Gloeilampenfabrieken.*

to apply it to the anode current feed only, using an artificial balance to avoid valve duplication[8]. As this idea is every now and then cropping up, it may be convenient here to give some explanation of the conditions required in such arrangements. A differential transformer is provided for the output circuit, with one half of the primary winding connected to the anode of the valve and the other half to a resistance imitating the internal resistance of the valve (see fig. 8). But now, owing to the difference between the A. C. and D. C. resistances of a valve, we are faced with special difficulties, which are perhaps best illustrated by a numerical example. A small output valve with an A. C. resistance of 2 500 ohms may have a D. C. resistance of about 10 000 ohms. As the A. C. resistance is the one to be matched to get rid of the mains hum, the balancing resistance *R* would consume four times as much "anode current" as the valve itself, while at the same time the transformer would be subjected to abnormal D. C. magnetization. If this is to be avoided by blocking the resistance *R* with a condenser, the capacity must be very large. Besides this there is another great drawback, in that *R* will be included in the external circuit of the output valve and thus consume a great part of the output power. An improvement in this respect would be to make the portion of the differential transformer primary connected to *R* smaller than that connected to the anode, and at the same time *R* correspondingly less, but this would on the other hand make things worse as regards the D. C. consumption and the magnitude of the blocking condenser. If, conversely, we try to improve the D. C. conditions, the output power will suffer accordingly.

If we are dealing with a valve in an earlier stage the difficulties will be less, as the internal resistance of this valve will be much higher. In such a case an alternative method already suggested is to employ another valve as a balance resistance, at the same time using it as a H. F. amplifier[9]. The power-handling capacity of this valve must be large enough to allow superposition of the L. F. current on the H. F. current without any risk of detrimental reaction effect on the modulation. This method cannot, therefore, be used for balancing the last stage. By choosing the balancing valve of the same type as the balanced L. F. valve, an additional advantage is gained in that the transformer is relieved of direct current, which is not possible when using an ordinary resistance for balancing. The loss of output power in the balancing valve is, however, unavoidable.

For the sake of completeness it may be pointed out that the modern pentodes are more suited for artificial balancing than triode valves. In contrast with the latter they (*a*) have a higher A. C. than D. C. resistance, so that the D. C. consumption in the balancing resistance will be small, and (*b*) require an external load impedance that is only a fraction of the internal resistance, with the result that the greater part of the output power will be utilized in the output circuit. The auxiliary grid voltage should of course be well smoothed. As far as is known, no experiments have yet been made with pentodes in such circuits. It should be borne in mind, however, that the need of smoothing here is not quite as great as with triodes. A certain permissible ripple current in the output circuit corresponds to 3 or 4 times as great variations in the anode voltage with a pentode as with a triode (provided here again that the auxiliary grid voltage is smoothed).

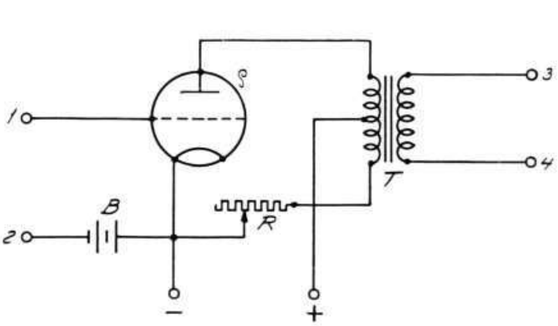


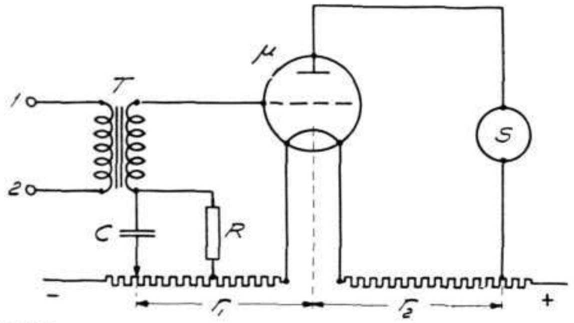
Fig. 8. Artificially balanced amplifier circuit.

2. Grid-Anode Compensation.

a) *Compensation of anode current.* Knowing how the compensation circuit shown in fig. 5 works, it is not a big step to superpose in the grid and anode circuits of the valve an incoming

[8] German patent 450 555. *W. Hahn.* — British patent 313 229. *N. V. Philips' Gloeilampenfabrieken.* — French patent 573 434. *R. E. C. Desoille.*

[9] French patent 564 508. *Compagnie pour la Fabrication des Compteurs et Matériel d'Usines à Gaz.* — French patent 573 434. *R. E. C. Desoille.* — U. S. patents 1 719 189 and 1 720 914. *B. F. Miessner.*



R 4135
Fig. 9. Grid-anode compensation with resistance voltage divider: adjustment on the grid side.

signal E. M. F. and the corresponding outgoing current, in other words to utilize the valve as an amplifier as well. For this purpose the only change we have to do is to break the grid circuit and interpose a suitable coupling element for supplying the signal E. M. F., while in place of the apparatus *A* we insert the telephone receiver, the loud speaker, or some other output circuit. Such circuit arrangements, with certain minor modifications, have been described in a number of Patent Specifications[10].

Unless the valve is intended to work as an anode-band rectifier, provision must be made to supply anode and grid D. C. voltages whose ratio is greater than that of the A. C. voltages, *i. e.* $\mu : 1$. The simplest method of doing this, though not perhaps suitable in practice, is to employ a supplementary grid battery in the same way as in fig. 5. A more economical alternative is offered by the arrangement, given in fig. 9, where all the currents and voltages necessary for operating the valve are taken from the mains *via* the voltage divider. For the grid there are two taps, one for the D. C. voltage *via* the resistance *R* and one for the ripple voltage *via* the condenser *C*. If the resistance is made large in comparison with the condenser impedance, which involves no difficulty since no direct current has to pass through it, the ripple potential at the grid will be equal to that at the condenser tap. The setting for compensating the mains hum can thus be found quite independently of the grid bias selected. The resistance *R* can of course be replaced by a large choke coil. The D. C. and ripple voltages can also be separated on the anode instead of the grid side (fig. 10), but this arrangement is not quite so good on account of the loss of anode voltage in the high-impedance D. C. arm.

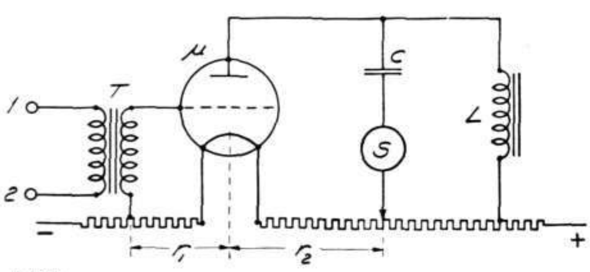
Compensation in both cases occurs when

$$r_2 : r_1 = \mu : 1$$

In figs. 9 and 10 the neutral point of the filament is shown in a central position, but in actual practice it is generally asymmetrically situated. We will return later to this and other factors affecting the compensation.

In multi-valve sets with transformer coupling the same idea may be applied to each separate valve stage, in which case the loud speaker will be replaced by the primary winding of the transformer in a preceding valve stage. In resistance-coupled receivers the matter is quite different, as there the anode voltage and not the anode current has to be compensated.

b) *Compensation of anode voltage.* If we imagine a resistance-coupled valve stage with compensated anode current, the A. C. potential drop across the anode resistance would vanish, and the anode would thus have the same ripple potential as the anode voltage terminal. This anode A. C. potential, which is applied to the grid of the following valve, would accordingly be larger than if the valve were not compensated. With resistance coupling one obviously ought instead to increase the ripple current in the anode circuit in order to get so great a ripple potential drop along the resistance as to make the remaining potential difference between anode and cathode disappear, in just he same way as in the case



R 4136
Fig. 10. Grid-anode compensation with resistance voltage divider: adjustment on the anode side.

[10] French patent 617 668. *Compagnie Lorraine d'Électricité.* — U. S. patent 1 541 311. *S. E. Anderson, Western Electric Co., Inc.* — British patent 240 851. *K. G. Jonasson and E. G. Eriksson.* — British patent 261 110. *G. M. Wright.* — British patent 305 940. *British Thomson-Houston Co., Ltd. (E. W. Kellogg).* — British patent 305 944. *Marconi's Wireless Telegraph Co., Ltd. (W. van B. Roberts).*

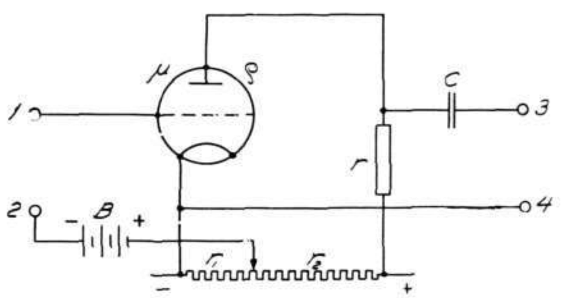
of fig. 6. The arrangement thus obtained[11] is shown in fig. 11. For practical purposes complete mains supply may be obtained in the same way as in fig. 9.

A method of coupling which has been commonly used, particularly for the last valve, is the choke-condenser coupling (fig. 12). If the impedance of the choke is large compared to the reduced impedance of the valve and the loading circuit, the choke will act as an effective suppressor of the mains hum as well. In other words, the anode current will be automatically smoothed and, if hum is to be completely eliminated, we have also to free the grid voltage from ripple, for which the simplest method is to use a resistance-capacity smoothing circuit, R—C in fig. 12. For the higher hum frequencies occurring with a generator-fed D. C. supply, this condition regarding the magnitude of the choke impedance will ordinarily be satisfied. But such is not the case with most rectifier mains, and still less where A. C. supply is used. The anode voltage variations due to pulsating grid voltage can then, however, be compensated in a relatively simple manner by means of a circuit $R'-C'$, shown by the dotted lines in fig. 12. The condencers C and C' are assumed to have small impendances in comparison with the resistances R and R', and further the choke resistance is neglected. If the ripple current in the voltage divider is denoted by I and that in the anode circuit by I_a the ripple voltages on anode and grid can be written respectively

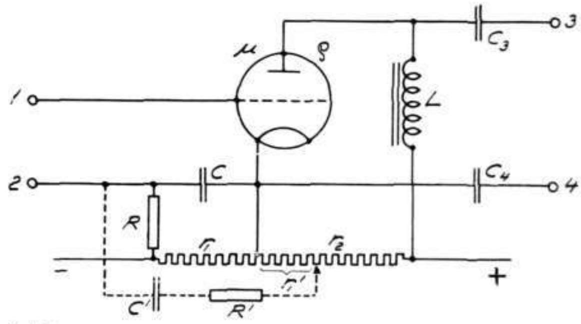
$$V_a = r_2 I - j\omega L I_a = 0,$$
$$V_g = \frac{1}{j\omega C}\left(\frac{r_1' I}{R'} - \frac{r_1 I}{R}\right).$$

In addition we have

$$\mu V_g = \varrho I_a,$$



R 4137
Fig. 11. Grid-anode compensation with resistance voltage divider: resistance-capacity coupling.



R 4138
Fig. 12. Grid-anode compensation with resistance voltage divider: choke-condenser coupling.

whence, with the help of the two foregoing equations:

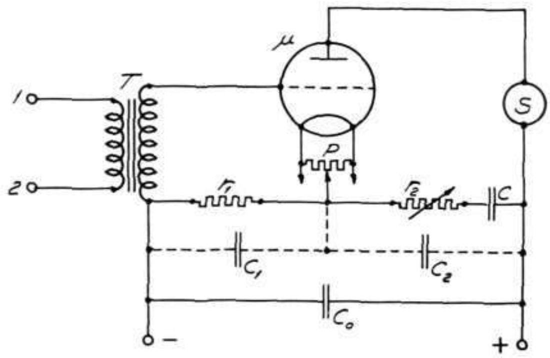
$$\frac{r_1'}{R'} - \frac{r_1}{R} = \frac{\varrho C}{\mu L} r_2 = \frac{C}{s L} r_2.$$

c) *Methods specially adapted for A. C. receivers.* All the methods so far described with grid-anode compensation have the characteristic point in common that they include a voltage divider across the current supply as an element essential for the compensation. This voltage divider is always required in D. C. receivers to serve as a series resistance for the valve filaments. But it is quite a different matter with A. C. sets. Here such a voltage divider would introduce an unnecessary load on the rectifier valve, and if we try to avoid this difficulty we are faced with others. This is perhaps best illustrated by a concrete example.

The arrangement shown in fig. 13[12] is a direct application of the same method of compensation as is shown in figs. 5, 9, and 10. The valve filament is assumed to be fed with alternating current, and, for this to have no disturbing effect on the anode and grid circuits, the latter are connected to the neutral centre point of a potentiometer P connected across the filament (for further details see under III : 4b). The positive arm of the voltage divider, r_2, is blocked against direct current by the condenser C. The negative arm, r_1, at the same time has to provide automatic grid bias, and its size is therefore settled. For compensation we must make $r_2 = \mu r_1$. The capacity C will then be fixed by the fact that the impedance corresponding to the fundamental frequency of the voltage pulsations has to be small

[11] British patent 261 110. *G. M. Wright.*

[12] U. S. patent 1 806 813. *B. F. Miessner, Radio Corporation of America.*



R 4139
Fig. 13. Grid-anode compensation with resistance voltage divider, the positive branch of which is blocked against direct current.

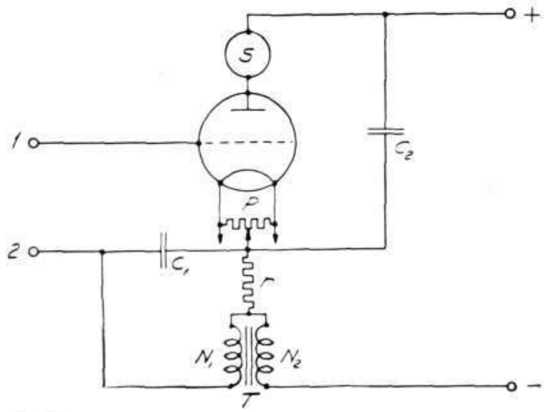
compared with r_2. On account of expense this condition cannot be satisfied as exactly as it can be, for example, in the circuit shown in fig. 9, in the case of the condenser C in relation to the resistance R, but otherwise there is no objection to the actual compensation. A serious criticism can, however, be made against the performance of the valve as an amplifier. The by-pass condenser which, particularly in A. C. receivers, is necessary as a return path for the anode alternating current is ordinarily connected to the cathode, but has here to be connected right across the whole voltage divider (C_o) in order not to upset the compensation. The result is that the anode alternating current will pass through the resistance r_1, setting up an extra alternating potential on the grid of approximately opposite phase to the original one. In other words, a negative reaction, a decrease in amplification, is produced, and with normal values of r_1 this effect will be fairly considerable. In a D. C. receiver the resistance r_1 is only about a tenth as large, and hence this bad effect is negligible.

From the point of view of compensation the only object of the voltage divider is to apply the ripple voltages to the anode and grid of the valve in opposite phases and in the ratio $\mu : 1$. It need not, however, necessarily consist of purely ohmic resistances, but may just as well be made up of any other impedances, provided the two arms on either side of the cathode have the same phase angle. For this reason it is possible, in the circuit shown in fig. 13, to avoid the reaction effect while maintaining the compensation, by introducing, in shunt with the two arms of the voltage divider, two condensers C_1 and C_2 with a capacity ratio of $\mu : 1$. The condenser C_2 may then suitably replace C_o as return path for the anode alternating current, but its size must be kept down as low as permissible, so as not to make C_1 excessively large. Although, if an electrolytic condenser were used, its size would be of less importance, it is not quite certain that such condensers are as constant as is necessary for compensation purposes.

Another attempt to obtain grid-anode compensation in A. C. receivers is represented by the arrangement[13], shown in fig. 14. Here a special transformer T has been employed to produce a compensating ripple potential on the grid. On the simplifying assumption that the impedances of the transformer windings are large compared to the impedances of C_1 and r, the following ratio of turns is required for compensation:

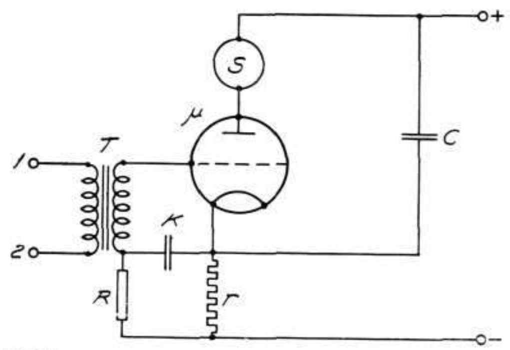
$$\frac{N_2}{N_1} = \frac{C_1}{\mu C_2}.$$

With the same number of turns in both windings, the transformer would only act as a choke in series with the resistance r, so that the condensers C_1 and C_2 would form a capacity voltage divider across the D. C. supply. Since the capacity of the condenser C_2 should amount to one or more μF, as it forms the return path for the anode alternating current, it is possible, by a suitable step-up ratio ($N_1 > N_2$), to gain the advantage that C_1 need not be made excessively large. In return, the need for a special transformer is a serious drawback in this circuit.



R 4140
Fig. 14. Grid-anode compensation with auxiliary transformer.

[13] U. S. patent 1 842 977. *B. F. Miessner, Radio Corporation of America.*



R 4141
Fig. 15. Grid-anode compensation with non-uniform voltage divider (C, r) and phase shifting (by means of K, R).

A simpler solution is provided by the arrangement shown in fig. 15[14]. In this circuit there is actually nothing to indicate compensation, for it contains nothing but the ordinary parts of an A. C. receiver. It is, however, the values of these that is decisive. The condenser K and the resistance R do not serve here, as they generally do, to smooth the grid bias, but to apply a compensating ripple voltage to the grid. If this circuit is compared with that given in fig. 13 we may explain its effect by saying that no attempt has been made here to obtain equality of phase between the two impedance arms of the voltage divider on each side of the cathode; instead, a phase shift of 90° has been produced in the ripple voltage taken out from the resistance r, this voltage in its turn being shifted 90° in phase from that across the condenser C. In this way the desired phase difference of 180° is obtained between the grid and anode voltages.

When the valve is compensated the whole ripple current I passes through the condenser C and the resistance arms r and R in parallel. The latter resistance, R, is assumed to be great compared with the impedance of the condenser K. The ripple voltages at the anode and grid respectively will then be:

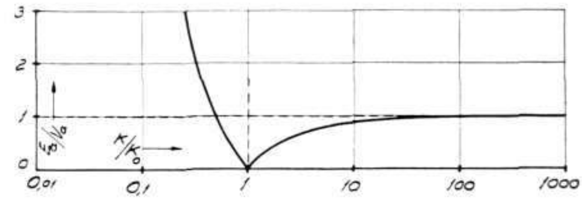
$$V_a = \frac{1}{j\omega C} I,$$

$$V_g = -\frac{1}{j\omega KR} \cdot \frac{Rr}{R+r} I.$$

For compensation it is required that $V_a = -\mu V_g$, which gives:

$$K(R+r) = \mu C r.$$

If the capacity K is varied while other quantities are kept constant, the resulting ripple E. M. F. in the anode circuit, E_a, will be as shown by the curve in fig. 16. At high values of K, the grid voltage is smoothed, and the said E. M. F. will then consist only of the ripple voltage across the condenser C. As the capacity K is reduced, the grid and anode ripple voltages partially neutralize each other, until complete compensation takes place at the value K_o. At lower values of K the grid ripple voltage prevails, rising very rapidly.



R 4142 Fig. 16. Effect of variation of capacity K on residual ripple voltage in the anode circuit of fig. 16.

In order not to have the capacity K unnecessarily large, the resistance R should be made relatively large, r being then negligible in comparison, so that approximately $KR = \mu C r$. In practice the accuracy of the compensation will depend on the degree, to which

$$\frac{1}{\omega_o K} \ll R,$$

where $\omega_o = 2\pi \times$ the lowest disturbing ripple frequency. If this relation is combined with the condition for compensation, it may also be expressed as follows:

$$C \gg \frac{1}{\omega_o \mu r}.$$

All three quantities on the right-hand side may be regarded as given. As an example we may choose the following typical values: $\omega_o = 2\pi \cdot 100$ rad./sec., $\mu = 10$, and $r = 1000$ ohms, which give $C \gg 0{,}16\ \mu F$. With the capacity value required for the return path of the anode alternating current, *i. e.* 2 to 4 μF, the compensation will thus be very marked. It is to be observed that if the above condition for the impedance of the condenser is satisfied for the fundamental frequency of the voltage pulsations, it will be even better so for the harmonics.

To a certain extent the compensation can be sharpened by making the resistance r larger than is necessary for the grid bias, which in that case should be taken out across only a portion of the

[14] British patent 339 851. *Telefon A.-B. L. M. Ericsson (E. O. Löfgren).*

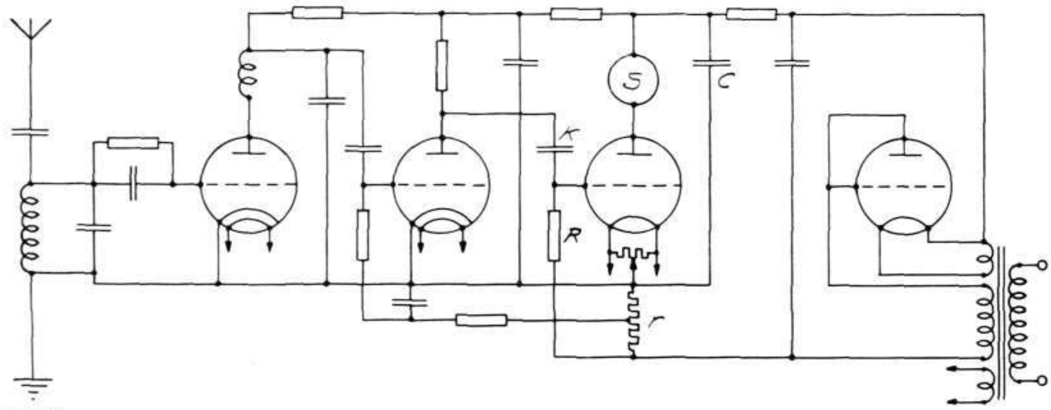


Fig. 17. Compensation as in fig. 15 applied to the last valve of a resistance-capacity coupled receiver.

resistance. At the same time a higher D. C. voltage will be required from the anode current supply on account of the voltage drop. If for some reason very sharp compensation is desired, there are better methods. The simplest way is to introduce in series with the condenser C a correction resistance, the size of wich is given by the following expression:

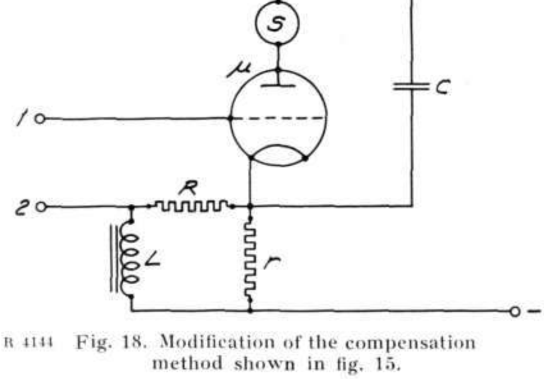
$$r' = \frac{1}{\omega^2 C^2 \mu r}.$$

Since the frequency enters into the formula, this method will only be suitable when there is a dominant pulsation frequency, as, for instance, the fundamental frequency from a rectifier. An even better, but at the same time more expensive, method of obtaining a phase correction in the compensation is to insert in parallel with the condenser C a resistance of the value μr, which, with regard to the direct current, should be connected in series with a condenser of about the same capacity as C. When deciding whether such sharp compensation is advantageous economically, one should not overlook that either there must be some possibility of adjustment or else very strict demands must be made on the constancy of the quantities involved.

An attempt to make this compensation arrangement even simpler is illustrated in fig. 17. In a resistance-coupled three-valve receiver the anode and grid voltages of the first two valves are smoothed by resistance-capacity filters, and there is a similar filter for the total anode current. But this will not be enough for the last valve if reasonable values of resistance and capacity are used, and to reduce the A. C. component through the loud speaker S, the last valve is therefore compensated by making the grid condenser and grid leak take the functions of the coupling elements K and R in fig. 15 respectively. Complete compensation cannot of course be obtained in this way, as the internal resistance of the preceding valve is in series with the grid condenser K, but this will only be noticeable at high frequencies, and these not only have lower amplitudes but are also more effectively attenuated by the resistance-capacity filter. When a higher degree of hum elimination is aimed at, it is advisable to substitute the resistance R with a small choke. In one commercial design[15] the total capacity of all the condensers in the receiver amounted to only 3,6 μF.

The phase shift of 90° in the A. C. voltage taken out from the resistance r can also be obtained by a combination of an inductance coil and a resistance, as in fig. 18. The condition for compensation will then be:

$$\frac{L}{R} = \mu C r.$$



R 4144 Fig. 18. Modification of the compensation method shown in fig. 15.

[15] *Svenska Radio A.-B. 1929.*

This circuit also can, in combination with a preceding valve, be made so that the components serve other purposes at the same time, namely by using the inductance coil as a grid choke, as in fig. 19. The resistance *R* here consists of the internal resistance of the preceding valve connected in parallel with its external anode resistance. In relation to this the coupling condenser *K* should be of low impedance.

Disturbing factors are the resistance and self-capacity of the coil. On this account the compensation obtained will not be as good as in the circuit given in fig. 15.

A grid-anode compensation of a very special kind is shown in fig. 20. This is an A. C. receiver where the anode voltage variations in the last valve are compensated, or rather reduced, by means of the filament A. C. voltage. The compensating A. C. grid voltage is obtained by displacing the tap on the potentiometer for the grid and anode circuits from its neutral centre point. For compensation to be possible the anode voltage rectifier must be of the half-wave type; the fundamental frequency of the voltage pulsations will then be equal to that of the A. C. supply. The phase conditions are rather troublesome. Already the A. C. voltage across the reservoir condenser C_1 following the rectifier lags slightly behind the A. C. voltage in phase, and the phase difference is still bigger after the filter $R-C_2$. To make the best of the situation a resistance instead of a choke has been chosen for the series impedance in the filter. Obviously no very great freedom from mains hum can be obtained with such an arrangement, but the commercial receiver[16] in which it was embodied was designed mainly with an eye to low cost. It does not seem impossible, however, to develop this method of compensation.

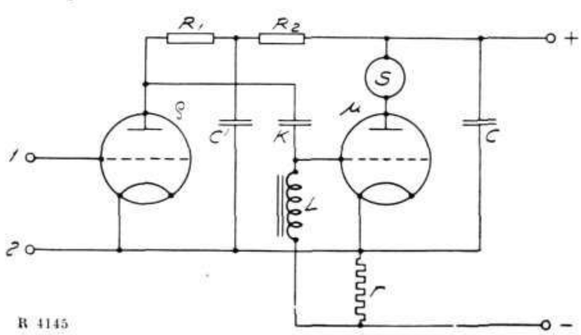

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Fig. 19. The compensation method of fig. 18 used in conjunction with resistance-capacity coupling.

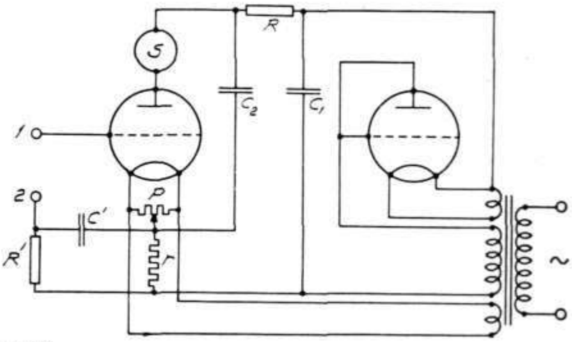
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Fig. 20. Compensation of anode voltage variations by means of a filament voltage potentiometer *(P)*.

d) *Compensation with multi-grid valves.* Most of the above remarks concerning compensation with triode valves can with little modification, depending on the number of electrodes, be applied to valves with two or more grids. For small changes of voltage, the resulting control A. C. voltage V_s in the anode circuit can, in terms of the anode and grid A. C. voltages V_a and V_{g1}, V_{g2}, etc. respectively, be written in the simple form

$$V_s = V_a + \mu_1 \; V_{g1} + \mu_2 \, V_{g2} + \ldots,$$

where the coefficients μ_1, μ_2, etc. are the amplification factors corresponding to the grids. For compensation we have as usual to make $V_s = 0$, which can here be done in various ways. Theoretically we can give all the A. C. voltages on the right-hand side in the above equation arbitrary values except one, which will then be determined by the condition for compensation. We can, for instance, in valves with two grids allow the anode and the auxiliary grid to retain their "natural" ripple voltages, and obtain the compensation by means of the control grid. It is also possible to let the control and auxiliary grids interchange their functions as regards compensation, or we may compensate on the anode side. Finally, we should also consider the possibility of smoothing the voltage for one of the three electrodes: the grid, auxiliary grid, or anode voltage, in that order from the point of view of expediency. This alternative is of great importance in cases where phase conditions are troublesome.

e) *The effect of the input impedance.* Among the various factors which may disturb the com-

[16] Manufactured by *Telefunken Gesellschaft für drahtlose Telegraphie, m. b. H.*

pensation we must first consider the impedance of the input circuit. Only if this is small compared with the grid impedance of the valve, the actual ripple potential at the grid will be equal to that at the grid voltage tap. A particularly favourable case is when the valve acts as an anode-bend rectifier with only a high-frequency coil in the path of the ripple current. Under all other conditions it will be necessary to give a certain amount of attention to the input circuit impedance.

It is fortunate that the grid impedance rises with decreasing frequency, for in A. C. receivers and where D. C. sets are connected to rectifier-fed mains, the strongest ripple voltages occur at low frequencies. The comparatively high ripple frequencies, up to several thousand cycles per second, occurring in generator-fed D. C. mains, become rather noticeable if an input circuit with too large impedance, *e. g.* a gramophone pick-up with an extremely large number of turns in the coil, is connected to a receiver with high amplification. But even for receivers with heavy smoothing such circuits are best avoided, as they are apt to pick up hum capacitatively from the mains. When necessary, a certain correction for high ripple frequencies may be made in D. C. receivers by introducing a condenser of very small capacity between the grid and a suitable point on the voltage divider.

3. Bridge Methods for Compensation.

A special class of compensation circuits is based on the bridge principle. The discharge path of the valve will then form one arm of a bridge, one diagonal being the pulsating D. C. source, the second the loud speaker or other corresponding output circuit. The grid voltage is normally assumed to be smoothed, but even if it is not, compensation is possible under certain conditions.

Fig. 21 shows a compensation arrangement[17] based on the simple Wheatstone bridge. This, of course, is chiefly suitable for D. C. receivers and then only for valves with low anode current. With an output valve the resistance *r* would either be so small as to considerably reduce the amplification, or else the voltage drop in it would be too great.

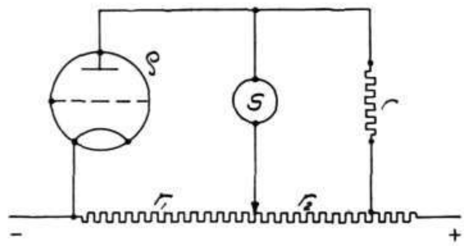


Fig. 21. Compensation by means of Wheatstone bridge.

The same disadvantages are inherent in the circuit shown in fig. 22, which is based on Wien's bridge. It has, however, certain features of value in A. C. receivers. More favourable properties are possessed by the circuit given in fig. 23, embodying the Maxwell-Hay bridge principle[18]. If the effective resistance of the choke coil is neglected, the condition for compensation will be

$$L = \varrho R C,$$

independently of the ripple frequency (the same, of course, applies also to the two previous bridge circuits). Besides this the circuit of fig. 23 satisfies in a convenient way the demands that may be made on a compensation circuit for A. C. receivers: small losses in anode voltage, low return impedance for the anode alternating current (*via* condenser *C*), few additional parts. As a matter of fact the resistance *R* is the only part that has been added for the compensation. If this is removed we come back to the well-known parallel-fed loud-speaker output circuit. Sometimes it may also be an advantage to have no D. C. load on the loud speaker. The parts *K* and *R'* in figs. 22 and 23 only serve to smooth the grid bias

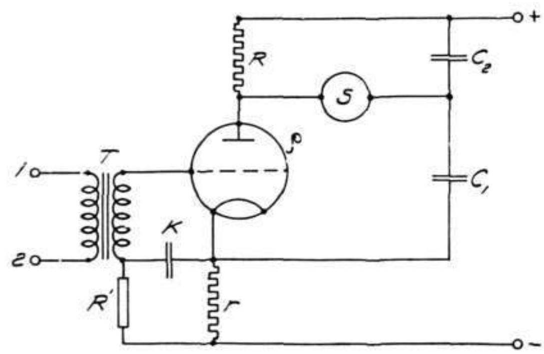
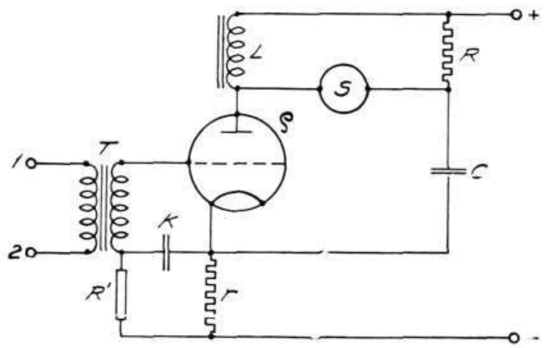


Fig. 22. Compensation by means of Wien's bridge.

[17] Swedish patent 65 981. *A. F. N. Lindsjö.*

[18] British patent 304 309. *Hazeltine Corporation (H. A. Wheeler).*



R 4149
Fig. 23. Compensation by means of Maxwell-Hay's bridge method.

voltage obtained through the voltage drop in the resistance *r*.

Compared with grid-anode compensation methods, the bridge circuits have few special advantages, rather the reverse. The compensation is here dependent on the valve resistance, which is not as constant as the amplification factor, either during the working life of the valve or in the course of one oscillation when the amplitude of the A. C. voltage is large.

4. The Effect of Variations in the Filament Current.

The foregoing is based on the tacit assumption that we can speak of a well-defined ripple potential on the cathode relative to anode and grid. It will be seen immediately that this is true where indirectly heated valves are concerned. Even with direct heating by a separate filament battery, conditions will be almost as simple, as the potential differences between various points on the filament caused by the variation in the anode current will be negligibly small. But in directly heated valves fed from D. C. mains with pulsating voltage, or from a pure A. C. supply, a definition of the cathode ripple potential will meet with difficulties. The various points on the filament will then have varying potentials relative to one another, and the question is whether it is possible to find on the filament, or on a voltage divider connected in parallel with it, any neutral point which corresponds as far as the A. C. potential is concerned to the cathode of an indirectly heated valve.

a) *D. C. feed.* Conditions are quite different with D. C. and A. C. feeds, and we will therefore consider the two cases separately. A theoretical investigation of a filament fed by pulsating direct current, indicates that if the wire were perfectly symmetrical and the emission of electrons varied linearly with the control voltage (straight-line characteristic), there would be a neutral point at the centre of the filament (*cf.* fig. 9). If the curvature of the characteristic is taken into account, we find that the position of the neutral point is displaced towards the negative side. If the control D. C. voltage is gradually reduced, the curvature of the characteristic becomes more pronounced, and the neutral point approaches towards the negative end of the filament.

Apart from their influence on anode and grid A. C. voltages, the variations in the filament current have also another effect of some importance in compensated D. C. receivers. A change in the filament current means a change in the power supplied for heating the filament, and therefore also a change in the temperature of the filament. This last, however, will show a certain slowness on account of the thermal capacity of the filament. This is manifested by the fact that the temperature variations lag behind the filament current variations in phase. With the hum frequencies that occur in practice the phase difference is very nearly 90°. In phase with the temperature variations, and proportional to them, corresponding variations arise in the anode current. Their effect may therefore be represented by an alternating E. M. F. E_t in the anode circuit which, regarded as a vector, bears the following relation to the vector V_f of the ripple voltage across the filament:

$$E_t = -j\,\frac{\Theta}{\omega}\,V_f,$$

where Θ is a constant, characteristic of the valve*). This E. M. F. is not considered in compensation methods of the usual kind, and it is too small to produce audible hum where only a single valve stage is concerned. If there is further amplification after the valve in question, it may become noticeable, but in that case it is possible to introduce a small correcting component in the compensation voltage.

The principle of such a method of compensating for the effect of the temperature variations

*) A full treatment of this subject will be given in a later paper.

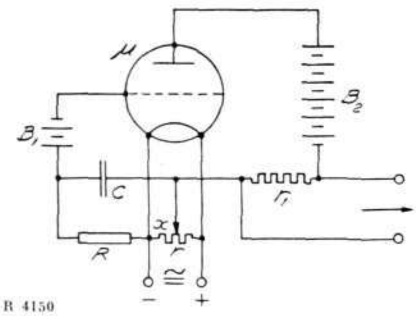


Fig. 24. Arrangement for investigation of the effect of filament voltage variations.

is shown in fig. 24. The idea of this circuit, which has been used to measure the magnitude of the E. M. F. E_t, is to apply to the grid an alternating E. M. F. of the same character as E_t but opposite in phase. The filament alone is fed with pulsating direct current; the grid and anode voltages are supplied by the batteries B_1 and B_2 respectively. The neutral point is tapped on a resistance r connected across the filament. Its distance from the negative end is called x. The anode circuit is connected direct to the neutral point, but the grid has two leads, one connected to the neutral point through a condenser C, and the other to one end of the filament through a high resistance R. The object of this disposition is to produce across the condenser a small A. C. voltage having a phase difference of 90° from the filament A. C. voltage, and being inversely proportional to the frequency, in exact similarity to E_t. To make the grid A. C. voltage opposite in phase to E_t, the resistance R must be connected to the negative branch of the filament circuit. We then obtain, if R is large compared to the impedance of the condenser C,

$$V_g = \frac{1}{j\omega CR} \cdot \frac{x}{r} V_f.$$

By varying the capacity C and listening in headphones connected *via* an amplifier to the resistance r_1 in the anode circuit of the valve, a distinct compensation point is found. This occurs when

$$V_g = -\frac{1}{\mu} E_t,$$

which gives the following value for the constant Θ:

$$\Theta = \frac{\mu x}{CRr}.$$

At one measurement (Philips' A 425) μ was 23, $R = 110\,000$ ohms, $r = 64$ ohms, $x = 17$ ohms (the neutral point being thus widely displaced from the centre) and $C = 0{,}36$ μF. This gives $\Theta \cong 150$. The value of the capacity remained unchanged at a frequency even as low as 50 cycles per second, which shows that the thermal slowness of the filament still prevails there. To illustrate the practical importance of variations in the filament current the following numerical example may be given: With the above valve a superimposed A. C. voltage of 1 per cent at 300 cycles per second would produce a disturbing E. M. F. E_t in the anode circuit of about 3 mV. For valves with larger filament current and lower filament voltage the constant Θ is only a fraction of the value mentioned above.

The method of compensating for filament temperature variations shown in fig. 24 may at will be applied at the same time as the usual grid-anode compensation. An instance of this is given in fig. 25*), where the circuit of fig. 9 is supplemented by a condenser C' and a resistance R', which have the same functions as C and R in fig. 24. For compensation it is required that

$$\frac{r_3 - r_1}{R'} - \frac{r_1 - r_o}{R} = \frac{1}{\mu} \Theta C r_f,$$

where r_f is the resistance of the filament.

As is seen here, the effects of these temperature variations are comparatively slight and do not by themselves justify the high degree of filtration used in D. C. receivers. The main object of smoothing the filament current actually is to reduce the ripple voltages impressed in the anode and grid circuits on account of these being connected with either end of the filament in-

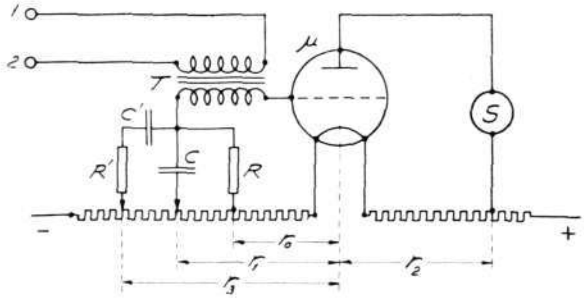


Fig. 25. Correction for filament current variations in the circuit of fig. 9.

*) Cf. also fig. 26.

stead of with the neutral point. The filament current in many cases would hardly need to be smoothed if these circuits were connected to taps at the neutral points on potentiometers across the filaments of the various valves, but such an arrangement would hardly be feasible in commercial receivers on account of the number of adjustments it would necessitate. In certain compensated receivers, however, these circumstances are of great advantage.

b) *A. C. feed.* When pure alternating current is used for the filament current feed, the temperature variations occurring in the filament will naturally be considerably larger, though not prohibitive. It is desirable to have a filament with high thermal capacity, and this has led to the production of special types of valves with low filament voltage and high filament current. These, however, have recently been largely superseded by the indirectly heated valves, and it is now only in the output stage that direct-heated valves are employed in A. C. receivers.

It is most important that the high A. C. voltage across the filament should be prevented from getting into the grid or anode circuits, and this is done by tapping a neutral point on a potentiometer connected across the filament[19] (see fig. 13). The position of this tap is generally very close to the centre, as is to be expected in consideration of the fact that, apart from slight irregularities in the structure of the valve, there is complete symmetry between the two filament current branches. In this way it is possible completely to eliminate anode current ripple of the same frequency as the filament current. Most of the remaining ripple is of double the frequency. This is due partly to the temperature variations of the filament and partly to the fact that the anode current, owing to the curvature of the valve characteristic, in the course of one half-cycle of the filament voltage increases more in one half of the filament than it decreases in the other half. Of the two corresponding E. M. F.s in the anode circuit, the former lags very nearly 90° behind the filament power curve, while the latter is in phase with it.

In order to compensate the alternating E. M. F. produced in the anode circuit by the filament current, an A. C. supply of twice the frequency of the filament current must be available. This is found in the rectifier for the anode current, provided this works with full-wave rectification. The problem will be first and foremost how to bring the anode A. C. voltage into exactly opposite phase to the resulting E. M. F. from the filament, and then how to adjust its size correctly. This method of achieving compensation first appeared in connexion with transmitters, where it was possible to resort to the very simple method of feeding the oscillator filament and the anode current rectifier from separate A. C. generators of the same frequency and with their shafts mechanically coupled[20]. The required phase shift could be obtained by adjustment of the shaft coupling. The anode A. C. voltage could be adjusted, for example, by properly choosing the capacity of the reservoir condenser following the rectifier. In a mains-operated receiver the phase shift of the anode A. C. voltage will naturally be rather difficult to vary. In that case one cannot, as in D. C. receivers, expect any very exact compensation. Both the anode A. C. voltage and the E. M. F. originating from the filament contain a series of harmonics, and these can hardly be made to compensate each other at the same time as the fundamental frequency.

IV. JOINT COMPENSATION OF SEVERAL VALVE STAGES.

In receivers with several valve stages it might be possible, as mentioned above, to compensate each stage separately, but this method is of hardly more than theoretical interest, as in practice it would be too inconvenient. It is therefore usual in multi-valve receivers to apply compensation only in that valve stage, generally the last, where filtration is most expensive, but otherwise to provide the apparatus with smoothing equipment. There is, however, another alternative, which may be very economical, namely, to design the receiver in such a way that a single adjustment is enough to compensate all the valves in common.

This method of compensation has been used chiefly in D. C. receivers. In A. C. receivers it

[19] French patent 575 508. *R. Depriester.* — Cf. British patent 218 681. *British Thomson-Houston Co., Ltd.*

[20] British patent 226 555. *Telefunken Gesellschaft für drahtlose Telegraphie, m. b. H.*

has only been suggested for reducing the hum caused by the A. C. feed to the valve filaments[21].

If we regard the hum current in the loudspeaker circuit as made up of components from all the different ripple voltages impressed on the various electrodes of the valves, it seems at least theoretically possible to adjust but one of these voltage components so that it neutralizes all the others. To keep this compensation voltage correct both in size and phase over the whole range of ripple frequencies without complicating the apparatus too much, the best plan is to maintain as far as possible, all the ripple voltages in the receiver in the same or directly opposite phases. In D. C. receivers with a voltage divider this condition is accurately satisfied with regard to the ripple voltages tapped on the voltage divider. But this alone is not sufficient, for the ripple voltage components transferred to the grid of the last valve are also dependent on the properties of the intervalve coupling units. Another essential condition is accordingly that the coupling units must not introduce amplitude variations or phase shift within the range of the ripple frequencies. For this reason resistance-coupled receivers are generally best suited for compensating in this way. Quite fair compensation can often be obtained with transformer coupling, but for the same amplification the remaining hum will be more prominent. Experience has shown that in a resistance-coupled three-valve receiver it is possible to compensate mains hum more completely than in a transformer-coupled two-valve apparatus.

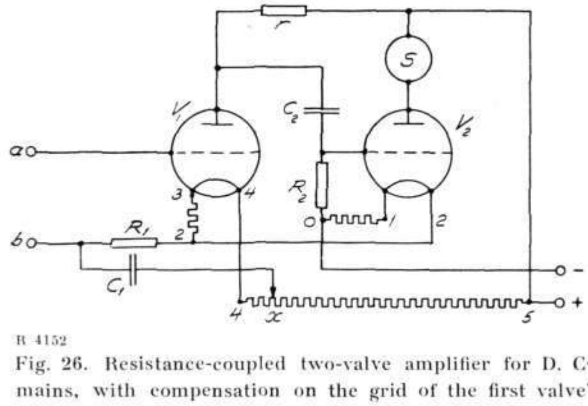
The factors which determine the correctness of amplitude and phase in a resistance coupling are, firstly the coupling capacities and grid leak resistances, secondly the stray capacities of the valve electrodes and connected leads. The former quantities only affect the lowest part of the frequency range, the latter only the highest. In D. C. receivers for generator-fed mains the coupling capacities and grid leak resistances may be of the usual sizes, but with rectifier-fed mains and in A. C. receivers they will have to be made larger. As far as stray capacities are concerned, there are hardly any other precautions at one's disposal, provided the wiring is properly done, than to avoid the excessively high anode resistances that are sometimes favoured.

1. Special Compensation Electrode.

Perhaps the most natural way of compensating a multi-valve set as a whole is to impress, by means of a circuit arrangement of the same kind as those used at the grid in fig. 9 or the anode in fig. 10, a ripple voltage on one of the valve electrodes direct from the voltage divider, independent of the D. C. voltage, and otherwise to dispose the feeders in whatever way suits the D. C. voltages best[22].

Two instances of this principle are given in figs. 26 and 27. The first shows a two-valve amplifier, where a voltage divider serves the double purpose of supplying the anode and grid voltages and of acting as a series resistance to the valve filaments. The taps of the voltage divider are so arranged that the last valve utilizes the full mains voltage. The compensation is applied on the grid of the first valve, the biassing voltage of which is taken through the resistance R_1 from the point 2, while the ripple voltage is obtained through the condenser C_1 from a variable tap x.

The simplest way of explaining how this circuit works is to start with the second valve. For this to be compensated, its grid must be given a ripple voltage $1/\mu_2$ times that of the anode, measuring both from the neutral point of the filament, but exactly opposite in phase (μ_2=is the amplification factor of the second valve). Although the ripple voltage at the point *0* is of the right phase, it will have very little influence on the resulting grid A. C. voltage, as the grid re-



R 4152

Fig. 26. Resistance-coupled two-valve amplifier for D. C. mains, with compensation on the grid of the first valve'

[21] British patent 299 089. *Gramophone Co., Ltd., and B. E. G. Mittell.* — British patent 299 908. *De W. C. Tanner.* — U. S. patent 1 790 874. *B. F. Miessner, Radio Corporation of America.*

[22] British patent 318 922. *Telefon A.-B. L. M. Ericsson (E. O. Löfgren).*

sistance R_2 is large compared with the internal resistance of the first valve. Even if the grid were connected direct to the voltage divider instead of to the anode of the first valve, it would be impossible to find a point on the voltage divider with the ripple potential required for compensation. We might, however, imagine a position of the connecting point on a fictitious extension of the voltage divider on its negative side, approximately as far from the point *0* as point *1* is on the positive side. The reason for this was given previously in reference to figs. 5 and 9. As in reality, however, the ripple potential for the grid of valve V_2 is taken from the anode of valve V_1, it can be varied over a very wide range by adjusting the tap x. If this is placed on the

An interesting feature of the compensation circuit in fig. 26 is the possibility of making a correction for the filament current variations solely by properly choosing the values of R_1 and C_1.

Another application, in some respects different, of the same compensation principle is illustrated in fig. 27. This is a three-valve amplifier, which is shown here chiefly to make it clear that compensation need not necessarily be dependent on the voltage divider serving as series resistance for the filament current and simultaneously offering a convenient means of tapping the required compensation voltage. Here the compensation is applied to the anode of the first valve, which gets its ripple voltage from a voltage divider formed by the resistance R in the anode circuit and the

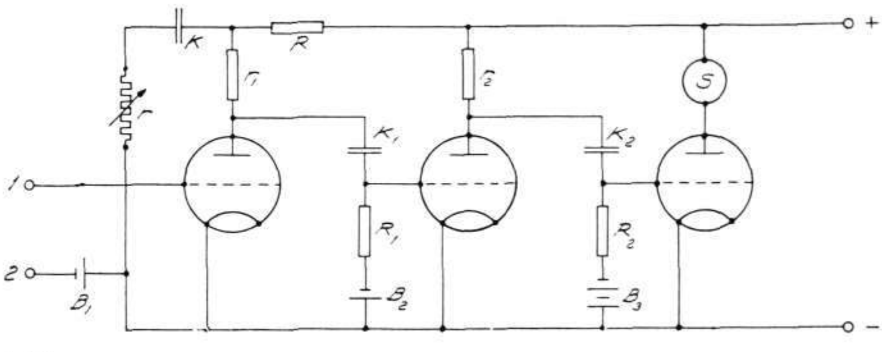


Fig. 27. Resistance-coupled three-valve amplifier with compensation in the anode circuit of the first valve.

positive side of the cathode of valve V_1, the ripple current in the anode circuit will be increased, and thus also the potential drop in the resistance r. The anode current of valve V_2 will be compensated when the ripple potential drop in the resistance r becomes so great as not only to cover the whole ripple voltage across the voltage divider, but to exceed it by as much as corresponds to the above imaginary extension of the voltage divider on its negative side.

In a multi-valve receiver of this kind the compensation can theoretically be applied to any of the valve electrodes. In practice, however, the choice is limited. In the circuit shown in fig. 26, the only suitable electrode is the grid of the first valve. For compensation to be possible by means of the anode voltage of either valve, the voltage divider would have to be tapped far out along an extension on the negative side of *0*. The grid tap of the second valve cannot be used because the resistance R_2 is too large.

variable resistance r in series with the blocking condenser K. The tap on this voltage divider being on the positive side of the valve cathodes, and the ripple voltage at the anode being a fraction of that at the tap, the effect in respect of the grid of the second valve will be the same as in the circuit in fig. 26.

A simple transformer-coupled receiver[23] is shown in fig. 28, where the compensation is applied to the grid of the first valve by means of the variable tap x. A remarkable feature in this circuit is that no arrangements need be made to separate the D. C. and ripple voltages of the compensation electrode. This may be understood from the following. The first valve is an anode-bend rectifier and the grid bias will, therefore, be correct as long as the point x is not too far from the position where the valve is compensated.

[23] British patent 309 902. *Telefon A.-B. L. M. Ericsson (E. O. Löfgren).*

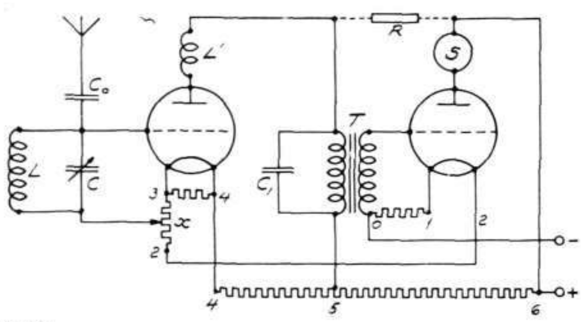


Fig. 28. Transformer-coupled two-valve receiver, with compensation on the grid of the first valve.

Actually, this is also the case. The object is here certainly not to compensate the first valve by itself, but to impress on the second valve through the transformer a ripple voltage which will so supplement the voltage from the resistance *0—1* that the resulting ripple voltage on the grid will be that required for compensation. On account of the step-up transformer, however, a relatively small primary voltage will be needed, which means that the first valve will not be far from compensation.

To give the ripple voltage induced in the secondary winding approximately the right phase, the impedance of the primary winding must be large in comparison with the internal resistance of the detector valve, which is also a condition for high-quality reproduction. Partly for this reason and partly in view of the reaction, it is desirable to get x displaced a little on the positive side of the compensation point of the valve itself. The rectification admits of some latitude in this respect. To get x on the positive side of the compensation point, the primary winding of the transformer has to be connected the right way. If required, it is possible to make x fall more towards the positive side, namely, by introducing between the anode and the positive end of the voltage divider a large resistance R. If this is made as large as several hundred thousand ohms, it will have very little influence on either the D. C. conditions or the amplification, but will affect the compensation position of x considerably.[24]

If owing to unfavourable properties of the transformer, compensation should not be quite satisfactory, there are nevertheless certain methods of obviating this deficiency. No general rules can be given, as transformers of different makes may vary widely. Each will have to be tried out separately. It has often proved effective to connect the by-pass condenser C_1 with some other point on the voltage divider than point *5*. Or it may be advantageous to insert a very small condenser between the grid of the second valve and a suitable point on the voltage divider. Another important factor is the point to which the iron-core of the transformer is connected. Finally, the hum may be considerably reduced by shunting a large resistance across the secondary winding of the transformer, though this will also affect the amplification.

2. Compensation by Cathode Displacement.

We now come to an interesting type of resistance-coupled multi-valve receivers, where, in contrast with those having a special compensation electrode, no special components are required solely for compensation. This is accomplished entirely by adjusting the various taps on the voltage divider, but in this case it is not only the anode and grid taps that are displaced but also the cathode taps. This method[23] is best suited for D. C. receivers, rendering their design extremely simple.

A two-valve amplifier on this principle is shown in fig. 29. A characteristic feature of this circuit is that the grid and filament connecting points of the two valves (*x, 1, 2,* and *3, 4, 5,* respectively) are not placed in immediate succession, as is usually the case, but part of the voltage divider is interposed between them. The reason for this may be explained as follows.

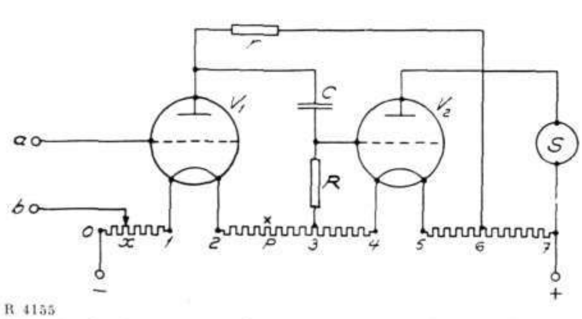
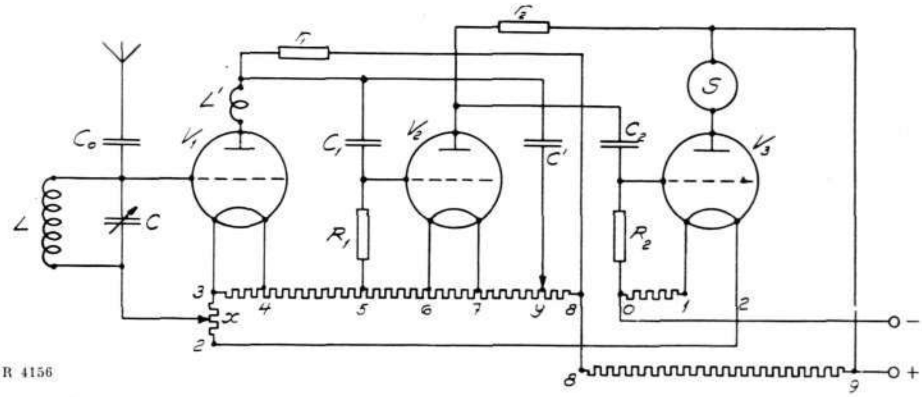


Fig. 29. Resistance-coupled two-valve amplifier, with compensation by cathode displacement.

[24] British patent 343 869. *Telefon A.-B. L. M. Ericsson (E. O. Löfgren).* Used by *Svenska Radio A.-B.*



R 4156

Fig. 30. Resistance-coupled three-valve receiver, with compensation by cathode displacement.

We assume that the taps for the valve V_1 are first placed on the voltage divider so that the cathode is as far towards the negative end as possible, with due allowance for the grid bias potentiometer, and that further the anode voltage is properly chosen. It will then be possible to find a point P on the voltage divider with the same ripple potential as the anode of valve V_1. This point may be referred to as the *anode point* of valve V_1. The connecting of a following valve will slightly alter its position, on account of the grid leak, but this does not affect the argument. It is now evident that with regard to the ripple current the valve V_2 will behave exactly as if its grid were connected to the anode point P on the voltage divider. It will, therefore, be compensated if its grid and filament taps on the voltage divider are located in such positions that the ratio of the resistance between the anode voltage tap 7 and the neutral point of the filament to that between the neutral point and the anode point P is equal to the amplification factor of the valve.

This disposition of the relative positions of the two valves on the voltage divider must of course be embodied in the design of the receiver. In a commercial set the connexions to the voltage divider must all be fixed, except one. Individual differences may occur in manufacture, both in the valves and in the other components. It must be possible to correct for these in the finished set, and for this reason one of the taps on the voltage divider is made adjustable. In practice the grid tap x of the first valve is usually the most suitable. If this is displaced sligthly towards the positive end, the anode point P will move rapidly towards the negative end, and *vice versa*. The grid bias of the first valve will, of course, also change at the same time as the ripple voltage, but if the receiver is properly designed the variations will keep within fairly small and quite permissible limits.

A practical disadvantage of the circuit shown in fig. 29 is that it does not take full advantage of the mains voltage, as only part of this is used for the anode voltage of each valve. With only two valves this is unavoidable, but with three we have more liberty in choosing the anode voltages, as will be seen from fig. 30. This shows a rather popular Swedish three-valve receiver[25]. The circuit diagram is obtained in the following way. The grid and filament taps of the last valve are first fixed nearest to the negative end of the voltage divider, and its anode circuit is connected to the positive end. It will thus utilize the full voltage of the mains. Immediately after the grid and filament taps of the last valve follow those of the first, for which a suitable anode voltage, not too high, is chosen. Between the first and second valves is interposed part of the voltage divider in the same way as in fig. 29, and finally the anode circuit of the second valve is connected directly to the positive end of the voltage divider. Assuming the connecting points 5, 6 and 7 of the second valve to be movable, compensation will be obtained in one distinct position.

With reference to the compensation circuits described above, the working of the apparatus in fig. 30 may be explained by saying that the first two valves function in relation to each other in much the same way as the two-valve circuit of fig. 29, while the second and third are

[25] Manufactured (with slightly modified circuits) by *Svenska Radio A.-B.*

together analogous to the two valves of fig. 26. To produce a compensating ripple potential on the grid of the last valve, we must increase the ripple current in the anode circuit of valve V_2 to such a value that the ripple potential drop in the anode resistance r_2 somewhat exceeds that in the whole voltage divider. This presumes that the grid of valve V_2 is given a ripple potential of the same phase as the anode circuit, which is done by connecting taps *5*, *6*, and *7* to the voltage divider *on the negative side of the anode point* of valve V_1. In consequence of the high amplification in valve V_2, the distance between its filament taps and the anode point of valve V_1 will be fairly small, and as further this last point lies somewhere between points *4* and *8*, though nearer the latter, it will be essential in this circuit for the second valve to be located on the positive side of the first one. Otherwise we are fully at liberty to place the valves how we like. The important point is, of course, that the full mains voltage can be used for the last valve. Apart from the economic aspects, there is another advantage in the disposition of fig. 20, namely, that the interaction between the feed circuits of the valves is neutralized. We will return to this phenomenon below.

As for adjusting the compensation by means of the potentiometer tap x, the same possibilities exist here as in the circuit in fig. 29. Another variable tap y is arranged for the by-pass condenser C from the anode of the detector valve. In order not to affect the compensation, the condenser should be connected to the anode point of the detector valve on the voltage divider. But there are also stray capacities from the anode and from the grid of the following valve to points having ripple potentials lower than that of the anode point (*3*, *4*, *5*, *6*, *7*, and the anode of V_2). We can compensate for such capacities by displacing the point y slightly towards the positive side of the anode point. As we are here dealing with a compensation of secondary importance, point y may be made fixed in commercial receivers.

With generator-fed D. C. mains, hum may be eliminated perfectly satisfactorily in a receiver like that illustrated in fig. 30, but with rectifier-fed mains compensation alone will hardly suffice.

Similar conditions as described above may arise in resistance-coupled receivers where the anode of one valve is connected directly to the grid of the next. If such a receiver is fed from a voltage divider, the cathode taps of the valves must be displaced relative to one another on the voltage divider so that the correct grid biassing voltages are obtained[26]. In this case, therefore, the principle of cathode tap displacement cannot be used specially to compensate the hum, but by choosing anode resistances and anode voltages suitably, it is *generally possible* to make the D. C. voltage conditions coincide more or less with those required for compensation of the hum.

[26] British patent 290 032. *J. F. Johnston.*

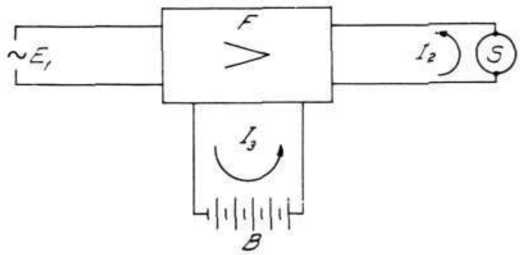
3. Elimination of Feed-Circuit Interaction.

In multi-valve receivers with a common current supply for the anode and grid voltages of the various valves, it is always a delicate problem to avoid interaction effects between the valves, or even between anode and grid circuits of the same valve. The usual way of achieving this is to employ separate smoothing devices for the various valve electrodes, so that, while suppressing the ripple currents, they also decouple electrically the various feed circuits. Sometimes this decoupling may lay more rigorous claims on the smoothing devices than the elimination of mains hum. Under such circumstances it is of great interest to learn how the compensated receivers are affected by this disturbing phenomenon.

We will first describe briefly, referring to fig. 31, how interaction arises owing to a common feed circuit. A multi-valve amplifier F is supplied from an anode battery B common to all the valves and having a resistance which is initially assumed to be negligibly small. Other sources of current, *e. g.* filament and grid batteries, are here left out of consideration. When an alternating E. M. F. E_1 is superimposed in the input circuit of the amplifier, a current I_2 will be produced in the output circuit, and another, I_3, through the battery. The amplitudes of currents and voltages are assumed to be small. Their relations can then be represented by linear equations:

$$I_2 = Y_{12} E_1,$$
$$I_3 = Y_{13} E_1.$$

The complex coefficients Y_{mn} correspond to mutual admittances in ordinary impedance networks, but differ from them in not following the law of reciprocity, or, otherwise expressed, $Y_{mn} \neq Y_{nm}$.



R 4157
Fig. 31. Amplifier circuit illustrating the appearance of interaction in common feed circuit.

Similarly, an E. M. F. E_3 in series with the battery B would produce a current $Y_{32} E_3$ in the output circuit. This may be applied when an impedance Z_3 is inserted in the battery circuit, since, as we know, this will have the same effect as a counter-E. M. F., which may be written

$$E_3 = - Z_3 I_3,$$

where I_3 denotes the original current. The current in the output circuit is assumed to be changed by this from I_2 to I_2'.

In computing this change in the current we must substitute for Y_{32} its value when the impedance Z_3 is connected in. If we call this Y_{32}', the change in the current will be

$$I_2' - I_2 = Y_{32}' E_3 = - Y_{32}' Z_3 I_3 = - Y_{13} Y_{32}' Z_3 E_1 = = - \frac{Y_{13}}{Y_{12}} Y_{32}' Z_3 I_2,$$

and the total current

$$I_2' = I_2 \left(1 - \frac{Y_{13}}{Y_{12}} Y_{32}' Z_3\right).$$

The last term within the brackets represents the change in amplification caused by interaction effect between the anode feed circuits *via* the common impedance Z_3. That quantity may have any direction in the complex plane, and the amplification may therefore, according to circumstances, be augmented or reduced. In the former case the apparatus may even go into self-oscillation, and in either case the change of amplification will in practice always vary with frequency and accordingly cause distortion.

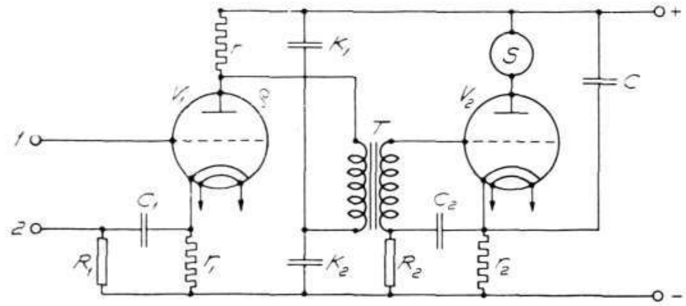
The object of separate smoothing devices would be to diminish the three quantities Y_{13}, Y_{32}' and Z_3. If instead the receiver were compensated for hum from the current supply B, an alternating E. M. F. connected in the feed circuit would produce no current in the output circuit, which means that

$$Y'_{32} = 0.$$

The interaction effect would thus be compensated at the same time as hum. The apparatus would behave as though it were fed from a battery having an internal resistance equal to zero.

The above argument can be applied directly to a circuit as in fig. 27. The matter is not quite so simple in receivers fed from a voltage divider with more than two taps, as in figs. 26 and 28—30. It is obvious, however, that if the alternating currents produced in various parts of the voltage divider by an incoming signal voltage are analogous to the respective ripple currents, the compensation of these latter will be accompanied by neutralization of the retroaction of the former on the output circuit. As an example, we will take the circuit of fig. 26. The signal alternating current in the voltage divider chiefly originates from the last valve. The anode alternating current of the first valve is so small as to be negligible in comparison. A point that deserves attention is, however, the fact that the anode alternating current of the last valve flows in opposite directions in the parts *0—1* and *2—5* of the voltage divider, while the direction of the ripple current is the same throughout the voltage divider. But provided the resistance R_2 is so large in comparison with the internal resistance of the first valve that the A. C. potential at point *0* does not considerably affect the grid A. C. potential, there is othervise no reason why the signal current through the voltage divider should differ from the ripple current in relation to the compensation. In both cases the effect on the loud speaker will be neutralized by the compensation. With regard to the signal current, this means that the A. C. potentials applied to the grid, cathode and anode of the first valve will, *via* the grid of the second valve, produce in the output circuit an E. M. F. which will exactly neutralize the voltage drop in the voltage divider. We might of course here again speak of a kind of interaction, but a useful one, since it eliminates the undesirable resistance of the feed circuit.

Quite similar conditions are found in the circuit shown in fig. 30, but a different case is represented by the transformer-coupled apparatus of fig. 28, as here the ripple voltage across the resistance *0—1* is a large proportion of the compensation voltage on the grid of the last valve. It would here be possible, however, to eliminate the feed-circuit interaction by smoothing the grid



R 4158
Fig. 32. Two-valve amplifier with compensated anode-feed interaction.

bias of the last valve and taking the whole compensation voltage *via* the transformer.

The conditions for the compensation of hum and of feed-circuit interaction are thus not necessarily identical, although frequently they may be. Cases are quite conceivable where the greatest importance attaches to the elimination of intervalve reaction and where, therefore, the apparatus is specially designed for this purpose[27]. As such a use of compensation methods is partly outside the scope of this paper, it may be enough to give only one more example here, fig. 32. This shows a circuit where, although both the mains hum and the anode-feed interaction are compensated, the latter compensation must be considered the more important. It is a transformer-coupled two-valve amplifier where the primary winding of the transformer forms one diagonal of a Wien's bridge, the arms of which are r, $\varrho + r_1$, K_1, and K_2, while the second diagonal consists of the anode voltage supply leads common to both valves.

Provided the bridge is compensated, any existing ripple in the anode voltage will, as we have seen before (fig. 22), be eliminated in the output circuit of the first valve, *i. e.* the primary winding of the transformer. The second valve can, for example, be compensated by the method shown in fig. 15. The interesting point here, however, is that the signal current from the second valve is also compensated in the primary winding of the transformer, while otherwise it would give rise to a reaction voltage on the grid of the second valve which would be troublesome at low frequencies. If the condenser K_1 is omitted, we return to the common parallel-feed circuit, but in this case the anode circuits of the valves will have to be provided with means for reducing the reaction effect. By means of the condenser K_1, whose capacity may be of the order of a tenth of a μF, the same effect is obtained in a more economical way.

[27] British patent 285 229. *Igranic Electric Co., Ltd., P. W. Willans and A. d'A. Hodgson.* — Cf. British patent 304 309. *Hazeltine Corporation (H. A. Wheeler).*

[28] British patent 354 453. *Gramophone Co., Ltd., and H. C. Atkins.* — U. S. patent 1 788 342. *B. F. Miessner, Radio Corporation of America.*

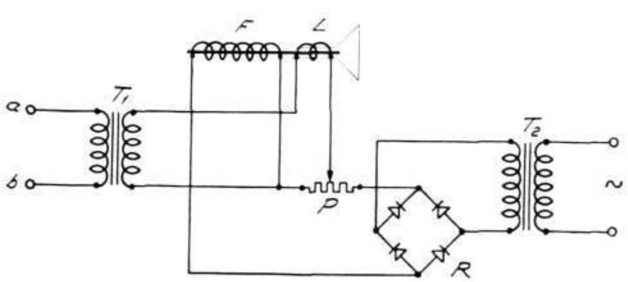
V. COMPENSATION METHODS OF PARTICULAR KINDS.

Apart from those generally applicable methods of compensation discussed above, there are in practice many opportunities of embodying the compensation principle by means of particular arrangements varying with the design of the apparatus. This is especially the case in A. C. receivers. One can, for example, make use of the external magnetic stray fields produced by the mains transformer, the choke coils, the field-coil of an electrodynamic loud speaker, etc.

To eliminate hum caused by stray fields, it is usual to place the parts sensitive to induction, *e. g.* coils, chokes and transformers, in a neutral position relative to the magnetic field. This method readily suggests the idea of slightly displacing them from the neutral position to one side or the other in order to obtain a compensation, or at least a reduction, of the remaining A. C. voltage components in the feed circuits[28]. Clearly, this method cannot, on account of the generally imperfect phase and frequency conditions, give any great freedom from hum, but as it hardly involves any additional expense its use is well justified.

Instead of the inductive coupling by means of the stray fields, other more or less ingeniously contrived impedance combinations may be used for the coupling between the alternating-current carrying feed circuit and the circuit in which the compensating counter-E. M. F. is to be introduced. It will then be possible, not only to give the compensating E. M. F. the right phase, but also to make it vary with frequency in the same way as the ripple voltage, which is usually made up of a series of frequencies. The design, however, is greatly complicated by the existing couplings between the circuits, necessitated by the D. C. conditions. In that respect the loud-speaker circuit is an exception when separated from the apparatus by an output transformer.

Fig. 33 gives a simple example of the use of the loud-speaker circuit for compensating purposes[29]. We have here an electrodynamic loud speaker, the speech coil L of which is coupled to the output valve of the receiver by a transformer T_1, while the field coil F is fed from the A. C. mains through a transformer T_2 and a rectifier R. This latter is assumed to be of the metal type, made up of four elements forming a bridge. With such an arrangement there will be no break in the magnetizing current, since, thanks to the high inductance of the field-coil, the current is drawn through the rectifier bridge even while the alternating current is passing through zero. The magnetizing current will thus be a relatively feebly pulsating direct current. With most of the usual models of loud speakers, however, the A. C. component is strong enough to produce an audible hum. It induces in the speech coil L an alternating current which, affected by the air-gap field of the magnet, causes the coil and the diaphragm attached to it to vibrate. To prevent this, the E. M. F. induced in the speech coil may be compensated, in the circuit shown in fig. 33, by means of an A. C. voltage introduced in the same circuit and obtained direct from the magnetizing circuit of the field-coil by means of a potentiometer P. The method is best suited for loud speakers with low magnetizing voltage, as the potentiometer resistance then required will be so small as not to affect appreciably the efficiency of the loud speaker.



R 4159

Fig. 33. Compensation of hum in a rectifier-fed electrodynamic loud speaker.

The now rather common method of using the field coil of an electrodynamic loud speaker as a choke for the anode direct current has drawn much interest to the compensation of the ripple voltage induced by the field variations. There are several other methods of doing this besides the one described above. Some form of compensation winding is frequently employed[30].

All these compensation methods should be applied with some care. Although the steady hum in the loud speaker is eliminated, the pulsations of the magnetic field in the air-gap will remain and give rise to a modulation of the driving force on the movable system due to the speech current. If this modulation exceeds certain limits, the quality of reproduction will suffer noticeably, a phenomenon which will be considered in the next section.

VI. THE APPLICATION OF COMPENSATION METHODS.

In the introduction to this paper it was observed that the elimination of mains hum is chiefly an economic problem, and that in most cases the compensation methods have originated in efforts to reduce the apparatus costs. This should not, however, be taken to mean that compensation can always take the place of smoothing. This may sometimes be the case, *e. g.* with generator-fed D. C. mains, but it is rather unusual. In general, a combination of smoothing and compensation is preferable. Thereby the advantages of both methods are obtained, while the drawbacks of carrying either method far enough to satisfy the present demands for feedom from mains hum are avoided.

As regards the smoothing method, it is certainly possible, with a reasonable expenditure of

[29] U. S. patent 1 733 232. *B. F. Miessner.*

[30] U. S. patent 1 105 924. *E. S. Pridham and P. L. Jensen, Commercial Wireless & Development Co.* — U. S. patents 1 830 401 and 1 830 402. *B. F. Miessner, Miessner Inventions Inc.* — U. S. patent 1 833 762. *L. W. Thompson, General Electric Co.*

capacity and inductance, to reduce the mains hum so far as to be hardly disturbing, but, as explained below, this is not under all circumstances enough to secure the quality of reproduction nowadays demanded. For further reduction of hum to the required extent by this method, the cost of the smoothing devices would be many times greater.

Compensation methods, on the other hand, frequently give a very high degree of hum elimination in a relatively simple and inexpensive way, but their use is generally restricted by the requirement that the disturbing ripple voltage must be small in comparison with the corresponding D. C. voltage, as otherwise the power-handling capacity of the receiver for signal voltage would be reduced. This condition is satisfied *a priori* by generator-fed D. C. mains, but in all other cases some preliminary smoothing will be more or less necessary.

In mains-operated receivers of older design it was generally considered sufficient if the mains hum was reduced far enough to be just unnoticeable in the reproduction of speech or music at normal loud-speaker strength. Since then, the demands for hum elimination have gradually risen, parallel with the improvement in the quality of reproduction, both in the receivers themselves and in the loud speakers, which are now usually of the electrodynamic type. The fact is that superimposed ripple, even if itself hardly audible as a tone, may nevertheless mar the quality of reproduction by making speech or music sound harsh and rough[31]. The effect will be similar when for some reason the sound is modulated by a disturbing tone. This phenomenon has not yet been adequately explained, but is probably caused by the generation of beats of very low pitch between the disturbing tone or tones and the partial tones of which the speech or music is composed. It seems to be the lower disturbing frequencies in particular that hazard the quality of reproduction, *i. e.* just that range of frequencies where effective smoothing requires large capacity and inductance values and therefore high costs. Here the compensation methods offer great advantages since, as we have seen above, they can easily be made practically independent of frequency.

[31] *W. Janowsky.* E. N. T. Vol. 6, p. 435, Nov. 1929.

The natural place of the compensation methods is thus as a supplement to the filtration method, *i. e.* to achieve complete elimination of mains hum after preliminary smoothing by means of a relatively inexpensive filter. The consequent advantages of such a combination are so great, that the practical solution of the problem of hum elimination will undoubtedly be found along these lines. In the last few years this method has been widely adopted, so that now most modern receivers employ compensation in some form or other. But the possibilities of applying the compensation principle are as yet hardly exhausted.

SUMMARY.

To eliminate disturbing sounds, known as mains hum, produced in mains-operated receivers by voltage pulsations in the valve feeds, two methods are used, based on different principles, namely filtration (or smoothing) and compensation. In the former a filter circuit is interposed between the current supply and the valve circuits, which will pass direct current, but more or less effectively suppress the disturbing alternating current. The latter method, which is discussed in the present paper, is based on the general principle of introducing artifically, in addition to the original disturbing voltage, another one of opposite direction and so adjusted as exactly to neutralize the effect of the former. This may be realized in a great variety of ways, amongst which the following groups and sub-groups are distinguished in the paper.

A. Smoothing the feed current by compensation methods.
   1. Impedance circuits.
   2. Separate smoothing valve.

B. Compensation methods for single valve stages.
   1. Balanced circuits.
   2. Grid-anode compensation.
   3. Bridge methods for compensation.

C. Joint compensation of several valve stages.
   1. Special compensation electrode.
   2. Compensation by cathode displacement.

D. Compensation methods of particular kinds.

The properties of the various methods are described, and their advantages and disadvantages discussed. The paper deals primarily with compensation of ripple in the grid and anode volt-

ages, but the effect of variations in the filament current is also considered. The possibility of neutralizing intervalve reaction in common feed circuits by means of compensation methods is pointed out. In contrast with smoothing, compensation can easily be made independent of frequency, a great advantage, especially where low frequencies are concerned. In general, compensation methods are less expensive than equivalent filtration. Their use, however, is confined to relatively feeble voltage pulsations. It is therefore usually preferable to combine compensation with some preliminary smoothing. In this way a high degree of hum elimination can be achieved by very simple means. The importance of this to the quality of reproduction is emphasized.

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